# The whetstone of witte,

## whiche is the seconde parte of Arithmetike: containyng thextraction of Rootes: The *Cossike* practise, with the rule of *Equation*: and the woorkes of *Surde Nombers*.

*Though many stones doe beare greate price,*
*The* whetstone *is for exercise*
*As needefull, and in woorke as straunge:*
*Dulle thinges and harde it will so chaunge,*
*And make them sharpe, to right good vse:*
*All artesmen knowe, thei can not chuse,*
*But vse his helpe: yet as men see,*
*Noe sharpenesse semeth in it to bee.*
*The* grounde of artes *did brede this stone:*
*His vse is greate, and moare then one.*
*Here if you list your wittes to whette,*
*Moche sharpenesse therby shall you gette.*
*Dulle wittes hereby doe greately mende,*
*Sharpe wittes are fined to their fulle ende.*
*Now proue, and praise, as you doe finde,*
*And to your self be not vnkinde.*

These Bookes are to bee solde, at the Weste doore of Poules, by Thon Kyngstone.

# DEDICATION.

# ¶ To the right worshipfull, the gouerners, Consulles, and the reste of the companie of venturers into Moscouia, Robert Recorde Philitian, wissheth healthe with continualle increase of commoditie, by their worthie and famous trauell.



Wil not, nother ought I so euilly to iudge of my countrie, that learnyng here can haue no libertie: but by aide of frendeshippe, or strength of power. For as Englande did neuer wante learned wittes, so at this tyme I doubt not, but there be a great multitude, that desirously embrace all kindes of knowledge, and frendely are affected toward the furtherance of it. And therfore I dare saie, thei can not malice me, whiche am so willyng to helpe the ignorannte, accordyng to my gifte and simple talẽte. Wherby also this moche praise I maie iustly craue, to haue the commendation and rewarde of a solliciter in this cause. For though my trauell can not moche profite them, that be well learned, yet doeth it excite the beste learned, to remember their duetie to their countrie: and to be a shamed, that thei hauyng so greate habilitie, shall be founde moare slacke to aide their coũtrie, then he that hath smaller knowledge,

and lesse occasion otherwaies. Accordyng as men haue receiued, so are thei bounde to yeld. These excellente giftes are not lente vnto mē, to be hidden. And there are a great multitude that thurst, and long moche for soche aide. For bothe these causes I saie, that naturalle bōde to our countrie doeth chalenge it: and for that the honeste desires of so many good natures so moche requireth it, I erhorte them that be beste hable, to take from me this chargeable woorke, and to further their countrie men, as equitie would. And in the meane season while I see them so slacke, let them not bee offended with me, for preuentyng them. For better it is that a simple Coke doe prepare thy brekefast, then that thou shouldest goe a hungered to bedde. yea better it is to haue some grosse repaste, then to sterue for honger. And the common sorte will finde smalle faulte of wante, as long as thei see any man serue their expectation. So that for this cause also, that my paines for a time, doe excuse other finer wittes, thei ought to render me some thankes again. But if thei staie for feare of tauntes, and barkyng of curres, their corage is smalle. If thei mis doubte the gratefull acceptation of their studies, thei doe iniurie to their countrie. For whoe cā doubt but so ciuile a coūtrie, will thankefully receiue, and moste frendly recompense the trauelle, of soche as studie for their

benifite,

benifite, and serueth their necessarie commo-
dities. This perswasion maketh me so bolde,
that I can not thinke it neadefull, to seke any
protector, for this or any like woorke. Sith
euery good man will offer hymself, to defende
that, wherby his natiue countrie is benifited.
Excepte at some tyme, by excitation of the fu-
ries, some naughtie natures doe practice their
fraude, to berefte the realme of some singu-
lare commoditie. But as I feare no soche, so
at this tyme I seke no soche aide against thē.
yet for testefiyng of frendeshippe, and grate-
fulle remembraunce, I could doe noe lesse, but
sende this Booke to soche as I thought, not
onely to deserue it, but also would gladly re-
ceiue it. And if I maie perceiue, that you doe
accepte it (as I doubte not) with as good a
wille, as I dooe sende it, I will for your plea-
sure, to your coumforte, and for your commo-
ditie, shortly set forthe soche a booke of Naui-
gation, as I dare saie, shall partly satisfie and
contente, not onely your expectation, but also
the desire of a greate nomber beside. Where-
in I will not forgett specially to touche, bothe
the olde attempte for the Northlie Nauigati-
ons, and the later good aduenture, with the
fortunate successe in discoueryng that voiage,
whiche noe men before you durste attempte,
sith the tyme of kyng Alurede his reigne. I
meane by the space of. 700. yere. Nother euer

any before that tyme, had passed that voiage, excepte onely Ohthere, that dwelte in Halgolande: whoe reported that iorney to the noble kyng Alurede: As it doeth yet remaine in aunciente recorde of the olde Saxon tongue. So that if you continue with corage, as you haue well begon, you shall not onely winne greate riches to your selues, and bryng wonderfull commodities to your coūtrie. But you shall purchase therewith immortall fame, and be praised for euer, as reason would: for openyng that passage, that shall profite so many. In that Boke also I will shewe certain meanes, how without greate difficultie, you maie saile to the Northe Easte Indies. And so to Camul, Chinchital, and Balor, whiche bee coūtries of greate commodities. As for Chatai lieth so farre within the lande, toward the Southe Indian seas, that the iorneie is not to be attempted, vntill you be better acquainted with these countries, that you must first arriue at. But these thynges come in this place vntimely. I praie you accepte frendely in the meane season this Booke, whiche will bee a greate aide to the well vnderstandyng of the reste that is behinde. And as I shall vnderstande your desire, so will I haste the other. God prospere well your endeuoure, and sende you suche good successe, as so worthie aduenture doeth deserue: Whiche I doubte not will insue,

insue, if cankered malice of some spitefull stomackes doe not preuaile, as thei can not cease to practice, to hinder your commoditie, and deface your trauel. But as it is euer seen, and therfore commonly knowen, that enuie doeth still repine at glorie, so ought all honeste hartes, to prosecute their good attemptes, and contempne the ballynge of dogged curres. So fare you well. And loue hym againe, that delighteth and studieth to farther your cõmoditie.

At London the .xii. daie of Nouember. 1557.

# Dedication.

# THE PREFACE
## to the gentle Reader.

ALthough nomber be infinite in increasyng: so that there is not in all the worlde, any thing that can excede the quantitie of it: Nother the grasse on the ground, nother the droppes of water in the sea, no not the small graines of Sande throughe the whole masse of the yearth: yet maie it seme by good reason, that noe man is so experte in *Arithmetike*, that can nōber the commodities of it. Wherefore I maie truely saie, that if any imperfection bee in nomber, it is bicause that nomber, can scarsely nomber, the commodities of it self. For the moare that any experte man, doeth weigh in his mynde the benifites of it, the more of them shall he see to remain behinde. And so shall he well perceiue, that as nomber is infinite, so are the commodities of it as infinite. And if any thyng doe or maie exceade the whole worlde, it is nomber, whiche so farre surmounteth the measure of the worlde, that if there were infinite worldes, it would at the full cōprehend them all. This nomber also hath other prerogatiues, aboue all naturalle thynges, for neither is there certaintie in any thyng without it, nother good agremente where it wanteth. Whereof no man can doubte, that hath been accustomed in the Bookes of Plato, Aristotell, and other aunciente Philosophers, where he shall see, how thei searche all secrete knowledge and hid misteries, by the aide of nomber. For not onely the constitution of the whole worlde, dooe thei referre to nomber, but also the composition of

*The excellencie of nomber.*

manne, yea and the verie substaunce of the soule. Of whiche thei professe to knowe no moare, then thei cā by the benifite of nomber attaine. Furthermoze, foz knowledge and certaintie in any other thynge, that mannes witte can reche vnto, there is noe possibilitie without nomber. It is confessed emongeste all men, that knowe what learnyng meaneth, that beside the Mathematicalle artes, there is noe infallible knowledge, excepte it bee bozowed of them. And emongeste them, it is sufficiently knowen, and well declared by *Nicomachus*, and diuerse other writers, that *Arithmetike* is the fountaine of all the other, and their ground and bonde, as he calleth it. If any man will saie, that Diuinitie, Lawe, and Physike, maie be had without it: oz that thei take litle aide therby. Although I haue befoze this tyme aunswered thereto, yet now I saie again: that in Diuinitie there are greate hidde secretes in nombers. So that diuerse excellente Diuines, haue written whole Bookes of the misteries of nombers. And some of their Bookes intituled: *The Diuinitie of Nombers.* But what Christen manne is ignoraunte, that betwene *Trinitie* and *vnitie*, doeth consiste the full grounde of al Diuinitie: Wherefoze I neade not to allege the other hollie and sacred Nombers. Saue that. 7. will not permitte me to passe it with silence. In whiche is contained, not onely the secretes of the creation of all thynges: and the consummation of the whole woalde againe, with the state of eternitie: But also by it is the Sabbothes reste, and therby the full life and conuersation of godlie persones, repzesented and insinuate. In Lawe twoe kyndes of Justice are the somme of the studie: *Iustice Distributiue*, and *Iustice Commutatiue*, whiche termes I vse, as beste knowen in that arte: But what is any of the bothe without Nomber: I haue said in an other place (as I learned of that noble Philosopher *Aristotell*)

*Diuinitie.*

*Lawe.*

that

that if the knowledge and distinction, of *Geometricalle* and *Arithmeticall* proportion bee not well obserued, there can noe Justice well bee executed. And how often the ministers of the Lawe vse aide of Nomber. I neade not repete, bicause none but madde men doubt of it. And as for Physike, without knowledge and aide of nomber is nothynge. Wee see that nature in generation, bothe of manne and beastes, yea and of al thynges els doeth obserue nomber exactly. As well in the tyme of formation, as in the monethes of quickenyng, and of birthe. The misteries of the seuenth and nineth monethes are sufficiente testimonies therein. Beside that from the fourthe monethe til the seuenth many thynges bee permitted, that els bee not conueniente. For the vse of the pulse, and for critticalle dayes, beside the proportion in degrees in simple medicines, and mixture of compounde medicines, and other infinite maters, what nomber can doe, and what aide it giueth, onely the ignoraunte doe doubte. *Physike.*

But where can there bee any better testimonie for Nomber, then that the celestialle bodies doe kepe an vnfallible nomber, in all their wonderfullie motions: By meanes whereof, mannes witte is habled to attaine the knowledge of them. As by the *Arithmeticalle* tables, of their motions it is easily knowen. Therefore and for that we see the yere, and all the distinction of tymes, beside the common vse of trafike betweene menne, to depende of nomber, wee muste neades not onely confesse it to bee, as it were the onely state of all natures woorkes, and of all ciuilitie: but we must also honoure and reuerence it, as often as wee duely remember the excellencie and benifite of it. Was not Nomber, thinke you, wonderfullie honoured, when noe name was thought moare meter for God, then the name of Nomber: I meane. 1. and. 3. the name of the Trinitie. But to come to moare familiare ma- *Astronomie.*

ters, I will saie, as Plato saieth in his Booke De summo bono. *Take awaie Arithmetike, with measure and weightes, from all other artes, and the reste that remaineth is but base, and of noe estimation.* Where although *Plato* doo name three thinges in appearaunce, that is Nomber, Measure, and Weighte. What are Measure and Weighte, but nomber applied to seueralle vses: For *Measure* is but the nombryng of the partes of lengthe, bredthe, or depthe. And so *weighte* (as here it is taken) is the nomberyng of the heuinesse of any thyng. So that if nomber were withdrawen, no manne could either measure, or weigh any quantitie. And therfore it must followe: that nomber onely maketh all artes perfecte, and worthie estimation: seyng that without it, all artes are but base, and without commendation. This maie suffice for the iuste cōmendation of *Arithmetike*. But yet one commoditie moare, whiche all menne that studie that arte, doe fele, I can not omitte. That is the filyng, sharpenyng, and quickenyng of the witte, that by practice of *Arithmetike* doeth insue. It teacheth menne and accustometh them, so certainly to remember thynges paste: So circumspectly to consider thynges presente: And so prouidently to forsee thynges that followe: that it maie truelie bee called the *File of witte*. Yea it maie aptly bee named the *Scholehouse of reason*. The like iudgemente had *Plato* of it, as appeareth by his woordes in the seuenth booke De republica. Where he saieth thus: *Thei that be apte of nature to Arithmetike, bee readie and quicke to attaine all kindes of learnyng. And thei that bee dulle witted, and yet bee instructed and exercised in it, though thei gette nothyng els, yet this shall thei all obtain, that thei shall bee moare sharpe witted, then thei were before.* What a benifite that onely thyng is, to haue the witte whetted and sharpened, I neade not trauell to declare, sith all menne confesse it to bc as greate as maie be. Excepte any witlesse per-

*Measure.*
*weighte.*

sone

sone thinke he maie bee to wise. But he that moste scareth that, is leaste in daunger of it. Wherefore to conclude, I see moare menne to acknowledge the benifite of nomber, then I can espie willyng to studie, to attaine the benifites of it. Many praise it, but fewe dooe greately practise it: onelesse it bee for the vulgare practice, concernyng Merchaundes trade. Wherein the desire and hope of gain, maketh many willyng to sustaine some trauell. For aide of whom, I did sette forth the firste parte of *Arithmetike*. But if thei knewe how farre this seconde parte, dooeth excell the firste parte, thei would not accoumpte any tyme losse, that were imploied in it. Yea thei would not thinke any tyme well bestowed, till thei had gotten soche habilitie by it, that it might be their aide in al other studies. And if *Plato* doe require *Arithmetike*, as a specialle and a necessarie qualitie in hym, whom he would admitte as a citezein in his politike toune: How maie wee thinke of our selues, that desire to gouerne other, and yet can scante skille of common nombers? So farre are many, yea moste parte of vs from cunnyng in nombers. *Plato* thinketh not manne hable to bee a good capitaine, excepte he bee skilfulle in this arte: And wee accoumpte it noe parte of those qualities, that bee required in any soche manne. Howbeit for the better trialle thereof, I haue in this Booke framed some of the questions in soche sorte, as thei maie approue the vse of this arte, not onely good for capitaines, but also moste necessarie for theim. So that without it, thei can not Marshall their battaile, nother vewe their enemies campe or forte. And if I shall saie as I thinke, without it a capitaine is noe capitaine. In this booke what I haue written, for the aide of all menne, and namely soche of my countrie menne, that vnderstand nothyng but Englishe, I neade not to repete particularely, but remitte them to the booke it self, to see it at

 large

large. Onely this maie I saie: that as I haue doen in other artes, so in this I am the first venturer, in these darke maters. Wherfore I trust thei that be learned, and happen to reade this worke, wil beare the moare with me, if thei finde any thyng, that thei doe mistike: Wherein if thei will vse this curtesie, either by writynge to admonishe me thereof, either theim selfes to sette forthe a moare perfecter worke, I will thynke them praise worthie. But if any manne will be so hastie, other to blame that, whiche he is not hable to amende, or to condempne that, whiche he did neuer vnderstande: As some ofte tymes doe of a fonde curiositie, I will wisshe hym a better witte, and moare modestie. And to preuente all soche seuere Judges, I thought it good to admonisshe you before, that by occasion of trouble vpon trouble, I was hindered from accomplishyng this worke, as I did intende. But yet is here moare, then any manne might well looke for at my handes, if thei did knowe and consider myne estate. And this moche moare I saie: that if I maie perceiue, that this Booke bee as well receiued, as the firste parte was, I will striue moche, to stele from my troubles so moche tyme, as to set out the reste of this arte, moare completely in Englishe, then euer I sawe it in any toungue, hetherto doen: trust thereto assuredly. And wisshe hym good, that traueleth for thy benifite.

One thyng is nothyng, the prouerbe is,
Whiche in ſome caſes doeth not miſſe.
Yet here by Woorkyng with one thyng,
Soche knowledge doeth from one roote ſpryng,
That one thyng maie with right good ſkille,
Compare with all thyng: And you will
The practice learne, you ſhall ſone ſee,
What thynges by one thyng knowen maie bee.

To the curiouſe ſcanner.

If you ought finde, as ſome men maie,
That you can mende, I ſhall you praie,
To take ſome paine: ſo grace maie ſende,
This worke to growe to perfecte ende.
But if you mende not that you blame,
I winne the praiſe, and you the ſhame.
Therfore be wiſe, and learne before,
Sith ſlaunder hurtes it ſelf moſte ſore.

# ¶ The seconde parte of Arithmetike,

*containyng the extraction of Rootes in diuerse kindes, with the Arte of Cossike nombers, and of Surdenombers also, in sondrie sortes.*

¶ *The interlocutors, Master. Scholar.*

The Master.

I See your desire can not bee satisfied, neither your requeste stated, vntill I maie iustly aunswere you, that I can teache you no more: whiche aunswere maie staie your requeste, although it content not your desire.

Scholar. I beseche God of his mercie, to withstande all suche occasion: except it maie be more to your owne contentation and profite, then it would be pleasaunt to the louers of learning.

Master. Yet a iuste excuse maie stande for my declaration: As if ignorannce doe inforce me to staie my trauell.

Scholar. Your owne ignorañce, I truste, you will not allege: and as for the ignorañce of other, it ought to bee no staie: sith the ignorannte multitude doeth, but as it was euer wonte, enuie that knowleedge, whiche thei can not attaine, and wishe all men ignoraunt, like vnto themself, but all gentle natures, contemneth suche malice: and despiseth theim as blinde wormes, whom nature doeth plague, to staie the poisone of their venemous stynge.

Master. We shall not nede to stande on this talke, but trauell with knowledge to vanquishe ignorañce: And beleue that the *pricke* of knoweledge, is more of force then the stynge of ignorannce: yea, the *poincte* in

Geometrie, and the vnitie in Arithmetike, though bothe be vndiuisible, doe make greater woorkes, & increase greater multitudes, then the brutishe bande of ignoraunce is hable to withstande.

*Vnitie.* Scholar. Our talke groweth well to our mater. I beseke you therfore, with that vnitie beginne, and builde on it your woorke, as a forte against ignorance

Master. Unitie is of it self vndiuisible, and yet is it in al partes of the woorlde, and in euery thing. Yea, the woorlde it self consisteth of vnitie, is named of vnitie, was made by vnitie, and is preserued by vnitie, and onely ignoraunce with her broode secluded from vnitie, so that of it to repete the fulle force, would occupie muche time, and make greate volumes.

Scholar. Sith vnitie is so mightie, and of suche force (as you saie) what maie be thought then of nom-

*Nomber.* ber, whiche containeth a multitude of vnities? And is nothyng els but a collection of vnities.

Master. Unitie is the fountaine and originalle of nomber, yea vnitie by addition onely shall make a greater nomber, then any nombers can doe by multiplicacion. But this is marueilouse, that no nomber repineth against diuisiō, till it come to an vnitie: and then will it permit no farther diuisiō. And therfore it is said, that vnitie doeth neither multiplie nor diuide.

And as al nombers maie be more or lesse, so the les-

*A parte.* ser is euer a *parte* or *partes* of the greater.

*Partes.* As 5 vnto 10 is a parte, named a halfe: but vnto 7. 5. is not a parte, but partes, and is called $\frac{1}{2}$. So 8 to. 24. is a parte that is $\frac{1}{3}$: but vnto. 36. it is partes, that is $\frac{2}{3}$.

Scholar. I perceiue, you call it a parte, when the numerator in the fraction (reduced to the smallest) is an vnitie. And when the numerator is a nomber, then that fractiō betokeneth partes of a nomber.

But I praie you, what varieties of nombers bee there principally to be considered in this arte?

Master.

Maister. Nomber is diuided into diuerse kindes, for some are *whole nombers*, and thei onely of *Euclide*, *Boetius*, and other good writers are called nombers. Other are *broken nombers*, and are commonly called *fractions*. Of these bothe I haue written in the firste and seconde partes of *Arithmetike*: So that I mighte seme to curiouse, to repete any parte of it again. *The firste diuision of nombers.*

But now in eche kinde of these, there are certaine nombers named *Abstracte*: and other called nombers *Contracte*. *The seconde diuision of nombers.*

*Abstracte* nombers are those, whiche haue no denomination annexed vnto them. And those that haue any denomination ioyned to theim, are called *Contracte* nombers. *Abstracte.* *Contracte.*

Scholar. This I see to be a reasonable distinctiō, and agreable to the signification of the names. For as that nomber is cōtracte, from his generall libertie of signification, whiche is bounde to one denomination, as in saiyng. 10. grotes (where. 10. is restrained frō the libertie of valowyng any other thing but grotes) so if it had no denomination adioined, it might then signifie the nomber of daies, or of miles, or any like thyng, as well as of grotes. For when I saie. 10. and doe not limitte any denominatiō, then is that. 10. abstracte and severed frō all specialties, and standeth free to any name of thing.

But this (me thinketh vnder your correction) can not extend to broken nombers: whiche euermore carry with them their denomination: seyng thei consiste of a numerator and a denominator. *whether broken nombers be contracte, or not.*

Maister. You seme to saie well. And the like iudgemēte doeth appere to be in some writers of this arte. But yet seyng that fractions maie haue all other artificiall denominations, that whole nombers maie receiue: and maie also bee without theim: therefore must wee either make a more curiouse distinction of

that name of denomination: or els wee must seclude fractions, frō the necessitie of that name: or els thirdly, to auoied contention, cal them nombers contracte improperly.

Scholar. I assente thereto as reason would.

*Why fractiōs be not called nombers properly.*

Yet one thyng more I must demaunde of you, why *Euclide*, and the other learned men, refuse to accompte fractions emongest nombers.

Maister. Bicause all nombers doe consiste of a multitude of vnities: and euery proper fraction is lesse then an vnitie, and therefore can not fractions exactly be called nombers: but maie bee called rather fractions of nombers.

Scholar. In deede now that I doe waie the matter more exactly, it appereth that a fractiō is not properly a nomber, but a connexion and conference of nombers, declaryng the partes of an vnitie. For the numerator doeth signifie one nōber, and the denominator an other: The denominator declaryng into how many partes the vnitie is diuided, and the numerator signifiyng that of those partes, not all, but so many onely are to be takē, as the numerator importeth.

*The diuision of nombers Abstracte.*

Maister. Well, then to procede, nombers abstracte are considered in. 3. principall varieties: That is, first without comparison to any other nomber or figure. And that nomber maie well be called *nomber absolute*.

*Nombers Absolute.*

Secondarily, some nombers bee vsed onely in relation to other, andtherfore ought to bee called *nombers relatiue*.

*Nombers Relatiue.*

Thirdly, many nombers are referred to some figure, that maie rise by multiplicacion of their partes together, and that diuersly. And those nombers therfore maie bee called *figuralle nombers*.

*Nombers Figuralle.*

Scholar. If I conceiue your wordes rightly, this is your meanynge: that when I saie. 10. 25. 100. or 200. &c. these nombers stand absolute from all denominacion

mination, and clere from all relatiō and comparison.

But when I saie. 6. is halfe of. 12. or. 15. is triple to 5. here the numbers beeyng compared together, are aptly called *nombers relatiue*: So if I saie, that. 16. is a *square nomber*, bicause it is made of. 4. multiplied by. 4 then is. 16. here to be called a *figuralle nomber*.

Master. You take it well. Therfore will I briefly touche the membres of euery kinde.

First of absolute nombres, some are *euen nombers*, and some are *odde*.

Scholar. All men knowe that. And farther, that *euen nombers* are those, whiche maie be diuided into equalle halfes: and so can not *odde nombers*, without a fraction. *Nombers, euen, & odde.*

Master. Of this plaine easie thyng, marke what foloweth: a greater doubte dissolued. For if an odde nomber (as. 7. for example) can not bee parted into. 2. equalle nombers, eche beeyng halfe of. 7. then. 3. $\frac{1}{2}$. whiche is commonly called the halfe of. 7. is no nōber

Scholar. It can not be denied. And so (I see now) no fraction can bee a nomber. This greate doubte is plainly dissolued, by a very certaine and moste knowen principle.

Master. Now farther. Of bothe these kindes of nombers, some bee *compounde*, and some bee *simple* and *vncompounde*. *Compounde* nombers are made by multiplication of. 2. nombres together, and not by additiō, though the name might seme to serue to bothe. *Nombers cō. pounde, and simple.*

Scholar. So I perceiue, that 5. is no *compounde* nōber, although it bee made by addition of. 2. and. 3. but 6. whiche is made by multiplication of. 2. and. 3.

Likewaies. 9. is *compounde*, bicause that. 3. multiplied by. 3. doeth make. 9.

And. 15. also is *compounde* by multiplyng. 5. and. 3. together.

And hereby I se that. 1. is not to be called a nomber *One is no nomber.*

for then all nōbers aboue it, must nedes be *compounde*, bicause thei consist all of vnities.

Master. But yet by multiplication of .1. no other nomber is *compounde*.

Scholar. By those wordes I am taught to knowe more, and speake better.

Master. *Euen nombers* are yet diuersly to be considered in their diuisions. For although the greate multitude of euen nombers bee *compounde*, yet .2. is accompted truely an euen nomber, originall, and *vncompoūde*. So that it maie make other nombers, & is made of no nōbers, but of vnities onely, as al odde nombers are.

*Twoo is vncompounde.*

All other euen nombers are *compounde*, and are diuersly diuided, for some are *euen nombers euenly*, and some are *euen nombers oddely*, and some are *euen nombers* bothe *euenly and oddely*. *Euen nombers euenly*, are suche nombers as maie bee parted continually into euen halfes, till you come to an vnitie. As for example. 32. first is diuided into. 16. as his euen halfe: and again, 16. into. 8. as his halfe: And. 8. againe by. 4. is parted into. 2. euen partes: Then. 4. also by. 2. And that. 2. is diuided into. 2. vnities, as his iuste halfes.

*Euen nombers, euenly.*

But *euen nombers vneuenly*, are suche nombers as maie bee diuided into. 2. equalle partes: whiche are odde nombers. As. 18. is diuided into. 9. and. 9. as his halfes, and thei are odde. So. 10. is diuided by. 5. And 30. by. 15. with a greate nomber more of suche sorte.

*Euē nombers vneuenly.*

*Nombers euen euenly and oddely*, bee commonly called suche nombers, as maie bee diuided into. 2. equalle and euen halues: but before you come to an vnitie, the halfes will be odde nombers. As. 60. maie be first parted into. 30. and. 30. whiche are euen: And thei againe diuided by. 15. whiche is odde.

*Euen nombers, euenly and oddely.*

Likewaies. 24. is diuided first by 12. And that 12. by 6. & lastly. 6. is diuided by. 3. whiche is an odde nōber.

So. 28. maie bee diuided into. 2. equalle and euen halues,

halues, that is into 14. And that. 14. into. 7. whiche is the halfe of. 14. but is an odde nomber.

Scholar. This I perceiue well. And, as I iudge, the distinction into those. 3. kindes, is not onely reasonable, but also nedefull. And yet you seme to speake doubtfully, of this laste membre. Bicause I remember not that you vse this worde commōly, but where you giue place rather to custome, then to reason.

Maister. Or els to custome of the common sorte of writers, rather then to the iudgemente of the moste aunciente writers.

And so in this case *Euclide* doeth not seme to admitte this thirde member. But accompteth it vnder the seconde kinde. As maie well appere in his. 9. boke, and 34. proposition, where he calleth suche a nomber, *euenly euen, and euenly odde* also, whiche place cōferred with the definitions in thesame booke, doeth approue in many wise mennes opinions, that *Euclide* minded but 2. onely kindes of those nōbers. And yet in this thing (I thinke) he did rather approue. 3. varieties by his propositiōs, then establishe onely. 2. sortes by his first definitions.

But herein I will spende no more tyme. But sale briefly that the distinctiō of. 3. kindes, serueth to good vse, and ease in teachyng.

And now for farther knowledge of nombers, some are called *nombers perfecte*, & some are *nombers imperfect*.

*Perfecte nombers* are suche ones, whose partes ioyned together, will make exactly the whole nomber. *Nombers perfecte.*

And therfore are. 6. and. 28. accompted perfecte nōbers: bicause the partes of eche of theim added together, doe make the ful and intere nomber, whose partes thei bee. As of. 6. the halfe is. 3. the thirde parte is 2. the sixte parte is. 1. As for a quarter, and fifte parte it hath not in whole nomber. Now put together. 1. 2. and. 3. and thei make iuste. 6. whose partes thei bee. 6.

And

And therfore is. 6. a perfecte nomber.

28. Likewaies. 28. hath for his halfe. 14. for his quarter. 7. for his seuenth parte. 4. and for his fowertenth parte. 2. and for his. 28. parte. 1. all whiche put together, that is. 1. 2. 4. 7. and. 14. doe make. 28. of this sort there are very fewe more in cõparisõ. And for an exãple, I sett here, as many as are vnder. 6000000000. and thei are these. 6. 28. 496. 8128. 130816. 2096128. 33550336. 536854528.

*Nombers imperfecte.* But now of the contrary kind, *imperfecte nombers* be suche, whose partes added together, doe make either more or lesse, then the whole nomber it self: whose partes thei bee.

*Nombers superfluouse.* And if the partes make more then the whole nomber, then is that nõber called *superfluouse*, or *abundaunt*. As 12. whose partes are. 1. 2. 3. 4. 6. whiche make 16.

So. 20. hath for his partes. 1. 2. 4. 5. 10. whiche make. 22. Likewaies. 120. hath these partes.

1. 2. 3. 4. 5. 6. 8. 10. 60. 40. 30. 24. 20. 15. 12. } whiche make 240.

*Nombers Diminute.* And if the partes make lesse then the whole nomber, whose partes thei be, then is that nomber called *Diminute*, or *Defective*. As. 8. hath these partes. 1. 2. 4. whiche make but. 7.

So. 16. hath these partes. 8. 4. 2. 1. and thei make onely. 15.

Likewaies. 32. whose partes are. 1. 2. 4. 8. 16. and make but. 31.

Scholar. In all these nombers I note that you reken one, for a parte of eche one of theim: whiche before I thought you had denied.

Master. 1. canne neither multiplie nor deuide, and therfore compoundeth no nomber. But one maie increase addition, and therefore where partes be added together, there. 1. maie well be called a parte.

And this shall suffice for the diuision of euen nombers

bers Abstracte.

Now to speake of *odde nombers*, some of thē are *compounde*, & some *vncompoūde*. Thei are *compounde*, whiche maie bee diuided into any other partes then vnities. As. 9. whiche is cōpounde of. 3. And. 15. that is made of. 5. and. 3. Also. 21. is compounde of. 7. and. 3. And so furthe. But. 3. 5. 7. 11. 13. 17. 19. 23. 29. and suche like, bee odde nombers *vncompounde*. For thei are not made of any other then of vnities. *Odde nōbers* *Compounde.* *Vncompoūde.*

Here must you vnderstande by *composition*, the multiplication of the partes of nombers together, as you remembre, before was declared.

Scholar. I consider it so. And I remembre all that you haue taught me, for the diuisiō of nōbers *abstracte* and absolute. What saie you now of nōbers *relatiue*? *Nombers Relatiue.*

Master. Some tymes their *relation* hath regarde to their partes, namely, whether these. 2. that bee so compared, haue any common parte, that will diuide theim bothe. For if thei haue so, then are thei called *nombers commensurable*: As. 12. and. 21. bee *nombers commensurable*: for. 3. will diuide eche of theim. *Commensurable.*

Likewaies. 20. and. 36. be *commensurable*, seyng 4. is a commō diuisor for theim bothe. But if thei haue no suche common diuisor, then are thei called *incommensurable*. As 18 and 25. For 25 can bee diuided by no nomber more then by. 5. And. 18. can not be diuided by it. *Incommensurable.*

In like maner. 36. and. 49. are *incommensurable*: For 49. hath no diuisor but. 7. And 7. can not diuide. 36.

Scholar. Doe you meane then, that *incommensurable nombers*, haue no cōparison nor *proportion* together?

Master. Naie, nothyng lesse. For any. 2. nombers maie haue comparison & *proportion* together, although thei be *incommensurable*. As. 3. and. 4. are *incommensurable*, and yet are thei in a *proportion* together: as shall appeare anon.

But first I will declare vnto you, the varieties of

*Proportion.* *proportion*, wherein there maie be double conferẽce: I meane of the lesser to the greater, or of the greater to the lesser.

Whẽ the greater is cõpared to the lesser, it is called a *Proportion of the greater inequalitie*. As 6 to 2. or 5 to 3.

*Of greater inequalitie.* And when the lesser is conferred to the greater, it is called a *proportion of the lesser inequalitie*. As. 3. to. 5. or. 2. to. 6.

*Of lesser inequalitie.*

Scholar. And what if I would cõpare two equalle nombers together?

Maister. That is accoumpted also a proportion of many men: and is called the *proportion of equalitie*. And then ought the first diuision of proportion to be, thus

Proportion of { Equalitie. / Inequalitie. { The greater. / The lesser.

So proportiõ of the greater inequalitie, is diuided into. 5. seuerall kindes: whereof. 3. be *simple*, and. 2. other *compounde*. The firste kinde is, when a greater nomber containeth the lesser diuerse times: as twise, or thrise, or oftener. So. 6. containeth. 3. twise: and it containeth. 2. thrise. This proportion is called generally, *multiplex*, that is to saie, many folde: but specially it is named, accordyng to the tymes that it conteineth the lesser. So that if it contein hym twise, then is it named *dupla*, or double. As 2 to 1 and 4 to. 2.

*Multiplex.*

And if it containe it thrise. As. 3. to. 1. and. 6. to. 2. it is called *tripla*, or triple.

If it containe it. 4. tymes, then is it *quadrupla*, or *quadruple*.

Of these and of diuerse other sortes in this kind also, here are the names briefly set doune with exãples.

*Dupla*

| | | | |
|---|---|---|---|
| *Dupla.* | 4.to.2:6.to.3:10. to. 5:18.to.9. | 2/1 | Double. |
| *Tripla.* | 6.to.2:9.to.3:12.to.4: 18. to.6. | 3/1 | Triple. |
| *Quadrupla.* | 4.to.1:8.to.2:12.to.3: 16. to.4. | 4/1 | Fower folde |
| *Quintupla.* | 5.to.1:10.to.2:15.to.3:20.to.4. | 5/1 | Fiuefolde. |
| *Sextupla.* | 6.to.1:12.to.2:18.to.3:24.to.4. | 6/1 | Sirefolde. |
| *Septupla.* | 7.to.1:14.to.2:21.to.3:28.to.4. | 7/1 | Seuenfolde. |
| *Octupla.* | 8.to.1:16.to.2:24.to.3:32.to.4. | 8/1 | Eightfolde. |
| *Noncupla.* | 9.to.1:18.to.2:27.to.3:36.to.4. | 9/1 | Ninefolde. |
| *Decupla.* | 10 to 1:20.to 2:30.to 3:40.to 4. | 10/1 | Tennefolde. |
| *Vndecupla.* | 11.to.1:22.to.2:33.to.3. | 11/1 | Aleuenfolde |
| *Duodecupla.* | 12.to.1:24.to.2:36.to.3. | 12/1 | Twelufold. |

And so infinitely.

Beside this there is an other kinde of proportion, when the greater containeth the lesser, more then ones, and not twise: and that maie bein 2 sortes. For if the greater containe the lesser, and any one parte of hym, that proportion is called *Superparticulare.* For example, take. 5. to. 4. Sith. 5. doeth containe. 4. and his quarter. Likewaies. 6. to. 5. is in thesame kinde of proportion: although, not of thesame speciall sorte. For 6. comprehendeth. 5. and his fifte parte. *Superparticulare.*

So that for a more speciall distinction, eche of these and many other, haue their seueral names, according to that parte, whiche thei doe containe. As if it containe the halfe more, it is named *Sesquialtera.* In whiche proportion are these nombers folowyng. *Sesquialtera*

3.to.2: 6.to.4: 9.to.6: 12.to.8: 15.to.10. | $1\frac{1}{2}$.

But if the greater comprehende the lesser, and his thirde parte, then is that named *Sesquitertia* proportion. As in these. *Sesquitertia.*

4.to.3: 8.to.6: 12.to.9: 16.to.12: 20.to.15. | $1\frac{1}{3}$.

And when the fifte, sixte, seuenth, or eight part doeth make the proportion, or any other part els, the name is taken of that same parte. As for briefnesse I will here sette examples.

| | | |
|---|---|---|
| *Sesquiquarta.* | 5. to. 4: 10. to. 8: 15. to. 12. | $1\frac{1}{4}$ A quarter moze. |
| *Sesquiquinta.* | 6. to. 5: 12. to. 10: 18. to. 15. | $1\frac{1}{5}$ a fifte moze. |
| *Sesquisexta.* | 7. to. 6: 14. to. 12. 21: to. 18. | $1\frac{1}{6}$ a sirte moze. |
| *Sesquiseptima.* | 8. to. 7: 16. to. 14: 24. to. 21. | $1\frac{1}{7}$ a seuenth moze. |
| *Sesquioctaua.* | 9. to. 8: 18. to. 16: 27. to. 24. | $1\frac{1}{8}$ an eight moze. |
| *Sesquinona.* | 10. to. 9: 20. to. 18: 30. to. 27. | $1\frac{1}{9}$ a nineth moze. |
| *Sesquidecima.* | 11. to. 10: 22. to. 20: 33. to. 30. | $1\frac{1}{10}$ a tenth moze. |
| *Sesquiundecima.* | 12. to. 11: 24. to. 22: 36. to. 33. | $1\frac{1}{11}$ a leuenth moze. |
| *Sesquiduodecima.* | 13. to. 12: 26. to. 24: 39. to. 36. | $1\frac{1}{12}$ a twelueth moze |

And so as farre as you liste to procede in suche proportion: where one parte of the lesser, is the iuste difference and excesse, betwene it and the greater.

But if the difference be. 2. partes. 3. partes, or moze partes: the proportiō is named *superpartiente*. As. 5. to 3. And. 10. to. 6. For as. 5. containeth. 3. and. $\frac{2}{3}$. of it: so 10. holdeth. 6. and. $\frac{2}{3}$ of it. *Superpartiēs*

Scholar. Now I perceiue some vse also, of the distinction betwene a parte and partes in nomber: Of whiche at the beginnyng you did speake. But how many kindes are there of this sorte?

Master. There are infinite kindes in this sorte of proporcion, as well as in the other. But for examples sake, I will set furthe some of the moste common nombers: that therby you maie gather the formes of the reste. And these be thei, with their names.

| | | | |
|---|---|---|---|
| *Superbipartiens.* | *Tertias.* | 5. to. 3: 10. to. 6: 15. to. 9: 20. to. 12. | $\frac{2}{3}$ |
| | *Quintas.* | 7 to 5: 14. to 10: 21. to 15: 28. to. 20 | $\frac{2}{5}$ |
| | *Septimas.* | 9 to 7: 18 to 14: 27. to 21: 36. to 28. | $\frac{2}{7}$ |
| | *Nonas.* | 11 to 9: 22 to 18: 33. to 27: 44. to 36 | $\frac{2}{9}$ |
| | *vndecimas.* | 13. to 11: 26 to 22: 39 to 33: 52 to 44 | $\frac{2}{11}$ |

| | | | |
|---|---|---|---|
| Suptripartiẽs | Quartas. | 7. to. 4: 14. to. 8: 21. to. 12: 28. to. 16. | 4/7 |
| | Quintas. | 8. to. 5: 16. to. 10: 24. to. 15: 32. to 20. | 5/8 |
| | Septimas. | 10 to 7: 20. to 14: 30 to 21: 40 to 28. | 7/10 |
| | Octauas. | 11. to. 8: 22. to. 16: 33. to. 24. | 8/11 |
| | Decimas. | 13. to. 10: 26. to. 20: 39. to. 30. | 10/13 |
| | Vndecimas. | 14. to. 11: 28. to. 22: 42. to. 33. | 11/14 |
| | Decimastertias. | 16. to. 13: 32. to. 26: 48. to. 39. | 13/16 |
| | Decimasquartas. | 17. to. 14: 34. to. 28: 51. to. 42. | 14/17 |
| Superquadru partiens. | Quintas. | 9. to. 5: 18. to. 10: 27. to. 15: 36. to. 20 | 5/9 |
| | Septimas. | 11 to 7: 22 to 14: 33. to. 21: 44. to 28. | 7/11 |
| | Nonas. | 13 to 9: 26. to. 18: 39. to. 27: 52. to 36. | 9/13 |
| | Vndecimas. | 15. to. 11: 30. to. 22: 45. to. 33. | 11/15 |
| | Decimastertias. | 17. to. 13: 34. to. 26: 51. to. 39. | 13/17 |
| | Decimasquintas. | 19. to. 15: 38. to. 30: 57. to. 45. | 15/19 |
| Superquintu partiens. | Sextas. | 11 to 6: 22 to 12: 33. to. 18: 44. to 24. | 6/11 |
| | Septimas. | 12. to. 7: 24. to. 14: 36. to. 21. | 7/12 |
| | Octauas. | 13. to. 8: 26. to. 16: 39. to. 24. | 8/13 |
| | Nonas. | 14. to. 9: 28. to. 18: 42. to. 27. | 9/14 |
| | Vndecimas. | 16. to. 11: 32. to. 22: 48. to. 33. | 11/16 |
| | Duodecimas. | 17. to. 12: 34. to. 24: 51. to. 36. | 12/17 |
| | Decimastertias. | 18. to. 13: 36. to. 26: 54. to. 39. | 13/18 |
| | Decimasquartas. | 19. to. 14: 38. to. 28: 57. to. 42. | 14/19 |
| | Decimassextas. | 21. to. 16: 42. to. 32: 63. to. 48. | 16/21 |
| Supersextu partiens. | Septimas. | 13. to. 7: 26. to. 14: 39. to. 21. | 7/13 |
| | Vndecimas. | 17. to. 11: 34. to. 22: 51. to. 33. | 11/17 |
| | Decimastertias. | 19. to. 13: 38. to. 26: 57. to. 39. | 13/19 |
| | Decimasseptimas | 23. to. 17: 46. to. 34: 69. to. 51. | 17/23 |
| | Decimasnonas. | 25. to. 19: 50. to. 38: 75. to. 57. | 19/25 |
| | Vicesimastertias. | 29. to. 23: 58. to. 46: 78. to. 69. | 23/29 |

Scholar. I vnderstande by these examples, somewhat of their reasons: but I perceiue, you doe not followe their naturalle order, without interruption, in

these of the laste kinde.

Maister. To thintente you maie the better vnderstande good ground in that omission, I wil set furthe here those omitted nombers: That you maie see how thei would expresse some other proportiõ here named And therfore thei doe seme rather to be omitted, then in deede so to be.

Marke theim well.

| | | | | |
|---|---|---|---|---|
| *Superbipartiens.* | *Secundas.* | 4. to. 2: | 8. to. 4. | $\frac{2}{1}$ |
| | *Quartas.* | 6. to. 4: | 12. to. 8. | $1\frac{1}{2}$ |
| | *Sextas.* | 8. to. 6: | 16. to. 12. | $1\frac{1}{3}$ |
| | *Octauas.* | 10. to. 8: | 20. to. 16. | $1\frac{1}{4}$ |
| | *Decimas.* | 12. to. 10: | 24. to. 20. | $1\frac{1}{5}$ |

Scholar. In deede here I see, the firste is double proporcion. The seconde *sesquialtera*, the thirde *sesquitertia*, the folwerth *sesquiquarta*, & the fifte *sesquiquinta*.

Maister. So marke these other.

| | | | | |
|---|---|---|---|---|
| *Supertripartiens* | *Secundas.* | 5. to. 2: | 10. to. 4. | $2\frac{1}{2}$ |
| | *Tertias.* | 6. to. 3: | 12. to. 6. | $\frac{2}{1}$ |
| | *Sextas.* | 9. to. 6: | 18. to. 12. | $1\frac{1}{2}$ |
| | *Nonas.* | 12. to. 9: | 24. to. 18. | $1\frac{1}{3}$ |
| | *Duodecimas.* | 15. to. 12: | 30. to. 24. | $1\frac{1}{4}$ |

Scholar. The firste of these I knowe not, but all the other are named before.

Maister. The firste is a compounde proportion (as anon I will declare) and is named *dupla sesquialtera*.

But now will I sette furthe all the other omitted names.

*Secundas.*

| | | | |
|---|---|---|---|
| superquadrupartiens. | Secundas. | 6. to. 2: 12. to. 4. | $\frac{3}{1}$ Tripla. |
| | Tertias. | 7. to. 3: 14. to. 6. | $2\frac{1}{3}$ Dupla sesquitertia. |
| | Quartas. | 8. to. 4: 16. to. 8. | $\frac{2}{1}$ Dupla. |
| | Sextas. | 10. to. 6: 20. to. 12. | $1\frac{2}{3}$ supbipartiẽs tertias |
| | Octauas. | 12. to. 8: 24. to. 16. | $1\frac{1}{2}$ sesquialtera. |
| | Decimas. | 14. to 10: 28. to 20 | $1\frac{2}{5}$ supbipartiẽs quintas |
| | Duodecimas. | 16. to. 12: 32. to. 24 | $1\frac{1}{3}$ Sesquitertia. |
| | Decimas quartas. | 18. to. 14: 36. to. 28 | $1\frac{2}{7}$ supbipartiẽs septimas |
| | Decimas sextas. | 20. to 16: 40 to 32. | $1\frac{1}{4}$ Sesquiquarta. |

| | | | |
|---|---|---|---|
| superquinquipartiens. | Secundas. | 7 to 2: 14 to 4. | $3\frac{1}{2}$ Tripla sesquialtera. |
| | Tertias. | 8 to 3: 16 to 6. | $2\frac{2}{3}$ Dupla superbipartiens tertias. |
| | Quartas. | 9 to 4: 18 to 8. | $2\frac{1}{4}$ Dupla sesquiquarta. |
| | Quintas. | 10 to 5: 20 to 10. | $\frac{2}{1}$ Dupla. |
| | Decimas. | 15 to 10: 30 to 20. | $1\frac{1}{2}$ Sesquialtera. |
| | Decimas quintas. | 20 to 15: 40 to 30. | $1\frac{1}{3}$ Sesquitertia. |

| | | | |
|---|---|---|---|
| supersextupartiens. | Secundas. | 8 to 2: 16 to 4. | $\frac{4}{1}$ Quadrupla. |
| | Tertias. | 9 to 3: 18 to 6. | $\frac{3}{1}$ Tripla. |
| | Quartas. | 10 to 4: 20 to 8. | $2\frac{1}{2}$ Dupla sesquialtera. |
| | Quintas. | 11 to 5: 22 to 10. | $2\frac{1}{5}$ dupla sesquiquinta. |
| | Sextas. | 12 to 6: 24 to 12. | $\frac{2}{1}$ Dupla. |
| | Octauas. | 14 to 8: 28 to 16. | $1\frac{3}{4}$ supertripartiens quartas. |
| | Nonas. | 15 to 9: 30 to 18. | $1\frac{2}{3}$ superbipartiens tertias. |
| | Decimas. | 16 to 10: 32. to 20. | $1\frac{3}{5}$ supertripartiens quintas. |
| | Duodecimas. | 18 to 12: 36 to 24. | $1\frac{1}{2}$ sesquialtera. |
| | Decimas quartas. | 20 to 14: 40 to 28 | $1\frac{3}{7}$ supertripartiens septimas. |
| | Decimas quintas. | 21 to 15: 42 to 30. | $1\frac{2}{5}$ superbipartiens quintas. |
| | Decimas sextas. | 22 to 16: 44 to 32. | $1\frac{3}{8}$ supertripartiens octauas. |
| | Decimas octauas. | 24 to 18: 48 to 36. | $1\frac{1}{3}$ sesquitertia. |
| | Vicesimas. | 26 to 20: 52 to 40. | $1\frac{3}{10}$ supertripartiens decimas. |
| | vicesimas secũdas. | 28 to 22: 56 to 44. | $1\frac{3}{11}$ supertripartiens vndecimas. |

Scholar. I see well that these proportions, bee agreable with some other name: and therfore might seme superfluouse in this place.

Master.

Maſter. Not onely ſuperfluouſely, but alſo falſely ſhould thei bee placed here: ſeynge thei doe belong to other places of right.

Scholar. Why doe you not name theim all by Engliſhe names?

Maſter. Bicauſe there are no ſoche names in the Engliſhe tongue. And if I ſhould giue theim newe names, many would make a quarrelle againſt me, for obſcuryng the olde Arte with newe names: as ſome in other caſes all redy haue doen.

Scholar. Yet I praie you declare thoſe doubtfull names of compounde proportions.

Maſter. As there is one kinde of proportion, that is named *multiplex*, or manyfolde, whiche doeth containe the leſſer diuerſe times exactly. And two other whiche doe containe the leſſer ones, and ſome parte or partes of theſame: So thoſe kindes maie be compounded together. As when the greater number containeth the leſſer, twiſe, or thriſe, or oftener: and yet more ouer ſome parte or partes of theſame. So. 8. containeth 3 twiſe, and his $\frac{2}{3}$. And 10 comprehendeth 3. thriſe and his $\frac{1}{3}$.

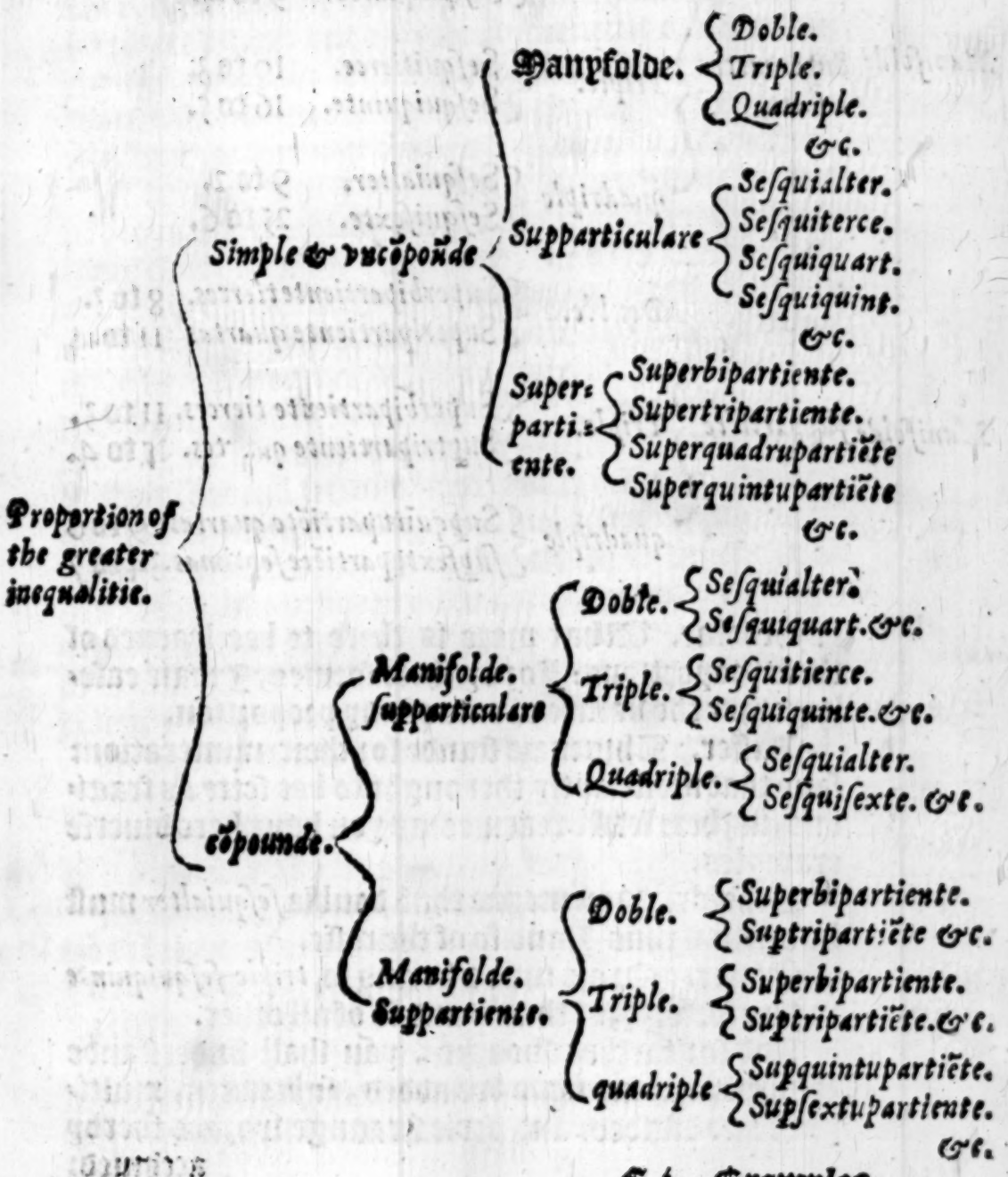
The firſte example is generally called *multiplex ſuperpartiens*: bicauſe the greater containeth the leſſer certaine tymes, and ſome partes of it beſides. But more ſpecially it is called *dupla ſuperbipartiens tertias*, that is, double with $\frac{2}{3}$ more.

The ſeconde example is generally referred to *multiplex ſuperparticularis*: bicauſe in it the greater comprehendeth the leſſer oftentymes, (as here thriſe) and his $\frac{1}{3}$ more. And therfore ſpecially it is called *tripla ſeſquitertia*.

But as I doe intende briefly to ouer runne this parte: ſo will I by tables ſet forthe the kindes of thē with their examples.

The

## The table of proportion of the greater inequalitie.



## Examples of eche compounde kinde, mentioned in the former table.

| | | | | |
|---|---|---|---|---|
| Manifolde | Superparticulare. | Double. | Sesquialter. | 5 to 2. |
| | | | Sesquiquarte. | 9 to 4. |
| | | Triple. | Sesquitierce. | 10 to 3. |
| | | | Sesquiquinte. | 16 to 5. |
| | | quadriple | Sesquialter. | 9 to 2. |
| | | | Sesquisexte. | 25 to 6. |
| Manifolde | Suppartiẽte | Double. | Superbipartiente tierces. | 8 to 3. |
| | | | Superipartiente quartas | 11 to 4 |
| | | Triple. | Superbipartiente tierces. | 11 to 3. |
| | | | Suptripartiente quartes. | 15 to 4. |
| | | quadriple | Supquītupartiẽte quartas | 29 to 6 |
| | | | Supsextupartiẽte septimas. | 34 to 7 |

Scholar. What more is there to bee learned of these proportions: For by these formes, I maie easely gather the valure or rate of any proportion.

Master, This maie stande for their numeration: saue that moste aptly thei ought to bee sette as fractions, in their leaste tearmes: as you haue here diuerse examples.

Scholar. You meane that double *sesquialter* must be written thus $\frac{5}{2}$, and so of the reste.

Master. Or els thus $2\frac{1}{2}$. and so *triple sesquiquinte* in this sorte: $3\frac{1}{5}$, or thus $\frac{16}{5}$ and so of all other.

And for farther woorke, you shall vnderstande that proportions maie bee added, subtracted, multiplied and diuided: and verie straunge woorkes therby

atchiued. For of the Arte of Proportios, dependeth all the subtilties, and fine workes, not onely of *Arithmetike*, but also of *Geometre*: besides farther mater that as now I will not touche. But as for the workes of *Proportions*, I will omitte them til an other tyme: considerynge not onely the troublesome condition, of my vnquiete estate: but also the conuenient order of teachynge, whereby it is required that the extraction of rootes, should go orderly before the arte of Proportions: whiche without those other, cā not be wrought.

Therfore will I now onely declare these kindes of proportion, whiche yet are not spoken of: to the intente that you maie haue here, the generall diuision of numbers, somewhat sufficiently touched.

As you see that betwene any two numbers, there maie be a conference of proportion: so if any one proportion be continued in more then .2. nombers, there maie be then a conference also of these proportions, in their seuerall termes: and that conference or comparison is named *Analogie*: whiche some delighte to call proportionalitie: As in this example, where 3 nōbers beare like proportion in their progression: 4. 6. 9. You see that 6. to 4. is in proportion *sesquialter*: and so is 9. to 6. and therfore is there a like proportion betwene the .2. laste, as there is betwene the .2. firste. *Analogie.*

Scholar. This I consider well by progression in *Arithmetike*.

Master. Likewaies where fower termes or more be set in order of proportion, as here 2. 6. 18. 54.

Scholar. I perceiue this wel: for here the proportiō is triple. But what saie you to this forme of comparison in Proportion? As 6. is to .2: So .30. is to .20. Is it not all one kinde of *Analogie*?

Master. It is one kinde of Analogie generalle, whiche maie be called *directe Analogie*: bicause the first is compared to him that doeth folowe nerte: & so eche other *Directe Analogie.*

other is still referred to that, that folo weth nexte. But this is the difference: that in the firste, there is a continuaunce of collation: and one terme is compared with twoo nombers: But in that forme of example, whiche you put, there is no nomber compared twise: For the first is referred to the seconde, and the second to the thirde. And so haue thei seueralle names to distincte theim a sonder.

*Continuall Proportion.*

Wherfore whē the first nomber is referred to the seconde, and that seconde to the thirde: the proportion is called *continualle*: and it maie consiste betwene 3. termes. As 5. 15. 45. doe procede in a continuall triple proportion. For as 5. is to 15: so is 15. to 45. as you doe see. But when I saie thus: as 5. is to 15. so 6. is to 18. Here is a triple proportion, but not continualle. For the seconde terme beyuge 15. is not compared with the thirde terme, that is 6. And therfore is it called a proportion *discontinualle*.

*Discontinual Proportion.*

Scholar. Now I perceiue certainly their distinction: For in twoo poinctes these examples doe agree, and differ in a thirde poincte.

Firste thei agree in that (as you saied) that the formoste is referred to the other that folo weth it nexte: And secondly, thei agree in this also, that bothe are compared in a triple proportion. But in this thei differ, that the seconde terme, doeth not beare like proportion to the thirde, as the thirde doeth to the fourth or the firste to the seconde.

Master. Farther more there is to bee noted, that in discontinualle proportion, there can bee no fewer then fower termes, or numbers: and so by euen formes still, as. 6. or. 8. and so forthe. Where as in continuall proportion, your termes maie bee of any number, euen or odde: aboue. 2.

And although I might saie more of the diuersities of proportion: as of *Proportion conuersed* or *indirecte*, *Proportion*,

*portion interchaunged, compounde Proportion, parted Proportion, reuerſed Proportion,* and *Proportion by equalitie.* Yet I thinke better to procede for this tyme, to the other kindes of nomber, and to reſerue the explication of proportions to their peculiare place.

Scholar. As you knowe the beſt order, ſo it ſhalbe mete that you doe vſe your owne iudgement therein.

## *Of figuralle nombers.*

Maſter.

THE nexte kinde of nombers are called *Figuralle nombers*: bicauſe thei doe, or maie repreſente ſome figure: And are euer conſidered in relation to thoſe formes.

Some of them haue a compariſon and relatiõ to length onely, and therefore are named *Linearie nombers*: whiche name, although it maie bee referred moſte aptly to ſuche numbers, as will make no other forme duely, yet maie it alſo be applied to any number abſtract. Sith all ſoche numbers maie be conſidered as the ſides of other figuralle numbers. *Nombers linearie.*

Secondly, numbers maie be conſidered, according to ſoche formes as thei make other in progreſſion, or in multiplication: And thoſe maie well be named *Superficiall nombers*, or *Flatte nombers*. Whereof there bee as many varieties, as there bee diuerſities of figures in *Geometrie*. As numbers *Triangulare*, *Quadrate*, *Cinkeangeled*, *Siſeangeled*: and ſo further. Alſo numbers *circulare*, *diametralle*, & *like flattes*, all whiche nombers haue bothe lengthe and breadthe: and thereof bee named *ſuperficiall nombers*. *Superficiall nombers.* *Flatte nombers.*

Besyde these there are other numbers, whiche are made of many multiplications, and thei are called *sounde nombers*: bicause that as by the firste multiplication, thei take lengthe and breadthe, like flatte numbers, so by the second multiplication, thei take depthe also: And thereof be thei named *bodily nombers*, or *sound nombers*.

*Sounde numbers.*

The leaste of them all is commonly called a *Cube*, or a *Cubike nomber*: And the other in their degrees seueralle named, as thei bee made by seueralle multiplications. For accordynge to the number of their multiplications, thei take their names. And all that haue like number of multiplications, are of one kinde, and bere one name: as well in flatte numbers, as in sounde.

But consideryng the infinite multitude of those figuralle numbers, I thinke beste to speake of theim onely in this place, whiche haue muche profitable vse in this arte. And, of those, emong infinite flatte numbers, I will take onely fower. That is to saie, *square nombers*, *longe squares*, *diametralle nombers*, and *like flattes*.

*Square nombers* are those, whiche maie be diuided by some one number, and haue thesame number for the quotiente: that is to saie, that a square nōber is made by the multiplication of any number into it self, as 10 multiplied by. 10. maketh. 100. That. 100. is a square number: whiche. 100. if I doe diuide by. 10. the quotiente will be. 10. also.

*Square nombers.*

Scholar. So, 4. multiplied by. 4. maketh. 16: and that must be a square number by like reason.

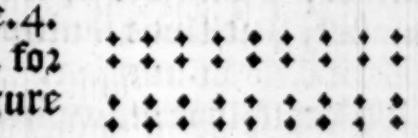
Master. So it is.

Scholar. And if I multiplie. 9. by. 4. is not that a square number? Seyng fower semeth to make all nōbers square by multiplication.

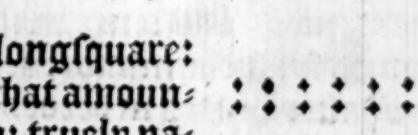
Master. Consider this well, that a square number doeth make suche a square in number, as a *iuste square* doeth make in *Geometrie*: That is suche a one whose

whose sides are equalle. For and if the one side be longer then the other, that figure in *Geometrie* is called a *long square*, and so it is named in number, a *long square* also.

Now if I sette doune the figure of your number, as you termed it, and sette. 4. for the one side, and. 9. for the other, this will the figure shewe.



Where you se a plain longsquare: yet is the whole number that amounteth of this multiplication: truely named a square number, as here you maie see. But then is the side or roote of it, neither. 4. nor. 9. but. 6.



Scholar. Now I vnderstande it: and the better by this figuralle example. And here also I haue learned what a *Roote* is: for you seme to expounde it, to bee the side of a figuralle number. *A roote.*

Master. Euery flatte nomber, and euery sounde number also haue their sides: But no flatte number, saue onely squares haue a roote: bicause a roote in flatte numbers, is a number multiplied by it self.

And in sounde numbers, thei onely haue rootes, whiche bee made by many multiplications, of some one nũber by it self: other by that, whiche riseth of it.

As when I saie, twoo tymes, twoo twise, maketh 8. that number is a sounde number: and is named a *Cube*. And so. 3. tymes. 3. thrise, doeth make. 27. whiche is also a *Cube*.

And generally, any number that is made by suche 2. multiplications, is called a *Cube*, or *Cubike* number. *A cube.* And the number of that multiplication, whiche commonly is named the multiplier, is in this poincte called the Cubike roote of that number. *A cubike roote.*

Wherfore, thus also maie you define a Cubike nũber, *A cubike nomber.*

ber: to bee suche a number, as beeyng diuided by his roote, shall haue for the quotiente the square of the same roote.

Scholar. Hereby I perceiue, that one multiplication, of any number by it selfe, doeth make a square number. And twoo multiplications in that sorte, doe make a Cubike number.

What if I doe multiplie any number thrise, or fower tymes, or oftener in that sorte, are there proper names for suche numbers?

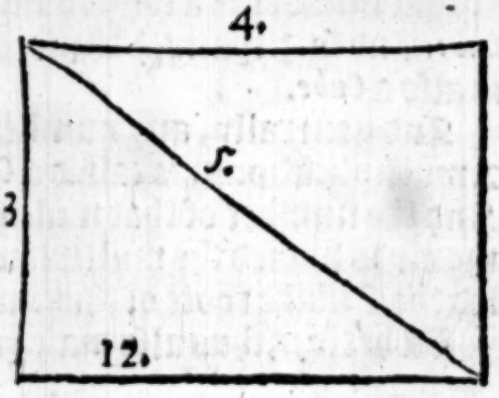
Maister. Yes in deede: as I will declare anon.

But firste before we attempte the other sounde nōbers, it shall bee mete, that we doe declare those twoo sortes of flatte numbers, whiche I named before: that is diametralle numbers, and like flattes.

*A diametral nomber.* A *Diametralle nomber*, is suche a number as hath twoo partes of that nature: that if thei bee multiplied together, thei will make thesaied *diametralle nomber*: And the squares of those twoo partes, beeyng added together, will make a square nōber also: whose roote is the *diameter* to that *diametralle nomber*.

*A diameter.*

As 12 is named a *diametralle nomber*, for that he hath twoo partes, that is. 3. and. 4, whiche beeyng multiplied together, doe make 12. that is the firste number. And if their squares be added together, thei wil make a thirde square: and the roote of that number will bee the *diameter* to that platte forme of 12. As in this example you see.



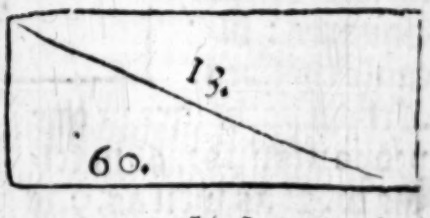

The one side is. 4. and the other side is 3 whiche bothe multiplied together, doe make 12. Then take the square of fower whiche is 16 and the square of. 3, whiche is. 9. and put them

together

together, and thei will make. 25. whose roote, beyng 5. is the *diameter* of that platte forme.

Scholar. That doe I perceiue well, bicause it is confirmed by the. 33. theoreme of the pathewaie.

Master. Yet take an other example. In this platte forme of. 60. you see the one side to bee. 5. and the other side to bee 12. Now take the square nomber of. 12. whiche is. 144. and the square of. 5. whiche is. 25. and put them together: so will it make 169. whiche is a square nomber: and hath. 13. for his roote.




Likewaies. 120. is to be accoumpted a *diametralle nomber*. For so muche as it hath twoo partes: that is 8. and. 15. whiche beeyng multiplied together, doe make the firste nomber. 120. And the square of those twoo partes (that is. 64. for. 8: and. 225. for. 15.) beyng bothe added together, doe make. 289. whiche is a square nōber: and hath for his roote. 17. And therfore that. 17. is the *diameter* to that *diametralle nomber*. 120.

Like examples infinite might I giue you. But these for erplication of the name, maie suffice.

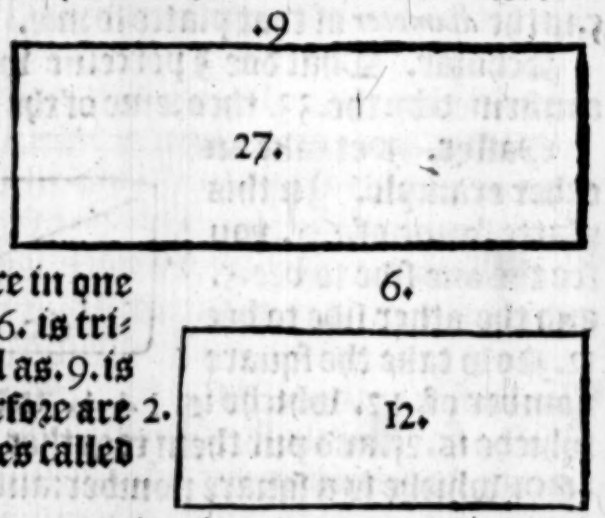
Scholar. I doe well vnderstande the examples: saue that I knowe not how to finde the roote of the laste square nomber, whiche amounteth by the addition of the former twoo squares together.

Master. That arte will I teache you anon. But we maie not forgette firste to ende all the difinitions of soche names, as I minde to write of.

Whereof yet there resteth *like flattes*: whiche maie bee as well taken for trianguler figures, as for quadrate figures. *Like flattes.*

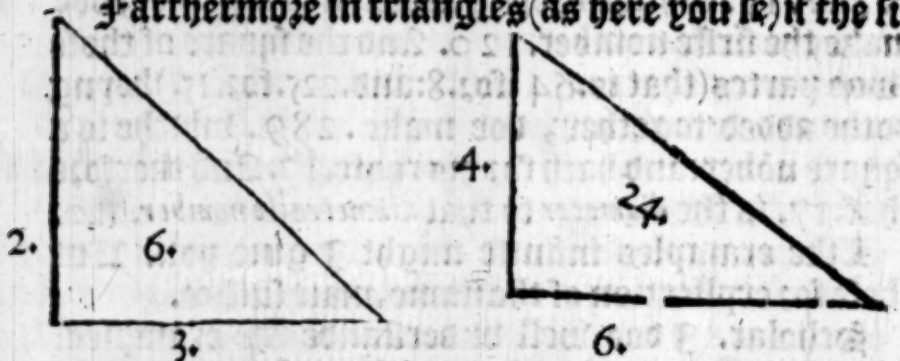
So that of any of them, when the sides of one platforme, beareth like proportion together, as the sides

of any other flatte forme of thesame kinde doeth, then are those formes called *like flattes*. As in these. 2. longe Squares: bicause the sides of them bothe, are in one proportion (for. 6. is triple to. 2: as well as. 9. is triple to 3.) Therfore are the whole figures called *like flattes*.




And so of due conueniencie, their nombers (that expresse their quantities, whiche here are. 27. and 12) be called by the like names, *like flattes*.

Farthermore in triangles (as here you se) if the sides of the one beare like proportion together, as the sides of the other doe: then are thei called *like flattes* also. And their nombers, that declare their quantities, in like sorte are named *like flattes*.




Scholar. I perceiue here: As 4 is to 2: so. 6. is to 3. bothe beyng in a double proportion. And therfore 6 and. 24. are to be called *like flattes*.

Master. You vnderstande it well.

And thus haue we briefly ouer runne the diuision of nomber, into his principalle kindes: And haue set forthe the definitiōs of eche of them, with examples.

The

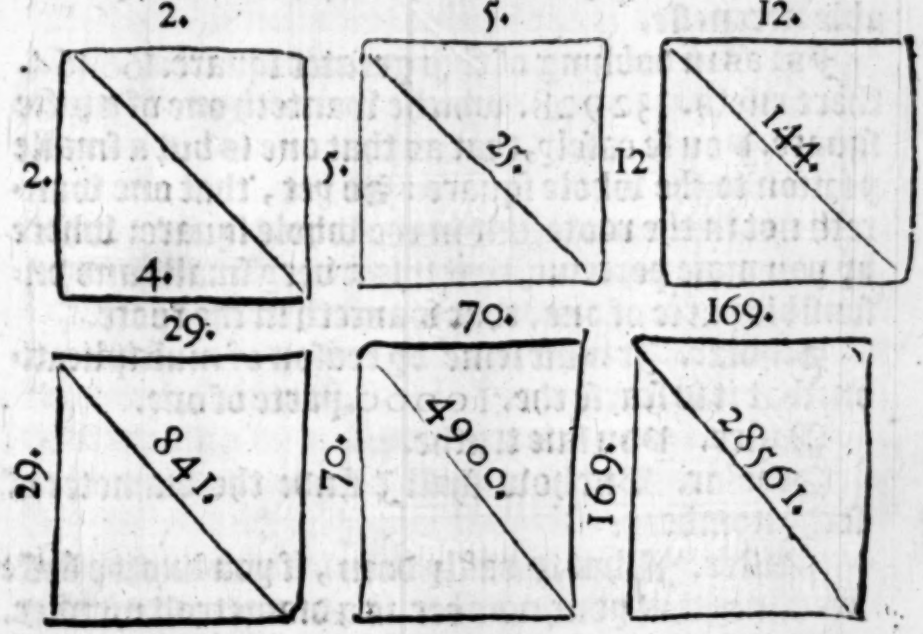
The vse of them you shall se largely in the practise of this arte.

But to the intent you maie the better obserue and regarde these twoo laste kindes of nombers: whiche are commonly neglected of artes men, I will shewe you some vse of them, with their properties.

Firste, all *diametralle nombers* doe sette forthe a triangle, hauyng all three sides knowen: whiche thyng as it doeth serue to many and wonderfull purposes: so can it be found in no other nombers, then onely in *diametrall nombers*. *Of diametralle nombers.*

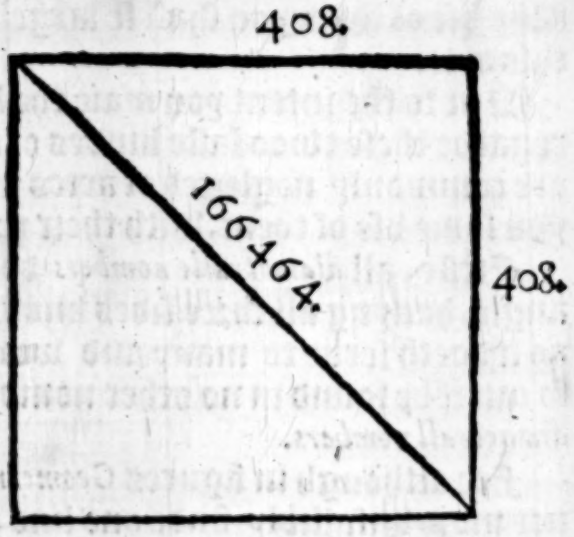
For although in figures *Geometricalle*, you maie euer more vnfallibly finde one line, that will make a square, equall to the twoo squares of any other twoo lines (as in the patthe waie you doe see it taught) yet the measure certaine of those sides, are not knowen.

Wherfore in nomber that is not possible alwaies to be doen: neither can it be doen with any other nōbers, then onely *diametricall nombers*. Yet maie other nombers go very nigh. As namely in these examples




of square nombers: whose double, I take for the squa-

res of the ſides, bicauſe thei are equall: and thei make. 8. 50. 288. 1682. 9800. 57122. &. 332928. All whiche differ onely by an vnitie, from a ſquare nomber.




For nine is a ſquare nomber and ſo are theſe other folowyng.

whoſe rootes be. 49. 289. 1681. 9801. 57121. &. 332929. 7. 17. 41. 99. 239. 577.

Whiche examples if you doe conſider well hereafter, thei will helpe you to geſſe at the nigheſte rootes of nombers that be not ſquare. And alſo for doblyng of ſquares, in a ſquare forme: within an vnſpeakeable nereneſſe.

For as in doblyng of this greater ſquare. 166464. there riſeth. 332928. whiche wanteth one of a iuſte ſquare. You ſe eaſely, that as that one is but a ſmalle portion to the whole ſquare: So yet, that one wanteth not in the roote, but in the whole ſquare: where by you maie perceiue, that it is a very ſmalle and vnſenſible parte of one, that wanteth in the roote.

Scholar. It muſt ſeme by reaſon of multiplication: that it is ſcarſe the. 10000. parte of one.

Maſter. You ſaie truthe.

Scholar. But how ſhall I finde the diameter of ſoche nombers?

Maſter. That is eaſily doen, if you knowe firſte certainly that your nomber is a diametrall nomber. And ſecondarily, if you knowe the true partes of it:

it: whiche you ſhould vſe in this caſe.

Scholar. Will not any twoo ſoche partes ſerue, whiche by multiplication will make the whole nomber?

Maſter. You maie by the former examples, eaſily ſe the contrary. For 12 is a *diametrall nomber*: and hath theſe partes (as it is ſone perceiued). 2. 3. 4. 6. Yet if you take. 2. and. 6. for the ſides of it, thei will not make a *diameter* in knowen nomber.

Scholar. That I vnderſtande: for the ſquare of 2. beyng. 4. added to. 36. whiche is the ſquare of 6. doeth make. 40. whoſe roote muſt bee greater then. 6. and leſſe then. 7. And therfore. 40. can haue no roote in whole nomber.

Maſter. Neither yet in broken nombers: for that is a generalle rule: that if any whole nomber haue a roote, that roote ſhall be a whole nomber. So that if the roote can not bee founde in whole nomber: you ſhall neuer finde it in broken nombers.

And for more certaintie of that I ſaied before, that all partes be not apte for the ſides of a *diametralle nomber*, to finde out the *diameter*: marke well the ſeconde erample, whiche is. 60. and hath theſe partes.

2. 3. 4. 5. 6. 10. 12. 15. 20. 30.

So that beginnyng with the two extremeſte, that is. 2. and. 30. thei will by multiplication make. 60.

And likewaies any two nombers, equally diſtant from thoſe extremes: As. 3. and 20. Likewaies. 4. and 15: other. 5. and. 12. And in like maner. 6. and. 10. All thoſe couples by multiplication doe make. 60. Yet none of them are apte ſides to finde the *diameter* by, but onely 5 and. 12. For of the other ſides beyng multiplied ſquarely (that is by thē ſelfes) and thoſe ſquares beyng added together, there wil not riſe a ſquare nomber. As you ſhall better vnderſtande, when you

haue learned to knowe square nombers, by extractiō of their rootes.

Yet in the meane ceason I will set forthe certaine notes, to knowe the *diameter*, and the apte sides, in all *diametralle nombers*.

1. And firste I saie: that as thei are three nombers in all (I meane the twoo sides, and the *diameter*) so all waies if the firste or leaste side bee odde, then shall there be twoo of them odde nombers. And the *diameter* shall euer bee the other of the odde nombers: that is to saie, the greateste of them.

2. Secondarily. It is true that all *diametrall nombers* are euen nombers. And no odde nomber can bee a *diametralle nomber*.

3. Thirdly. I saie, that all odde nombers aboue one, maie be the lesser side in soche *diametralle nombers*.

But euen nombers doe not serue so generally: for thei onely maie stand in soche place, whiche be greater then. 4: As. 6. 8. 10. 12. 14. 16. 18. 20. &c. And none other euen nombers then soche as maie be diuided by 4. maie be the greater side in any *diametralle nomber*.

4. Fourthly. If the lesser side bee an odde nomber, then ordinarily, the square of it is iuste equalle with that that amounteth by the addition of the *diameter*, to the greater nomber. As in the firste example, 3. is the lesser nomber, and. 4. is the greater: vnto them bothe the *diameter* is .5. Now. 3. hath for his square 9. and so moche is made by the addition of. 4. and. 5.

Again in the seconde example, the lesser nomber is 5. and his square is 25. The greater nomber is 12, and the *diameter*. 13. Put. 12. and. 13. together, and thei make. 25. whiche is equalle with the square of the lesser.

Likewaies. 7. and 24. multiplied together maketh 168. whiche is a *diametralle nomber*. And bicause the square of the lesser side (whiche here is. 49.) must bee

equalle

equalle to the greater ſide, and the *diameter* added together: therfore ſeyng. 25. added to. 24. maketh. 49. that. 25. muſt nedes bee the *diameter* to the foreſaied nomber.

By theſe rules (if you doe marke them well) you maie ſone perceiue, how to make any *diametralle nomber*: if the leſſer ſide bee giuen vnto you, and bee an odde nomber. Yet for your caſe, I will giue you this plaine rule.

When any odde nomber is propounded: as the leſſer ſide of a *diametralle nomber*, and you would finde the other ſide, and the *diameter* alſo: or els the *diametralle nomber*, that maie haue ſoche a ſide: multiplie that propoſed nomber by it ſelfe, and it will make a ſquare nomber, and will be an odde nomber: ſo that of it you ſhall finde no iuſte halfe. Therfore take you thoſe twoo nombers, that are nerte vnto the halfe of it: The leſſer ſhall alwaies bee an euen nomber, and ſhall be the ſeconde ſide of the *diametralle nomber*: The other nomber whiche is the greater, ſhall alwaies be an odde nomber: and ſhall bee the *diameter* of that nōber whiche you deſire. For example marke wel theſe formes that doe folowe.

If three bee propounded as the one ſide of a *diametralle nomber*: And you would knowe, what maie bee the other ſide: and what is the *diametralle nomber*: And thirdly, what is the *diameter* to that nomber: Doe, as I ſaied before: multiply. 3. by it ſelf, and it will make 9. whiche is a ſquare nomber, and an odde nomber: and therfore hath no iuſte halfe. But the nigheſt nōmbers to the halfe, are. 4. and. 5.

Therfore I ſaie, that. 4. whiche is the leſſer of thē twoo, is the ſeconde ſide of the *diametralle nomber*: and 5. beyng the greater of them, is the *diameter* it ſelf.

Scholar. Now is it light inough to perceiue that the *diametralle nomber* is. 12: ſeeyng. 3. multiplied by

folwer

4. maketh. 12.

Master. So is it.

Again, if. 5. be assigned for one side of a *diametralle nomber*, and you obserue the former worke you maie easily finde the other side, and the *diameter*.

First you see, that the square of 5. is. 25. and it hath no halfe. But. 12. and. 13. are the. 2. nombers nighest his halfe: wherfore. 12. shall bee the seconde side: and 13. must be the *diameter*. And the *diametralle nōber* is. 60.

Likewaies, if. 7. be set for the lesser side, the greater side shall be. 24. and the *diameter*. 25.

Scholar. Touching this I nede no more instruction: the thyng is so manifeste.

Master. Then shewe your knowlege by an exam ple, or twoo.

And first I appointe 9 for the lesser side of a *diametralle nomber*, whereunto I would haue you to assigne the other side, and the *diameter*. &c.

Scholar. I followe your precepte, and multiplie 9. by it self, whereof commeth. 81. whose halfe is betwene. 40. and. 41. Therfore must. 40. be the other side: and 41. the *diameter*. And here the *diametralle nomber* is. 360.

Master. Proue the like: where. 15. is the lesser nomber.

Scholar. 15. multiplied square maketh. 225: whose nighest halfes are. 112. and. 113. of whiche the first is the seconde side, and the later is the *diameter*: and the *diametralle nomber* is. 1680.

Master. What shall be the other nombers: where 21. is the lesser side?

Scholar. 21. yeldeth in square. 441. whose portions nighest his halfe, are. 220. and. 221: And so appereth their offices, and the *diametralle nomber* is 4620

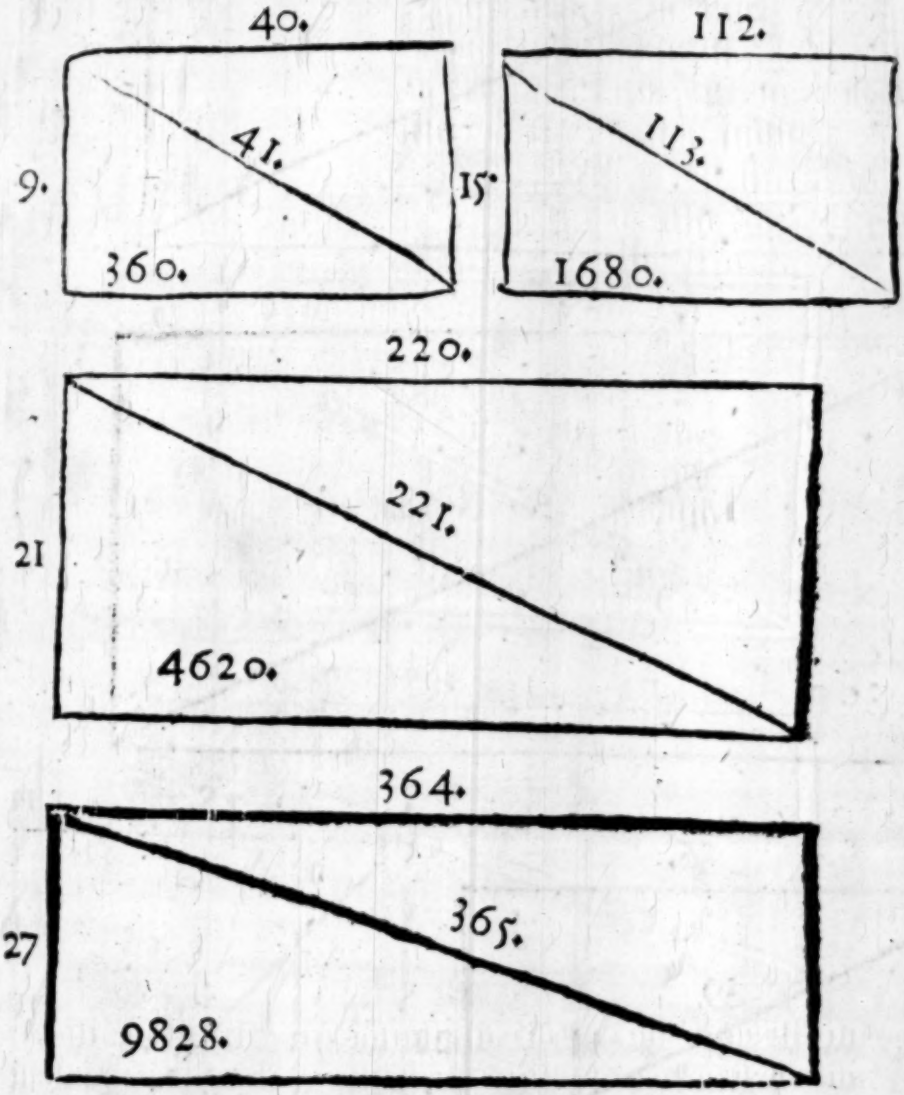
Master. So maie you saie that vnto. 27. being the lesser side: the greater side shall be. 364. and the *dia-*

*ter*

ter. 365. bicause the square of. 27. is. 729. And the *diametralle nomber* is. 9828.

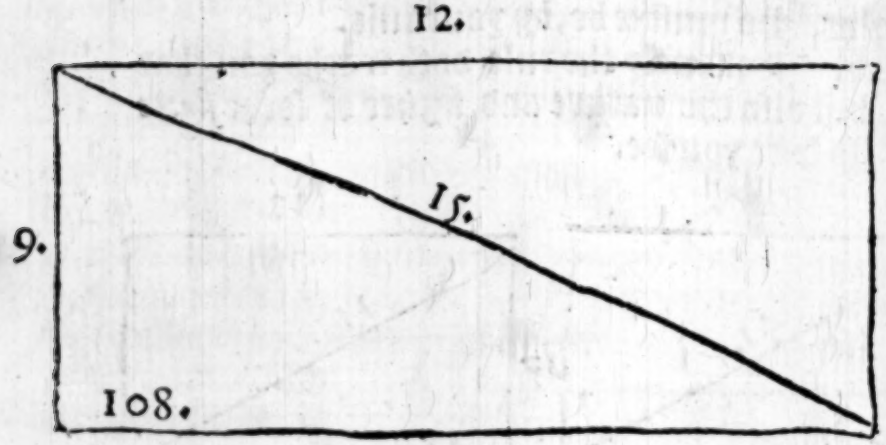
Scholar. So must it be, by your rule.

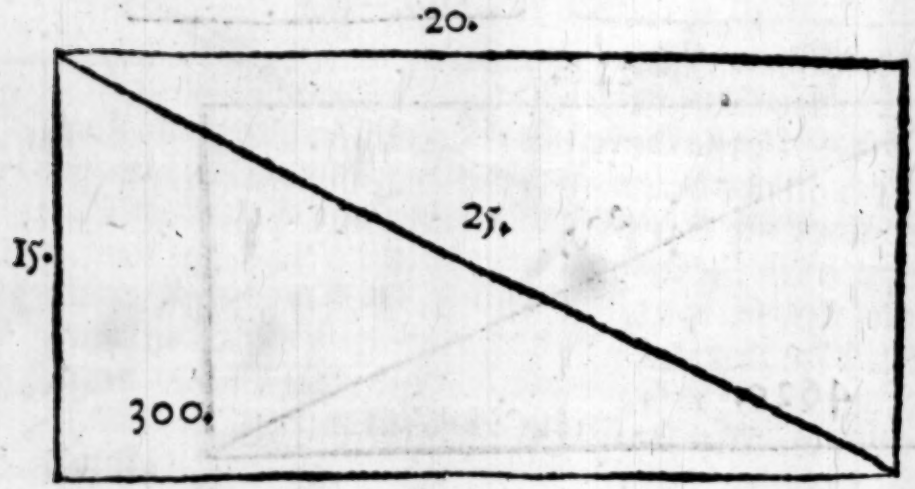
Master. Not onely the rule doth teache you that it is so, but also the nature and figure of soche *flatte nombers*. As here you see.

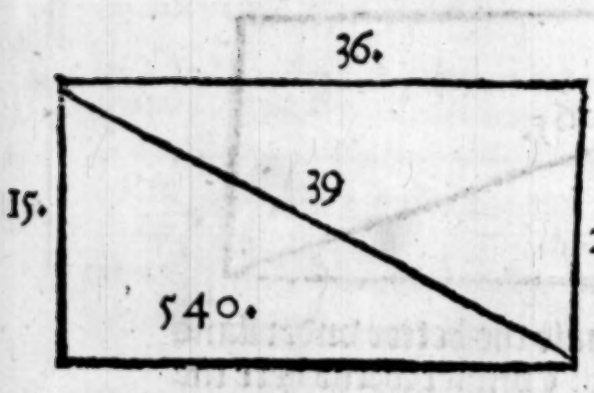


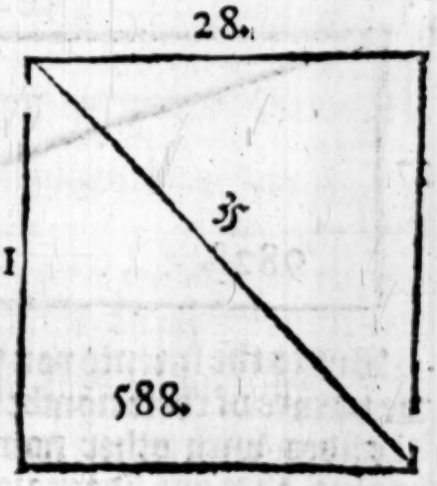

But to the intente you maie the better vnderstand the nature of these nombers: I wil set forthe here the like sides with other nombers: whereby you maie knowe, that one side maie serue to diuerse *diametralle*

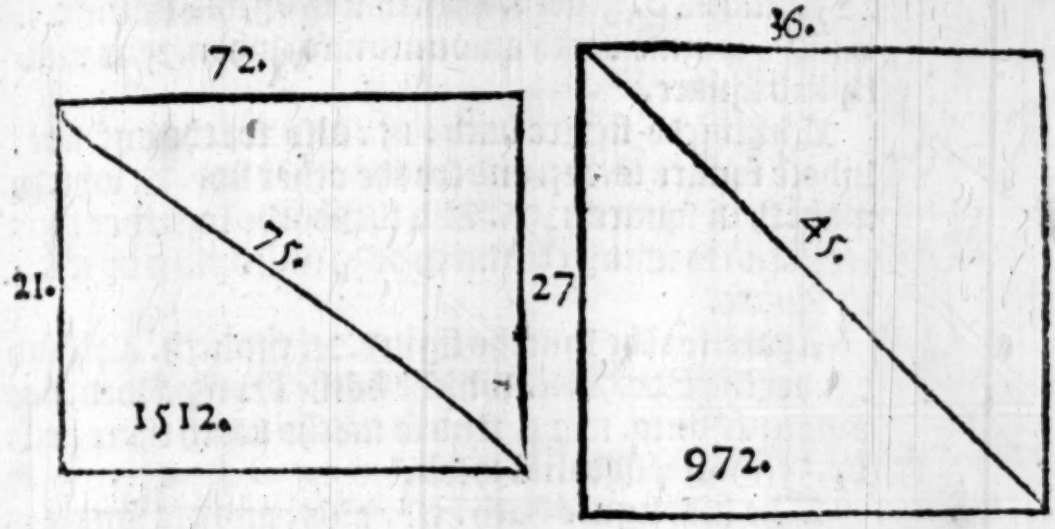
numbers. Therfore marke these formes well.

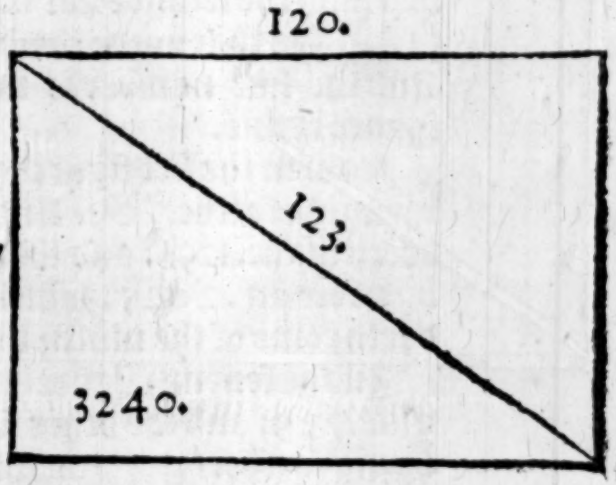





Scholar. Here I see the same. 4 nombers. 9. 15. 21. and. 27. set as the lesser sides: And their greater sides are soche as disagree frō the former rule. And in. 15. 21. and. 27. I see twoo varieties, vnlike to the former example. But seeyng the sides doe disagree, I doe not maruel that the *diametralle nombers* are diuerse from the former.

Master. Examine these nombers, whether thei be true.

Scholar. I must multiplie eche side by it self, and then adde thē together: and if thei make as moche iustly, as the *diameter* beyng multiplied square, then are thei true nombers. So I see, that. 9. maketh. 81. and 12 doeth yelde 144 whiche bothe added doe make. 225. And so moche doth 15 make, being multiplied square.

Likewaies, for the second figure 15. bryngeth forth

225. and. 20. giueth. 400. that is by addition. 625. whiche somme doeth amounte also, when. 25. is multiplied square.

The thirde figure hath. 15. also for the one side, whose square is. 225. and for the other side. 36. whiche maketh in square. 1296. And thei bothe together giue 1521. And so many commeth of 39 multiplied by it self in square.

Again for the fourthe figure. 21. maketh. 441. and 28. doeth yelde. 784. whiche bothe beyng added, doe amounte vnto. 1225. And so moche doeth there arise by. 35. multiplied into it self.

The fifte figure hath. 21. also, and his square is 441. and the seconde side beyng. 72. maketh in square 5184. So that bothe those squares doe make. 5625. And the like nomber is made by. 75. multiplied in square forme.

Now in the firt figure 27 beyng multiplied square bryngeth forthe. 729. And. 36. likewaies multiplied doeth make. 1296. and that with the other will make by addition. 2025. whiche somme (as is well seen) doeth come of the multiplication of. 45. by it self.

In the seuenth figure. 27. multiplied square, doeth giue. 729: and the other side (whiche is. 120.) doeth bryng forthe. 14400. These bothe ioyned together doe make. 15129. And the like somme is gathered by the multiplication of. 123. squarely.

So that all those figures doe appere true.

But how thei maie agree with your former rule, I can not see.

Maister. That rule did I make for nōbers vncompounde. For nombers compounde haue not onely in their owne name, the vse of that rule, but also thei folowe the forme of those nombers, of whiche thei bee compounde.

So. 9. beyng compounde of. 3. foloweth the forme

of

of.3. And therfore as.3.hath.4.for to make the second side with hym,so. 9.(beeyng thrise.3.) shall haue.12. (whiche is thrise.4.)for a matche side with hym.

Likewaies.15. beyng compounde of.5. and.3.shall haue their formes in the makyng of the *diametralle nõbers*. For as.3.hath. 4. so.15.(beeyng fiue tymes.3.) shall haue. 20. (whiche is fiue tymes . 4.) for the seconde side.

Again,as.5.hath.12.so shall.15.(beeyng three tymes.5.)haue. 36.(that is three tymes.12.) for his seconde side.

Likewaies.21. beyng compounde of.3.and.7.shall haue bothe their formes.

And. 27. whiche is compounde of. 3. and. 9. shall haue all the varieties of their formes.

Scholar. I see it is euen so,and that in the *diameter*,as well as in the seconde side. But the *diametralle nomber* doeth varie moche in them.

Master. Yet doe those nombers agree in a marueilouse good proportion. For if you doe consider the proportion of bothe the sides in one figure, to bothe the sides in an other figure:and adde those twoo proportions together, the addition of theim doeth make the nomber that representeth the proportiõ betwene their twoo *diametralle nombers*. Whiche thynge I will now onely touche,as briefly as maie bee,to giue you occasion to marke it better hereafter: Sith this place doeth not fully serue for it. As.3. and.4. beeyng the twoo sides of a *Diametralle nomber*,doe make.12. So if 9.and 12 be the sides of a *diametralle nomber*,that nomber must be.9.tymes.12.that is.108.For.9.is triple to.3:and.12. is triple to. 4. And bicause the addition of proportions, is like the multiplication of fractiõs, I must multiplie. 3. by. 3. or els $\frac{3}{1}$ by $\frac{3}{1}$, whiche is all one,and that will maie.9.

Likewaies,if 3.and.4.be taken for the sides of the

lesser nomber *diametralle*, and. 15. and. 36. for the sides of the greater nomber: As the lesser nomber shall bee 12. so the greater must be. 540. that is. 45. tymes. 12.

For. 15. vnto. 3. is in a *quintuple* proportion, and is written thus. $\frac{5}{1}$: and. 36. vnto 4 is a *noncuple* proportion, and is written thus $\frac{9}{1}$. Now if you multiplie these nombers together, thei will make 45: whiche declareth the proportions of the twoo *diametralle nombers*. And so of all the reste, as you maie easily consider.

Scholar. I praie you, let me examine one or twoo of thē, in comparison to that firste *diametrall nomber*. 12.

I see that 15 beyng the lesser side, and 20. the greater side, doe make. 300. as their *diametralle nomber*: and that. 300. is. 25. tymes so moche as. 12. is. Therfore by your saiyng the proportion of 15. to. 3. and of. 20. to 4. must make. 25. And so it doeth. For eche of them is a *quintuple* proportion. And it is quickely gessed, that 5. multiplied by. 5. doeth make. 25.

For farther proofe, I take the *diametralle nomber* 1680. whose sides are. 15. and. 112. First I see, that. 15. to. 3. beareth a *quintuple* proportion: and. 112. to. 4. is as. 28. to. 1. Therfore I multiplie. 28. by. 5. and it maketh. 140. Then if I multiplie that nomber by. 12. it will make. 1680.

This is a sufficiente trialle for these nombers.

*Of euen sides* But of soche *diametralle nombers*, as haue euen nombers for their lesser side, you haue giuen no rule, nother examples, saue onely of. 8. Wherfore I praie you tell me, how shall I finde out the *diametralle nomber*, with his other side, and the *diameter* in soche euen nombers.

Master. You shall make it square, as you did in the other nombers, that wer odde: And of that square you shall take twoo quarters, whiche you shall alter in soche sorte, that you shall abate. 1. frō the one quarter, and put it to the other quarter. And so haue you

twoo

twoo nombers, differyng onely by .2. and bothe beyng odde. The lesser of them twoo, is the greater side of the *diametralle nomber*: and the other is the *diameter* to it. As. 8. beyng your lesser side, the square of it is 64. whose quarter is. 16. from whiche I abate. 1. and there resteth. 15. and that is the seconde side. Also I adde 1. to 16. and it maketh. 17: whiche is the *diameter*.

Scholar. This is no thyng harde. As by example I will proue. If. 12. bee the lesser side: his square is 144. and the quarter of it is. 36. Then abatyng. 1. I see there will bee. 35. for the other side of the *diametralle nomber*. And addyng. 1. to. 36. it maketh. 37. to be the *diameter*. And if I multiplie. 35. by. 12. it bryngeth forthe. 420. whiche is the *diametralle nomber*.

Now for proofe of these nombers, I multiplie. 12. by it self, and it maketh. 144. Then I multiplie the other side, that is. 35. by it self, and it yeldeth. 1225. Those bothe together doe make. 1369. And seyng 37 multiplied by it selfe, doeth make thesame nomber. Therfore are thei all true nombers.

An other example. 10. beyng set for the lesser side, I doe multiplie it squarely: and there riseth. 100. whose quarter is. 25. For whiche I take (as you taught me). 24. and. 26. And so the whole *diametralle nomber* is. 240. For proofe of the other nombers, I take. 100. whiche commeth of. 10. multiplied square, and to it I adde. 576. whiche is the square to. 24. and thei bothe doe make. 676. And so muche amounteth by the multiplication of. 26. squarely.

Master. This maie suffice for this presente: if you marke that the euē nombers haue not onely one generalle forme, whiche I did expresse in the former rule, but also soche as be compounde of any other nōbers, euen or odde: Haue the like nombers in proportion, for the greater side, and for their *diameter* as the nombers haue, of whiche thei bee compounde. And

bicause

bicause I will not staie to long on this matter, I will here set forthe diuerse varieties of *diametrall nombers*, whereby you maie gather not onely the true vnderstandyng of the former rules: But also in theim you maie see other notable cõclusions: and straunge workes of the natures of nombers.

Marke well this table forme, with the titles ouer it: whiche declare the true meanyng of it.

And where you see one nomber in the firste columpne against twoo, three, or fower in the other columpnes, you shall vnderstande that that nomber is the side to so many seueralle nombers *diametralle*.

## *The table of diametralle nombers.*

| The lesser side. | The greater side. | The diameter. | The nober diametrall |
|---|---|---|---|
| 3. | 4. | 5. | 12. |
| 5. | 12. | 13. | 60. |
| 6. | 8. | 10 | 48. |
| 7. | 24 | 25 | 168 |
| 8. | 15 | 17 | 120 |
| 9 | 12 | 15 | 108 |
| | 40 | 41 | 360 |
| 10 | 24 | 26 | 240 |
| 11 | 60 | 61 | 660 |
| 12. | 16 | 20 | 192 |
| | 35 | 37 | 420 |
| 13 | 84 | 85 | 1092 |
| 14 | 48 | 50 | 672 |
| 15 | 20 | 25 | 300 |
| | 36 | 39 | 540 |
| | 112 | 113 | 1680 |
| 16 | 30 | 34 | 480 |
| | 63 | 65 | 1008 |
| 17 | 144 | 145 | 2448 |
| 18 | 24 | 30 | 432 |
| | 80 | 82 | 1440 |
| 19 | 180 | 181 | 3420 |
| 20 | 48 | 52 | 960 |
| | 99 | 101 | 1980 |
| 21 | 28 | 35 | 588 |
| | 72 | 75 | 1512 |
| | 220 | 221 | 4620 |
| 22 | 120 | 122 | 2640 |
| 23 | 264 | 265 | 6072 |
| 24 | 32 | 40 | 768 |
| | 45 | 51 | 1080 |
| | 70 | 74 | 1680 |
| | 143 | 145 | 3432 |
| 25 | 60 | 65 | 1500 |
| | 312 | 313 | 7800 |
| 26 | 168 | 170 | 4368 |
| 27. | 36 | 45 | 972 |
| | 120 | 123 | 3240 |
| | 364 | 365 | 9828 |
| 28. | 96 | 100 | 2688 |
| | 195 | 197 | 5460 |
| 29 | 420 | 421 | 12180 |
| 30 | 40 | 50 | 1200 |
| | 72 | 78 | 2160 |
| 31 | 480 | 481 | 14880 |
| 32 | 60 | 68 | 1920 |
| | 126 | 130 | 4032 |
| | 255 | 257 | 8160 |
| 33 | 44 | 55 | 1452 |
| | 180 | 183 | 5940 |
| | 544 | 545 | 17952 |
| 34 | 288 | 290 | 9792 |
| 35 | 84 | 91 | 2940 |
| | 120 | 125 | 4200 |
| | 612 | 613 | 21420 |
| 36 | 48 | 60 | 1728 |
| | 105 | 111 | 3780 |
| | 160 | 164 | 5760 |
| | 323 | 325 | 11628 |
| 37 | 684 | 685 | 25308 |
| 38 | 360 | 362 | 13680 |
| 39 | 52 | 65 | 2028 |
| | 252 | 255 | 9828 |
| | 760 | 761 | 29640 |
| 40 | 75 | 85 | 3000 |
| | 96 | 104 | 3840 |
| | 198 | 202 | 7920 |
| | 399 | 401 | 15960 |

This table maie you ertende infinitely. And these thinges maie you se, as thinges of greate admiratiō.

1. There is no *diametralle nomber*, but it maie be diuided by. 12. Wherfore thei be all euen nombers euenly and oddely.

2. Again, there is no *diametralle nomber*, but it endeth in. 0. in. 2. or in. 8.

3. Thirdely, there is no *diametralle nomber*, that can haue any more *diameters* then one.

4. Yet maie one nomber bee the *diameter* to diuerse other.

As you se 25. is the *diameter* to. 168. and also to. 300.
So. 65. is the *diameter* to. 1008. and also to. 1500.
Likewaies. 145. is the *diameter* to. 2448. and to 3432.

5. Fiftely: No square nomber can bee a *diametralle nomber*.

Scholar. These properties be notable.

*To knowe a diametralle nomber.*

But how shall I knowe, when a nomber is proponed, whether it be a *diametralle nomber*, or not?

Master. In that thyng I finde a tediouse trauell, by any rules, in those that write of it. But I wil ease you of moche paine therein.

Firste remember the properties of those nombers.

And if you haue any other figure in the first place, then. 0. 2. or. 8. it is no *diametralle nomber*.

Secondarily, if it maie not bee diuided by. 12. although it ende in one of those. 3. figures, it is no *diametrall nomber*.

Wherfore if it haue bothe those twoo properties (whiche an infinite multitude of nombers doe want) and be no square nomber (as none be that ende in. 2. or. 8. or with odde cyphers) then sette out all the partes of it, in soche sorte, that the lesser parte doe stande direcly ouer those greater partes, which beyng multiplied together, will make the whole nomber.

And

And then examine those partes, whiche seme to haue any likelihod: accordyng to the former doctrine.

As for example: if. 72. be proponed to be examined in that sorte, I sette his partes in order thus.

| | | | | |
|---|---|---|---|---|
| 2. | 3. | 4. | 6. | 8. |
| 36. | 24. | 18. | 12. | 9. |

Howbeit I neded not to set doune. 2. nother. 4. for lesser partes, nother those other greater partes, that aunswere to them: For, as I said before, thei can not bee the lesser side in any *diametralle nomber*. Wherfore thei nede no examination.

Farthermore, for them that you shall nede to examine, if the lesser nomber bee an odde nomber, the square of it must contain double to that greater nomber (that is coupled with it) and one more.

And if the lesser be an euen nomber (of them twoo that you would examine) then must the square of it containe the greater nomber (that standeth by it). 4. tymes, and. 4. more. And this is not onely a shorter waie, then I see to be taughte by other artes menne: but it is also more certaine, for all nombers not compounded of other *diametralle nombers*.

Scholar. By this doctrine it appeareth quickely, that. 72. is no *diametrall nomber*.

For although it doeth ende in. 2. and maie be diuided by. 12. yet no couple of nombers here haue those properties that is required.

For vnder. 3. is. 24. whiche is to greate: and vnder 6. there is. 12. whiche is to greate also.

But vnder. 8. standeth. 9: whiche is to litle, by a greate deale.

Master. Then proue in this other nomber. 132.

Scholar. His partes will stande thus.

| | | |
|---|---|---|
| 3. | 6. | 11. |
| 44. | 22. | 12. |

Where I see quickely that it can not bee a *diametrall nomber*. For the nombers vnder. 3. and. 6. be to greate: sith no nomber that should bee sette vnder. 3. maie be aboue. 4.

Nother vnder. 6. maie any nomber bee set greater then. 8. As it doeth sufficiently appeare by that that is taughte before.

And vnder. 11. there can bee no lesse nomber placed then. 60: and therfore. 12. is to smalle.

And herein I perceiue greate helpe by this table, whiche you haue set forthe.

Master. It is well marked of you. But yet trie this other crample. 6072.

Scholar. I set doune his partes in order, thus.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3. | 6. | 8. | 11. | 12. | 22. | 23. | 24. |
| 2024. | 1012. | 759. | 552. | 506. | 276. | 264. | 253. |

| | | | | |
|---|---|---|---|---|
| 33. | 44. | 46. | 66. | 69. |
| 184. | 138. | 132. | 92. | 88. |

And here I see a greate sorte of nombers, whiche can not serue to my purpose, bicause those that bee euen, and are lesse then. 44. make to litle a square, to be 4. times so moche as the nomber vnder any of thē.

And. 44. maketh to greate a square: wherfore it can be none of the euen nombers.

Again, those that be odde vnder. 23. doe make to litle a square, to bee double to the greater nomber vnder it. And those that bee odde aboue. 23. doe make to greate a square. So that. 23. doeth remain to bee the true nōber for the lesser side: and 264 the greater side.

Master. Bicause exercise is the beste instrument

in learnyng: therfore will I propounde to you one example more.

What saie you of. 5460? Is it a *diametralle nomber* or no?

Scholar. I will trie it, by settyng doune his partes thus.

| 3. | 5. | 6. | 7. | 10. | 12. | 13. | 14. | 15. |
|---|---|---|---|---|---|---|---|---|
| 1820. | 1092. | 910. | 780. | 546. | 455. | 420. | 390. | 364. |

| 20. | 21. | 28. | 30. | 35. | 42. | 52. | 60. | 70. |
|---|---|---|---|---|---|---|---|---|
| 273. | 260. | 195. | 182. | 156. | 130. | 150. | 91. | 78. |

And here I se diuerse and many nombers, whiche at the firste sighte, appere nothyng mete for this purpose. For. 20. is to smalle a nomber, as I maie sone iudge: and therfore all other nombers vnder it, must nedes be to smalle, of force.

Againe, I see that. 30. is to greate a nomber, and therfore, of necessitie, all other nombers aboue it, must nedes be to greate. So that. 21. other. 28. must be the true nomber, or els none.

Wherfore I examine first. 21. whose square is 441 whiche should bee one more then double, to the nomber vnder it, that is to saie, it should bee. 521. And so it is not: Therfore I refuse it, and examine. 28. whose square is. 784. And that should bee fower tymes so moche as. 195. (whiche is the nomber vnder it) and 4. more. Therfore I doe *quadriple*. 195. and it maketh. 780. And then I see that it wanteth, but fower of the other square: wherfore I take those twoo nombers, I meane. 28. and. 195. for the true sides of. 5460. whiche I finde to be a *diametralle nomber*.

Master. By the waie, remeber that you could easily perceiue, that all nōbers vnder. 20. were to small for your purpose: and contrary waies, all aboue. 30,

*A ſhorte meane in working.*

to be to greate. So that you neded not to ſette doune ſo many partes of your firſte nomber.

Wherfore if your nomber bee ſoche a one, as hath many partes, you maie chose one by geſſe, which you thinke will go nigh to ſerue your purpoſe: and if you finde it to ſmalle, then ſet theim doune onely that bee greater then it, til you finde one other iuſte: and then haue you your purpoſe. Or if you finde any to great, after that whiche was to ſmalle, and betwene theim none iuſte. then is not your nomber a *diametrall nõber*.

But and if the parte whiche you tooke by geſſe, be to great, you ſhall refuſe all partes aboue it, and take onely leſſer partes, til you finde a iuſte parte for your purpoſe: or els one that is to litle.

And if in deſcendynge orderly, you finde no iuſte parte, before you come to one that is to litle, then is your nomber no *diametralle nomber*.

Scholar. This is a greate eaſe in ſhortenynge of worke: whiche I will proue in this nomber. 9786.

Maſter. If you remembred well your former rules, you would not admitte this to be examined for a *diametralle nomber*: bicauſe it endeth in none of the thre peculiare terminations: that is. 0. 2. or. 8.

Scholar. I cõfeſſe my faulte. And therfore I take this nomber. 9780. whoſe. 20. parte is. 489. But ſeyng. 20. doeth make in ſquare but. 400. therfore is it very moche to litle.

Then I take the. 30. parte of it, whiche is. 326. and finde it alſo to litle.

Thirdely, I take the. 40. parte of it, whiche is $244\frac{1}{2}$: and ſeyng. 40. maketh in ſquare. 1600. I ſee that it is almoſte. 7. tymes ſo moche as. $244\frac{1}{2}$: and therfore is it to greate.

So muſt the true nomber be betwene. 30. and. 40: or els there is none at all.

Therfore firſte I take. 35. whiche is the middelle nomber,

nomber (as the moste apte for a coniecture) and it yeldeth. 279 3/7. And the square of. 35. is. 1225. whiche is farre more then the double of. 279 3/7.

Therfore, again I proue with. 32. whiche giueth 305 5/8. And seyng the square of. 32. is. 1024. it is not 4. tymes so moche as. 305 5/8. for that is. 1222 1/2.

Wherfore I take a greater nomber, betwene it and. 35. And first I take. 33. whiche bringeth forthe 296 1/11. wherby I male see that. 33. is to greate. And seyng there is no nomber lefte betwene. 32. and. 33. therfore I iudge that firste nomber. 9780. to bee no *diametralle nomber*.

Master. Examine this nomber. 43200.

Scholar. Bicause I see it to be a greate nomber, I will begin with a greate parte of it. And therfore, I take. 100. whiche yeldeth. 432. And considerying that the square of. 100. is. 10000. whiche is farre to greate, I must seke a lesser nomber.

Master. I will ease you of your paines in that. For bicause here is more to bee considered. You remember that I tolde you before, in makyng of *diametralle nombers*, how that some nombers doe followe the rules of other, of whiche thei be compounde. And farthermore, that soche compounde *diametralle nombers*, did beare proportion to the lesser, as the proportion was of bothe their sides added together.

Scholar. That is true.

Master. Of like reason all soche *diametralle nombers*, must bee excluded from these rules, whiche bee made peculiarly for nombers that haue their owne proper formes, and depende not of other.

And yet some common rule must bee giuen, that male ertende as well to them, as to any other.

Wherfore let this be it.

That the twoo sides of all *diametralle nombers*, haue soche a proportion together, as here you see erpressed

in

in some one of these formes: if thei bee continued as here thei be begon.

¶ The firste order.

$\frac{3}{4}$: $\frac{5}{12}$: $\frac{7}{24}$: $\frac{9}{40}$: $\frac{11}{60}$: $\frac{13}{84}$: $\frac{15}{112}$: $\frac{17}{144}$: $\frac{19}{180}$: $\frac{21}{220}$:

$\frac{23}{264}$: $\frac{25}{312}$: $\frac{27}{360}$: $\frac{29}{420}$: $\frac{31}{480}$: $\frac{33}{544}$: $\frac{35}{612}$: $\frac{37}{684}$: $\frac{39}{760}$. &c.

¶ The seconde order.

$\frac{2}{11}$: $\frac{12}{35}$: $\frac{16}{63}$: $\frac{20}{99}$: $\frac{24}{143}$: $\frac{28}{195}$: $\frac{32}{255}$: $\frac{36}{323}$: $\frac{40}{399}$: $\frac{44}{483}$:

$\frac{48}{575}$: $\frac{52}{2704}$: &c.

Here haue I setts the lesser side as the numerator, and the greater side as the denominator. Whereby you maie perceiue the cause of their distinction.

For the first order is, when the lesser side, or nomber, is odde.

The seconde order is, when that lesser side is an euen nomber.

*Stifelius* doeth set them so, that the numerator standeth for the seconde, or greater side: and the denominator for the firste nomber, or lesser side. And for the more delectable contemplation, to behold their forme of progression, he setteth doune as many whole nombers, as the fraction will giue.

And this is his forme.

¶ The firste order.

$1\frac{1}{3}$: $2\frac{2}{5}$: $3\frac{3}{7}$: $4\frac{4}{9}$: $5\frac{5}{11}$: $6\frac{6}{13}$: $7\frac{7}{15}$: &c.

¶ The seconde order.

$1\frac{7}{8}$: $2\frac{11}{12}$: $3\frac{15}{16}$: $4\frac{19}{20}$: $5\frac{23}{24}$: $6\frac{27}{28}$: $7\frac{31}{32}$. &c.

Where

Where in the first order, you se bothe in the whole nombers, and also in the numerators of the fraction, the naturalle order of nombers. And in the denominators, the naturalle progression of odde nombers.

But in the seconde order, you see that the whole nombers go in their naturalle order, and the numerators and denominators, kepe an *Arithmeticalle* progression, by equalle distaunce of. 4. saue that in the numerators, all the nombers bee odde: and in the denominators, thei be all euen.

Now by this generalle rule, if you finde any twoo partes of any nomber, in one of these former proportions, you maie bee sure that it is a *diametralle nomber*. But for the more apte conference of the partes, you shall doe beste to reduce them to their leaste nombers: as you haue learned in the firste parte of *Arithmetike*.

So in your laste nomber, whiche was 43200. you shall finde his. 180. parte, to bee. 240. whiche beyng reduced to their smalleste nombers, will bee. $\frac{3}{4}$: wherfore I am assured, that it is a *diametralle nomber*.

Yet one thyng more shall you marke.

If any nomber ende in Ciphers, abate euen Ciphers, as often as you can (I meane. 2. 4. or. 6. &c, and if the reste be a *diametralle nomber*, so was the first. And therfore in this laste exampie. 432. is a *diametralle nō-ber*, as well as. 43200.

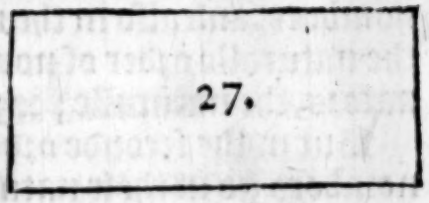
Also if any nomber beeyng diuided by any square nomber, doe make a *diametralle nomber* in the *quotiente*, then was the firste nomber a *diametralle nomber* also.

And this, for this tyme, shall suffice for *diametralle nombers*.

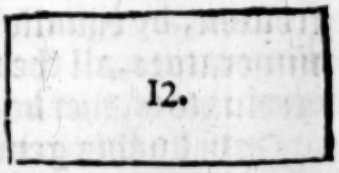
Now will I speake somewhat briefly of *like flattes*: and then procede to other *figuralle nombers*. *Of like flattes.*

Scholar. I remember you defined them before, to bee soche flatte nombers, as had one forme of proportion betwene their sides.

As here 27. and 12. be *like flattes*: bicause their sides be in one proportion. For as. 9. is to. 3. so 6. is to. 2. bothe beeyng in triple proportion.




Master. You saie well. And that is the cause why thei be called like: for the likenesse in the proportiō of their sides. Although some menne delite more to call them *squarelike figures*: bicause thei haue some properties agreable with square nombers (for as *Euclide* saieth in his. 8. booke, and. 18. proposition:




*Squarelike figures.*

*Euery twoo nombers, beeyng like flattes, haue one meane nomber betwene theim in proportion. And the one flatte nomber beareth vnto the other flatte double that proportion, that their sides doe.*

For declaration of whiche proposition, marke the twoo flatte nombers before: I meane. 27. and. 12. whose sides are in proportion *Sesquialter*: And the flat nombers themselfes be as $\frac{9}{4}$. or. 9. to. 4: that is double *Sesquiquarte*. Now doe you double the proportion *Sesquialter*, and it will make *double Sesquiquarte*.

Scholar. Thus doe I sette them in order. $\frac{3}{2}$: $\frac{3}{2}$. And I multiplie the numerators together, and the denominators also. (For I remember, you tolde me before, that proportions are added, as fractions are multiplied) and then will it be. $\frac{9}{4}$: euen as you saied.

Master. Again *Euclide* saith in the twenteth proposition of thesame booke.

*If any nomber stande as a middle nomber in*

*proportion,*

*proportion, betwene other twoo nombers, thoſe twoo are like flattes.*

That is to ſaie: if any twoo nombers, beyng mulplied together, doe make a ſquare nomber (ſo̱ none but ſoche can haue a middle nomber betwene theim) then are thei *like flattes.*

As. 3. and. 12. multiplied together doe make. 36. whiche is a ſquare nomber: and. 6. therby appeareth to bee the middell nomber betwene theim. And therfo̱re are. 3. and. 12. *like flattes*

Likewaies. 3. and. 27. fo̱ thei make. 81. whiche is a ſquare: and their middle nomber is. 9.

And ſo are. 2. and. 8: 2. and. 18: 2. and. 50. 2. and. 72 3. and. 48: 3. and. 75: 4. and. 9. 4. and. 16: 4. and 25. 5. and. 20. 5. and. 45: 6. and. 24: 6. and. 54.

And ſo of infinite other.

This erpoſition is confirmed by the firſte and ſeconde pro̱poſition of the nineth boke of *Euclide*, where he ſaieth thus.

*If twoo nombers beyng like flattes, bee multiplied together, the nomber that thei make, ſhall be a ſquare nomber.*

*And if. 2. nombers beyng multiplied together, do make a ſquare nō ber, then are thei like flattes.*

By whiche rules it doeth appere, that you cā haue no pro̱greſſiō *Geometricalle*, but it muſt be made either of ſquare nombers, o̱ els of *like flattes*, wherby there appeareth a greate agreablenes, betwene *like flattes*, and ſquare nombers. And therfo̱re ſaieth *Euclide* alſo in the. 26. pro̱poſition of the eight booke.

*Nombers that bee like flattes, haue ſoche proportion together, as one ſquare nomber bea-*

reth to an other.

This maie you proue by any of the former examples. For. 12. to. 3. is in like proportion, as. 16. to. 4. or. 36. to. 9.

Also. 27. to. 3. hath like proportion as. 36. to. 4: or 144. to. 16. other. 81. to. 9.

And farther, if you deuide the one of theim by the other, the *quotiente* will be a square nomber.

Scholar. That doeth appeare euidentely at the firste bewe.

For. 12. diuided by. 3. doeth make. 4. And. 75. diuided by. 3. giueth. 25.

So. 54. by. 6. maketh. 9. And. 72. by. 2. yeldeth. 36. And so I see in the reste, that all the *quotientes* will be square nombers.

*how like flattes be made.*

But I desire moche to knowe, how those nombers be produced. For that I knowe not yet.

Master. Take any twoo square nombers, what so euer thei bee, and multiplie them by any one nomber, that you liste: and thei will make. 2. *like flattes.*

So. 4. and. 9. multiplied by. 2. doe make. 8. and. 18: whiche bee *like flattes.*

Again, if you multiplie them by. 5. thei make. 20. and. 45. whiche be also *like flattes.*

Scholar. I am perfect inough in this, if that be al.

Master. An other waie you maie make them also: If you take any twoo square nombers, that will admitte one diuisor, and diuide them bothe by it.

As for example. Seyng 9. and. 36. will be bothe diuided by. 3. I doe so diuide theim: and their *quotientes* are, 3. and. 12. whiche are *diametralle nombers.*

So in like maner, if I diuide 196 and 49 (whiche bothe are square nombers) by. 7. the *quotientes* will be 28. and. 7.

Again, 16. and. 100. beyng bothe square nombers

and

and diuided by. 4. doe make. 4. and. 25. as their *quotiente*, and thei be *like flattes*.

Scholar. And in these I see an other straunge worke: that if those twoo *like flattes* bee multiplied together: thei will make the greater square, of whiche thei came.

For. 3. tymes. 12. maketh. 36: and. 7. tymes. 28. giueth. 196: And so. 4. tymes. 25. bryngeth forthe. 100.

Master. It doeth so happen often times: but it is not alwaies so.

For if you diuide. 16. and. 100. by. 2. the *quotientes* will be. 8. and. 50. whiche twoo nombers multiplied together, doe make. 400. farre differyng from. 100. So. 36. and. 196. beyng bothe square nombers, and diuided by. 2. doe make. 18. and. 98. whiche be *like flattes*: and those *like flattes* multiplied together, doe yelde 1764. whiche is a square nomber, but it is. 9. tymes so greate as is. 196.

Scholar. Yet one doubte I haue: whether all square nombers be *like flattes*, and so bee not distincte from them?

For although in the diuision of figuralle nombers you did distincte them, yet in the examples of *like flattes*, you put certain square nombers emongest other.

Master All square nombers are *like flattes*, beyng compared together: and els not. For as any. 2. square nombers maie be compared together: so maie thei be referred to their rootes, without comparison together. Or els thei maie be compared to other nombers that bee not square.

Therfore marke these two rules well. that no one nomber can bee called a *like flatte*: but in comparison to some other. For. 2. by hymself is not called a *like flatte*, excepte he bee compared to. 8. or to. 18. other to 32. or. 50. or some other soche.

So likewaies. 4. whiche by nature is a square nō-

ber, and alwaies ſhall bee ſo: yet is it not accepted as a *like flatte*, onles it bee referred to ſome other ſquare nomber.

Scholar. What if it be compared with. 12. which you named before to be a *like flatte*?

Maſter. You remember: one of *Euclide* his rules (whiche I repeated before) is, that *like flattes* beeyng multiplied together, will make a ſquare nōber. And ſo doeth not. 12. beyng multiplied by. 4.

Scholar. Now I doe vnderſtande your woordes better. So. 3. and. 8. compared together, bee not *like flattes*: yet eche of them compared to other nombers, maie be *like flattes*. As. 3. compared to. 12. or to. 27: and 8. compared to. 18. or to. 50.

*Of rooted nombers.*

Maſter. Now will we lette theſe *like flattes* alone for a tyme: And intreate more of rooted nōbers. And firſt I will tell you ſomewhat of the names and natures of ſoche nombers as haue rootes: Then ſecondarily I will trache you the order to extract their rootes: And afterwarde will I ſhewe ſome parte of the vſe of theim.

*A roote.*

Wherfore to begin, where we lefte a litle before, the explicatiō of rootes: I ſaie, that the roote of nomber, is a nomber alſo: and is of ſoche ſorte, that by ſondrie multiplications of it, by it ſelf, or by the nomber reſultyng thereof, it doeth produce that nōber, whoſe rooe it is. And accordyng to the nomber of times that it is multiplied, the nomber that reſulteth thereof, taketh his name.

So that one multiplication maketh a *ſquare nomber* And twoo multiplications doe make a *Cubike nomber*.

Likewaies. 3. multiplications, doe giue *a ſquare of ſquares*. And. 4. multiplications doe yelde *a ſurſolide*.

And ſo infinitely.

For as multiplication hath no ende, ſo the nombers amountyng of them be innumerable, and their

rootes

rootes as infinite. But their names thei take certainly, of the nombers that thei doe make.

So the roote of a square nomber, is called a *Square roote*: and the roote of Cubike nomber, is named a *Cubike roote*: In like sorte that roote is called a *Squared square roote*, whiche maketh a square of squares in nōber. And that roote is a *Surfolide roote*, that yeldeth a *Surfolide nomber*: in whiche sorte of multiplication, you maie procede infinitely, as I saied.

*A square roote.*
*A cubike roote.*
*A squared square roote.*
*A surfolide roote.*

Notwithstandyng for your ease, I haue set foorthe here in a table, certain of the moste notable kindes of rooted nombers.

And to the intente you maie partly conceiue the reason of their names, I will after the table, set forth a brief explication of their names, with the protracture of the figures, that thei doe resemble in multiplications *Geometricalle*: where pointes, lines, platte formes, or soundformes bee multiplied: and bryngeforthe other formes agreable to soche multiplications.

But first marke the table well: And it will giue you greate lighte, and aptnesse to vnderstande all that foloweth, moche the better.

For examples are the lighte of teachyng.

The

# The table of rooted nombers.

| | The vulgare names. | | | | | | | | | | The authors names. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | Rootes. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Rootes. |
| 2. | Squares. | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | Squares. |
| 3. | Cubikes. | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 | Cubes. |
| 4. | Squares of Squares. | 1 | 16 | 81 | 256 | 625 | 1296 | 2401 | 4096 | 6561 | 10000 | Longe Cubes. |
| 5. | Surſolides. | 1 | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 | 100000 | Squares of cubes |
| 6. | Squares of cubes | 1 | 64 | 729 | 4096 | 15625 | 46656 | 117649 | 262144 | 531441 | 1000000 | Cubike Cubes. |
| 7. | Seconde Surſolides. | 1 | 128 | 2187 | 16384 | 78125 | 279936 | 823543 | 2097152 | 4782969 | 10000000 | Longe Cubike Cubes. |
| 8. | Squares of ſquared ſquares. | 1 | 256 | 6561 | 65536 | 390625 | 1679616 | 5764801 | 16777219 | 43046721 | 100000000 | Squares of Cubike Cubes. |
| 9. | Cubes of Cubes. | 1 | 512 | 19683 | 262144 | 1953125 | 10077696 | 40353607 | 134217728 | 387420489 | 1000000000 | Cubes of Cubike Cubes. |
| 10 | Squares of Surſolides. | 1 | 1024 | 59049 | 1048576 | 9765625 | 60466176 | 282475249 | 1073741824 | 3486784401 | 10000000000 | Longe Cubes of Cubike Cubes. |

Here you see diuerse rewes of nombers, and against euery rowe twoo names written: one on the right hande, and the other on the lefte hande, whiche serue for all the nombers in that rewe.

The names on the lefte hande bee those names, whiche bee commonly vsed, and attributed to those nombers.

The names on the righte hande, are names of my addition, whiche doe aptly expresse the very natures of the nombers, vnto whiche thei bee assigned: as anone I will declare.

And now concernyng the nombers, you see firste in the hedde of the table, a rewe of nombers set in order, as thei followe in common nombryng, from one forward. And thei bee called rootes, for that the multiplication of eche of them, by theimselfes, or by that, that thereof amounteth, bryngeth forthe all thother, that bee set vnder them. Of the whiche, the seconde rewe is called *Square nombers*: bicause that their length and their bredth (whiche I vnderstand by the .2. nombers of their multiplication) is equalle. *Square nombers.*

As .2. tymes .2. doeth make .4. whiche is a square nomber, and maie bee figured thus.



Likewaies .3. tymes .3. maketh .9. whiche is a square nomber, and is represented thus.

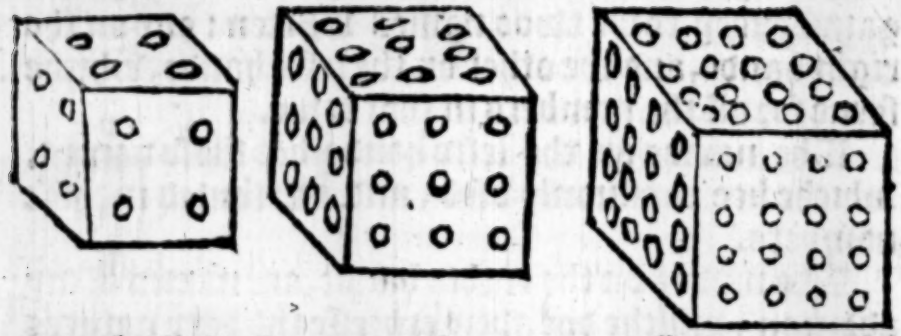


And here you se, thatif you diuide the *Square nomber* by his roote, the *quotiente* will be thesame nõber also.

Scholar. That must nedes be so.

Master. Then in the thirde rewe are placed *Cubike nombers*: whiche are produced by triple multiplication. As .2. tymes .2. twise, maketh .8. And .3. tymes .3. thrise, yeldeth .27. So .4. tymes .4. fower tymes, giueth .64. These nombers can not be expressed aptly in flatte, but prospectiuely, as Dice maie be made in protracture. *Cubike nombers.*

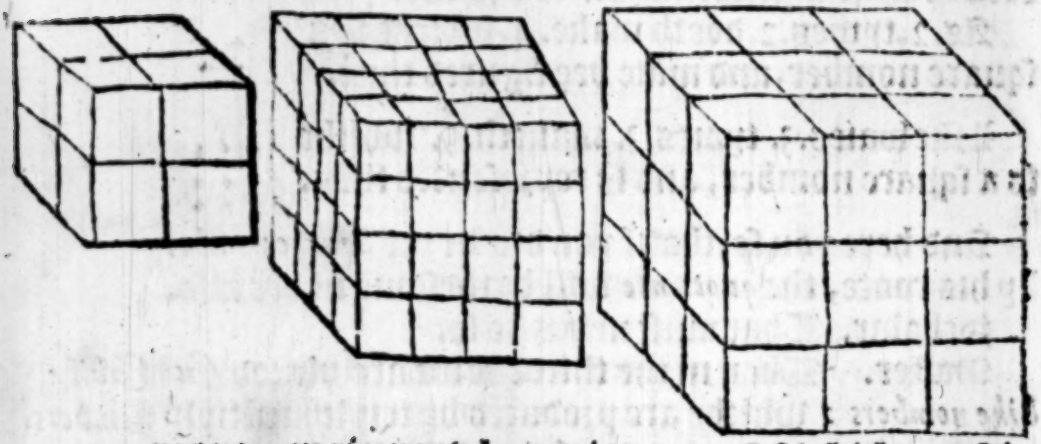
And these are their formes.



In the firste figure you see. 2. expressed in lengthe bredthe, and depthe. And in the second forme. 3. is representeed in all those. 3. dimensions. In the. 4. figure 4. is the roote, and is drawen agreably to that forme.

Scholar. This is manifeste inough to sighte.

Master. Yet reason ought to waigh it more exactly, then sight can comprehende it. For as their triple multiplication doeth resẽble the nature of sounde bodies, so it might appeare more iuste expressyng of their figures, agreably as sounde bodies ought: in whiche euery parte can not appeare to sighte, sith diuerse of them loke inwardly. As by these. 3. laste figures you maie partely coniecture. Of whiche at this tyme and in this place, some men will thinke it an ouersighte to speake, and moche more ouersighte to write of them any thyng largely. Saue that we maie vse theim for the apter explication of that triple mul-



plication,

tiplication, wherby thei be made.

So that as it is multiplied thrise, so the nomber that doeth amounte thereof, hath gotten. 3. dimensiones, whiche properly belongeth to a bodie, or sound forme. And therfore is it called a *Cube*, or *Cubike nomber* Whiche nomber if you diuide by the roote, the *quotient* wil be the square of thesame roote. As I said afore.

But to procede, if you doe multiplie that *Cubike nomber* by his roote, the nomber that riseth of it, is called a *Square of squares* commonly: bicause that not onely it is a *Square nomber*, but the roote of it also is a *Square nomber*. As you maie perceiue by examination, of all those nombers that be in the fourth rewe, whiche nombers I doe call *longe Cubes*: bicause thei make a line of Cubes. And hath in lengthe so many Cubes, as the firste roote doeth containe vnities. *Squares of Squares.* *Long Cubes.*

This line of Cubes, although it haue for his bredthe, and depthe also, the thickenesse of one Cube, yet bicause it hath no nomber of Cubes, in bredthe, nor in depthe (or generally no nomber of that thyng, whereof it is called a line) therfore maie it tollerably beare the similitude and name of a line. And so doe we commonly call lines, those smalle cordes, whiche are onely long, and haue litle bredthe to their length. But yet are thei not without all bredthe.

Scholar. And thereof (I thinke men call a line of Buckes, and a line of Assheles stones, when many bee laied in a rowe, in lengthe: and but one (or fewe) in bredthe.

Master. You saie truthe. And that name doeth continue still, emongest all our countrie menne: saue that moste menne doe not call it sharply a line, but more broder (after tholde Englishe language) a laine And so men vse to saie, a laine of wine buttes, and a laine of brode clothes: and soche other like.

And vse hath so largely applied this name, that it

maie seme no greate absurditie, to name any thynge a line or laine, that hath moche more lengthe then bredthe: and is made by often addition, or multiplication of any one quantitie. But yet for auoidyng of errour, it ought to bee limited, whereof that line is named. As in our mater to saie, *a line of vnities: a line of Cubes: a line of Cubike Cubes*: and *a line of Cubike Cubes Cubikely*: and so forthe.

In likewaies must we iudge of platte formes, that thei haue no depthe or thickenesse. When one nomber is multiplied by an other, onely twise: that is to saie, in bredthe and lengthe onely: and is not multiplied the thirde time by any nomber, to make it beare depthe.

And this must be considered generally, though the nomber so multiplied bee a *Cube*, or any other sounde nõber. For in soche case, that *Cube*, or sounde nomber, what so euer it be, standeth but as an vnitie.

Scholar. Sir, I doe very well vnderstande the meanyng, and reasonablenesse of those names, line, and square, in any thing. But I knowe not those termes, *Cubike Cubes*, and *Cubike Cubes Cubikely*. Although I se them set in the table, whiche you haue giuen me.

Maister. No more then doe you vnderstande diuerse other names there, whiche I will therfore declare vnto you.

If you agree to the vse of the name, of a line and a square, in that sorte that you haue consented vnto: then if I multiplie a Cubike nomber by his roote. As to saie. 8. by. 2. or. 27. by. 3. other. 64. by. 4. then shall I haue a line of *Cubes*. whiche I doe therfore call *longe Cubes*: but commonly thei bee called *Squared Squares*, or *Squares of Squares*: and of some men thei are named *Zenzizenzikes*, as square nombers are called *Zenzikes*. Whiche name although in sounde bodies, it hath no vse, yet in practice of sounde nombers, it

*Squares of squares.*

maie

maie and doeth expresse some properties aptly. As namely that all those nombers, whiche rise of 4 multiplications, maie be as well made by twoo multiplicatiõs. But then the roote of that multiplication shal be a square nomber also.

Scholar. So I understande that. 16. is a nomber of that sorte, which here is called *Square of squares*. And yet maie it bee called a square nomber: and is so in deede, in comparison to. 4. And therfore, I perceiue, it is set twise in the table: ones emongest square nombers, vnder 4 whiche then is his square roote: And again it is set emongest *squares of squares*, vnder 2 which in that place standeth as his squared square roote.

Likewaies. 6 4. is twise set in thesame table, ones emongest *squares*, vnder 8. whiche is his square roote: And again emongest *Cubike nombers*, vnder. 4. whiche is his Cubike roote.

Master. You saie truthe. Although the laste exãple be not to your purpose, concerning *Squared squares* or *Zenzizenzikes*. And if you did note it onely, forbicause it is twise set in the table: then maie you see it thrise sette in thesame table, for it is in the firte rewe vnder. 2.

Scholar. So I see, wherfore I might rather haue take. 8 1. whiche is a *Zenzizenzike nomber*, and so hath for his roote. 3: And also it is a square nomber, and hath. 9. for his roote.

Master. Farther to procede, if I multiplie those *squares of squares* by their roote, thei will make *Surfolide nombers*. *Surfolides.*

Scholar. I perceiue by the nombers in the table, that you meane the leaste roote of the twoo: bicause vnder. 16. I see. 32. in the rewe of *Surfolides*.

Master. Reason maie driue you to thinke so. For the nomber and his roote, muste beare alwaies one name. So that if I name. 16. as a square nomber, I

muſt referre it to his ſquare roote. And if I name it as a *Zenzizenzike nomber*: it muſte bee referred to his *Zẽzizenzike* roote. And in like ſort of al other names.

As when I call. 64. a ſquare nomber, & demaunde what is his roote: you muſte nedes aunſwere by his *Square roote*, whiche is. 8. But if I name. 64. as a *Cube*, and doe then feke for his roote: you muſt vnderſtande his *Cubike roote*, and that is. 4. But if I name it to bee a *Square of Cubes*, or *zenzicube*: then is. 2. his *roote*. As you maie by the table perceiue. And alſo by the orderly multiplication of euery rewe, or order of nombers by their roote. For therby amounteth the nexte rewe.

And ſo maie you increaſe the nombers of thoſe rewes, or orders, accordyng to the tymes of your multiplicatiõ, as moche as you liſt. And euery order ſhall beare ſoche names, as agreeth to the nature of their rootes.

Wherfore thei appeare to bee ouerſene, that call thoſe formers nombers *Surdeſolides*, ſeing thei are not any waies *Surde nombers*, but haue their rootes. And yet, to confeſſe the truthe, I cannot well tell you the true *etymologie* of their name: except thei be ſo named, as it were *ſolide* vpon *ſolide*. And that interpretation were to ſtreightly racked. But the name beyng receiued and well knowen, wee maie more eaſily with libertie vſe it, then with ſcrupuloſitie, curiouſly ſeã it.

Theſe nombers are ſimple nombers in their kind. For thei riſe of. 5. multiplications. And if their roote bee a digite nomber, then is it theſame nomber, that ſtandeth in their firſte place. And if their roote be an article, then hath that *Surſolide*. 5. tymes ſo many Cyphers together in the firſte places, as his roote hath: and the nexte figure after thoſe Cyphers, is the firſte figure ſignificatiue of his roote.

Scholar. I ſee it ſo in all theſe nombers, that bee

in

in the table.

Master. And so shall you finde it in all others.

And farther if the roote bee a nomber mixte, then the firste nomber of the *Surſolide*, is the first nomber of the roote. And this I doe tell you for some helpe, in gessyng at their rootes.

This name therfore of theim, I meane *Surſolides*, in *Arithmetike*, maie serue to admonishe you of their roote. But in *Geometrie*, or in composition of sounde bodies, it serueth to no vse: and therfore I doe call thē agreable to their figure, *Squares of Cubes*: bicause thei make a square forme: but so that euery vnitie of that square, is in it self a *Cube*: As by the figures that followe, you maie well coniecture. *Squares of Cubes.*

And also thei are made by multiplication of a *Cubike nomber*, and a *Square nomber* together, bothe hauyng one roote: and the *Surſolide* hauyng thesame roote. Wherfore reason with the nature of their founde figure, inforceth me to call thē *ſquares of cubes*.

Yet other menne attendyng more to the nature of their rootes, then to their owne formes and nature, doe giue that name to the nexte rewe of nombers, bicause thei maie be made of multiplication, of any *Cubike nomber* by it self, that is to saie squarely.

Scholar. It is so. For. 8. whiche is a *Cubike nomber* multiplied squarely maketh. 64. And that. 64. is set emongeste the *Squares of Cubes*.

Master. And this commoditie commeth by that name: that it putteth menne in remembraunce of the spedie and easie extraction of their roote: As you shall learne hereafter.

But I consideryng their owne nature and makynge, as sounde nombers or bodies: doe call theim *Cubes of Cubes*, or *Cubike Cubes*.

After these nombers in the seuenth rewe, there do followe those nombers, whiche commonly are called

*Surſolides*,

*Seconde surfolides.*

*bsurfolides*, or *bissurfolides*, that is, *seconde surfolides*, or *double surfolides*. But I maie call them *seconde squares of cubes*, alludyng at the same name. Howbeit if I looke to their forme and nature, I shall be inforced to call thē, *longe cubes of cubes*, or *longe cubike cubes*.

And so by like reason, doe I cal the nexte nombers *square cubes of cubes*, or *square cubike cubes*: whiche other men doe cal *zenzizenzizenzikes*, that is *squares of squared squares*.

*Squares of squared squres.*

*Cubes of Cubes.*

The nineth rewe of nombers, is commonly called *Cubike Cubes*, or *Cubes of Cubes*: bicause the *Cubike* rootes of those nombers are *Cubike* nombers also. But I after their true nature, doe call them *Cubes of Cubes Cubikely*: or *Cubes* of *Cubike Cubes*.

*Squares of Surfolides.*

The tenth rewe of nombers is named vulgarely, *Squares of surfolides*, bicause thei haue a Square roote, whiche is of it self a *surfolide nomber*. And for their figure *Grometricalle*, I name thē *long cubes of cubike cubes*.

So that I considryng their nature, that thei be figuralle nombers, am constrained to name theim, accordyng to their figure, I meane in this place, where I doe make explication of their natures and names.

But other men for aide of woorke, in extraction of rootes, haue giuen theim sothe names, as maie beste put menne in remembraunce of redy woorke therein. Whiche names I will vse also hereafter, in my writynges, bicause I will not bee an author of vnnedefull singularitie. And yet bicause truthe in nature is as well to be regarded, as ease in woorkyng, and rather more, I could not omitte in this place, the declaration of their true nature and very formes.

And so bothe of vs hauyng good reasons, for those names, neither maie contempne other, neither contende together.

*A generalle reason for na*

And although the names that I doe giue, maie seme to some menne (whiche are scarse apte iudges)

more

more odiouse, for the newe inuention (as thei maie thinke) then needfull to the practise of thartc: yet shal you see in theim a naturall sequele, and orderly propagation. *mes of these nombers.*

For all those nombers are considered, in one of. 2. formes firste. That is to saie, other thei bee taken as nombers absolute, without any cõsideration of multiplication. And so thei maie be named nombers onely, without name of relation. Or els thei bee considered as nombers multiplied, and that can be but in. 3. varieties.

If thei be multiplied but ones, then doe thei make a line of nombers, or a liniarte nomber. And that nõber hath onely lengthe, without bredthe, or depthe: And therfore maie be the roote to a *Square*,, or a *Cube*. But is of it self, in that consideration, nother *Square* nor *Cube*.

Secondarily, it maie bee multiplied twise, the one nomber stãdyng for the lengthe, and the other for the bredthe: and so is it a *Square nomber*, and therfore a *flat nomber*.

Thirdly, it maie bee multiplied thrise, and therby gette lengthe, bredthe, and depthe: wherby it is made a *founde nomber*. And bicause the sides bee equalle, it is specially a *Cube* or *Cubike nomber*.

Now can there be no folwerth waie, that any multiplication maie increase: for there are no more dimẽtions in nature.

But if any manne doe multiple the fourthe tyme, then must he accoumpte that he maketh a *line of Cubes*: and the fifth multiplication maketh a *Square*, in whiche euery vnitie is a *Cube*: So the firte multiplication maketh a *Cube* of *Cubes*, accoumptyng euery lesser *Cube* for an vnitie. And there is a staie again.

Wherfore if any man multiplie the seuenth time, he retourneth againe to the firste nature of nombers

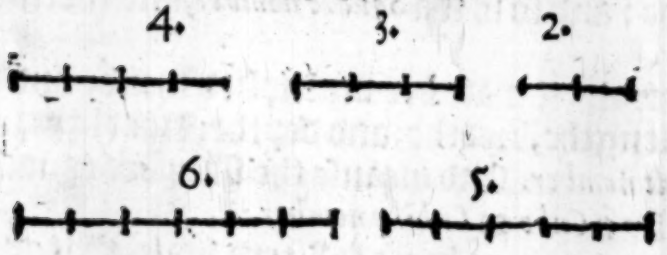
multiplied, whiche are *liniarie nombers*: And the 8. multiplication, wooꝛketh as the ſeconde did, and maketh *flatte nombers*. The nineth multiplication agreably with the thirde, doeth make *Cubes*.

And ſo infinitely theſe. 3. wooꝛkes maie bee reiterate, but a fourthe foꝛme can neuer be deuiſed.

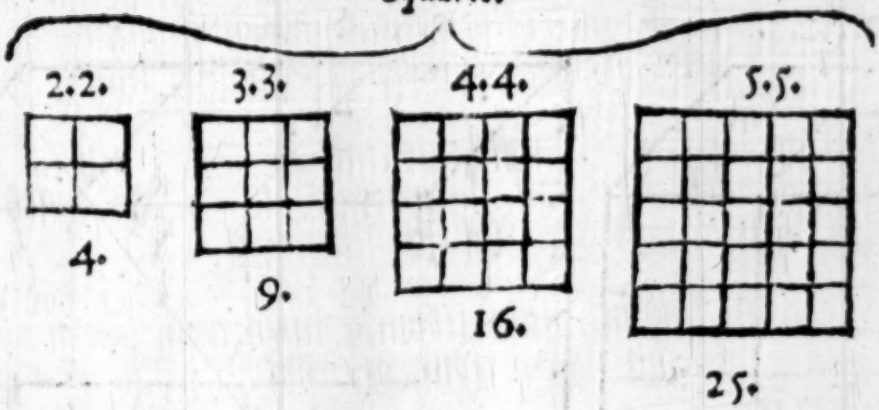
And thereforꝛe doe I, as reaſon doeth compell me, reduce all nombers to thoſe. 3. foꝛmes, as their verie oꝛiginalle ſpꝛynges and fountaines.

But to the intente that you maie the moꝛe aptly iudge of theim, and their natures, I haue here ſette fooꝛthe the foꝛmes, whiche thei make in figures *Geometricalle*, oꝛ ſounde quantities. Admoniſhyng you to remember this well. That after any nomber is become a ſounde nomber, it is againſt reaſon, to reduce him to an abſolute flatte nomber again, and moſte of all by multiplication. But nowe marke theſe figures.

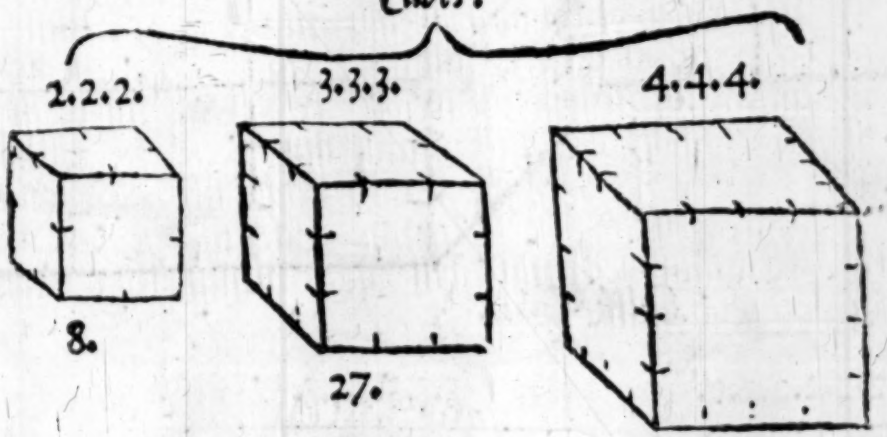
Rootes, oꝛ Lines.



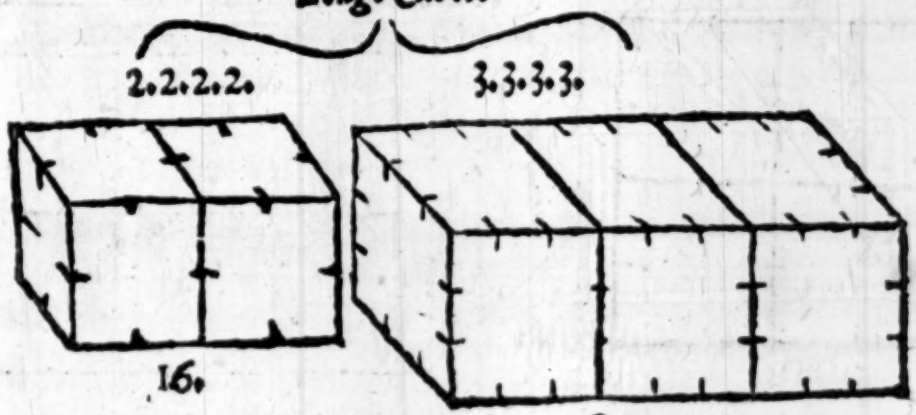
Squares.




Cubes.

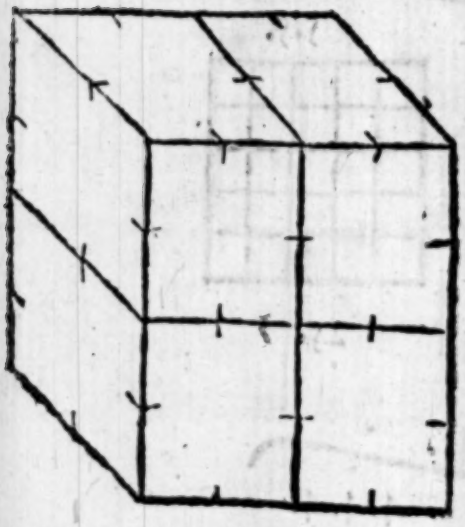



Longe Cubes.

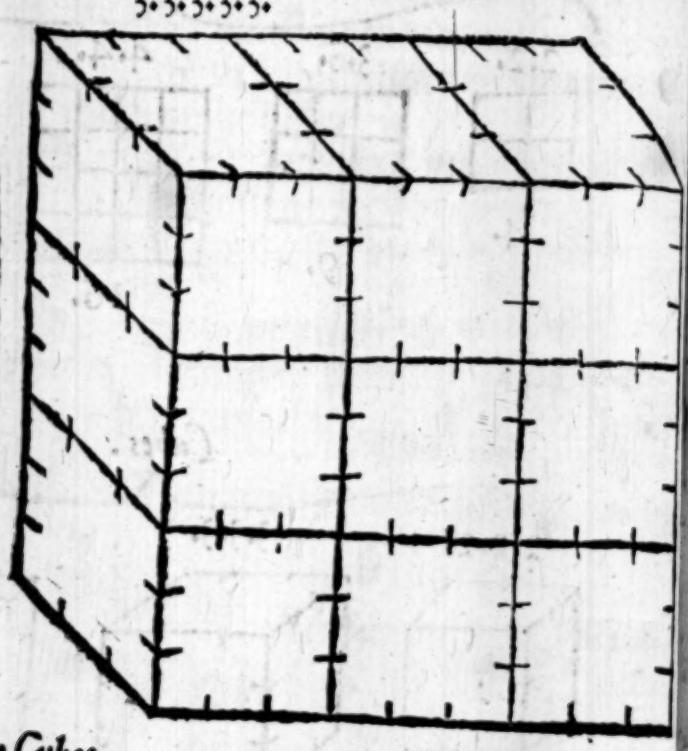


*Squares of Cubes.*

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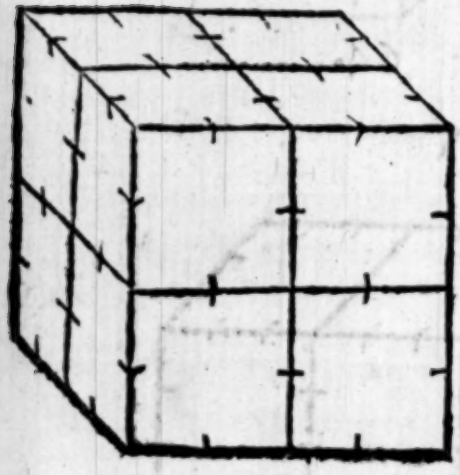


3.3.3.3.3.3.

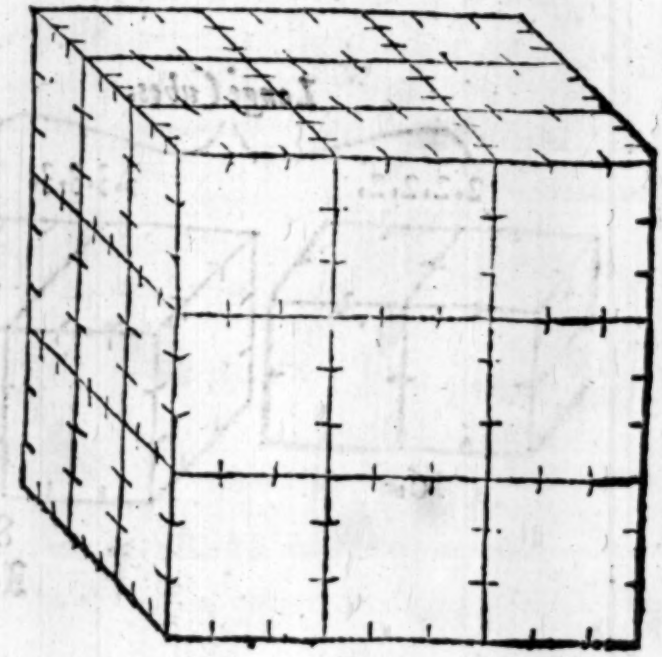


*Cubike Cubes.*

2.2.2.2.2.2.2.2.



3.3.3.3.3.3.3.3.



Here

Here, as you see, I haue set first certaine lines, containyng soche partes as thei bee made of by multiplication: that is to saie, 2. 3. 4. or. 5. And these bee produced by the firste multiplication, where an vnitie of any thyng is multiplied by a nomber.

And so an inche multiplied by. 3. maketh. 3. inches: And a foote multiplied by. 6. maketh. 6. foote: and so of other measures and quantities, in like sorte. All whiche multiplications, doe make onely longe lines, or measures in lengthe onely, without bredth or thickenesse.

And in this multiplication, nother the nomber, nother yet the vnitie, is accoumpted or called a roote. But the line that is made therby: maie bee a roote to any of all the other kinde of nombers before rehersed, and sette forthe in the table. For if you multiplie the same line, by the nomber that his lengthe doeth include, then there will be made thereof, by this seconde multiplication, a square figure, containyng a square nomber in it: As you see emongest those figures, the firste fower to be, whiche are marked with these nombers. 4. 9. 16. and. 25.

Scholar. I perceiue well in eche of thē, that their lengthe is agreable with their bredthe, and so thei make square figures, but I knowe not what those nombers doe meane, that be set ouer their heddes.

Master. The quantitie of the nomber, doeth betoken the valewe of their roote. And the multitude of thesame nomber repeted, doeth declare the nomber of multiplications, for eche figure.

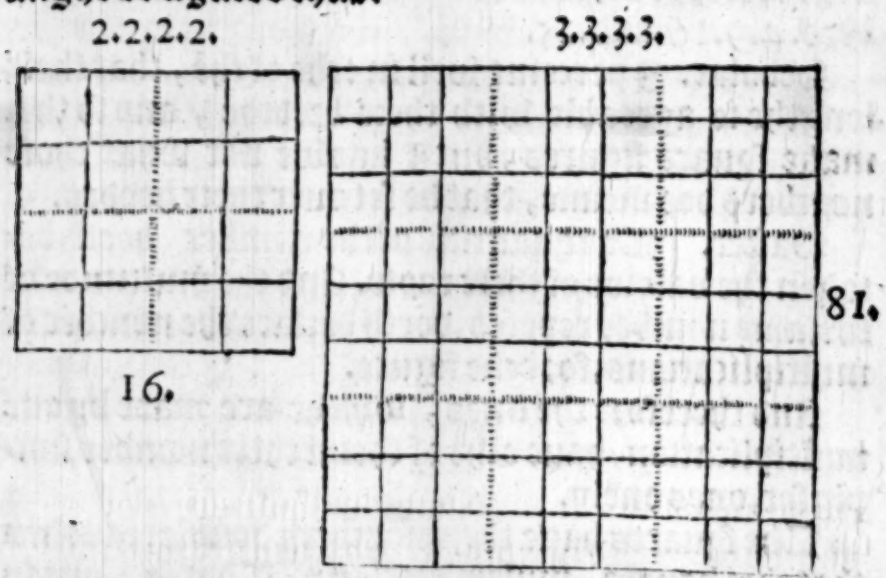
And therefore the lines, whiche are made by one multiplication, haue eche of theim their nomber simply set, ones onely.

The squares haue their nombers double: in token that thei haue. 2. multiplications. That is, one in lengthe, and an other in bredthe.

The third formes, whiche be *Cubes*, and are made of. 3. multiplications, haue their roote repeted thrise. And the like nombers did I ſette, in the ſide of the former table, againſt the like quãtities. Whiche ſhall helpe you ſomewhat in the extraction of rootes.

Scholar. Now doe I perceiue not onely their names, and multiplications, moche better then I did before: but alſo I vnderſtande better the difference of your names, and their reaſons. For by thoſe figures, whiche you haue ſet in the fowerth place, and doe call theim *longe Cubes*, I ſee their forme doeth agree to that name. For thei are longer, then thei are other brode or depe. And ſaue for their depthe, I might liken theim to *longe Squares* in *Geometrie*. Howbeit, other men neglectyng their forme, and lookyng onely to their rootes, doe call them, *Squared ſquares*.

But if you will permitte me, to ſpeake in the defence of theim, as a ſimple ſcholar maie ſpeake for affection, in the defence of his maſter, it appereth to me, that thei maie well bee called squared ſquares: and might be figured thus.




Where the ſmalleſt ſquares, whiche be contained within the pricked lines, beyng taken as rootes, and multiplied

multiplied by thesame nomber again, whiche thei do containe (other els twise by their rootes) will make the whole greater squares.

And by this figuryng of theim, there doeth appere no inconuenience nor absurditie, in their vulgare names: but rather a iuste expressyng of their naturalle formes.

For in the first figure. 2. standyng as the side of the lesser square, and multiplied by it self, doeth make. 4. whiche is the quantitie of the lesser square. Then if I multiply that lesser square. 4: by his owne nomber, it maketh 16. whiche is the greate and whole square: and is a *Square of squares*.

So in the seconde figure. 3. standeth for the roote of the lesser square, contained within the pricked lines, and if it bee multiplied by it self, it maketh. 9. whiche is the quantitie of thesame lesser square. Then if I multiplie that. 9. by it self, it will make. 81. whiche is the quantitie of the greate Square, and is a *Square of squares*.

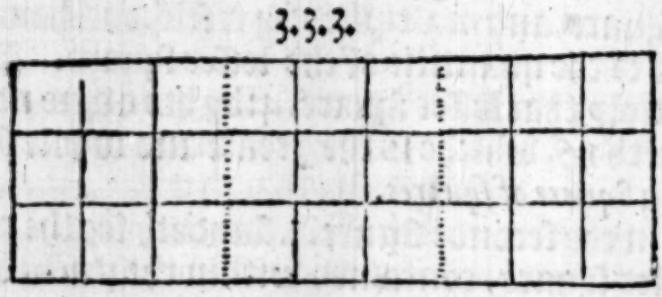
Master. I commende you well: not onely for so diligente excusyng of theim, whiche for their honeste trauell, deserue moche thankes, but also for that you seke to bryng manifest reason, and some shewe, at the least, of linearie demonstration for your purpose. So that you will not seme to speake, without some good grounde.

But as in deede, your figure doeth truely expresse a square of squares, so it doeth suppose the other nomber, whiche by order of multiplication, doeth go nert before it, to be a flatte nomber also. For it is not possible that a sounde nomber (as a *Cube* is alwaies) beyng multiplied by any other nomber, maie lese the nature of a sounde nomber: But shall continue a sounde nomber still. And therfore seeyng the nerte nomber, before a *Square of squares* was a *Cube*, it is not possible

poſſible that a *Square of ſquares* can be a mere *flatte nomber*, as you haue drawen it.

Wherfore if thei had intended, that a *flatte nomber* ſhould occupie the .4. place, then ſhould thei haue ſet ſome *plat forme* in the third place alſo. Whiche might haue been made in this ſorte.

And then will it be a *longe Square*, and not a *Cube*.




But in as moche as thei doe not admitte this *longe Square* (whiche by that name hath no roote) therfore mate not the nomber that foloweth it, bee any other then a *ſounde nomber*. For euery *Cubike* forme, beeyng multiplied by his roote, doeth make a *Square piller*. Whoſe length beareth vnto his bredth theſame proporcion, that his roote doeth vnto an vnitie.

Scholar. I am very well ſatiſfied now: cõcerning the names and formes of thoſe nombers. And by this that you haue ſaied, I doe farther perceiue, that .5. multiplications doeth make the *ſquare of Cubes*, whiche be ſet in the fifte place, emongeſte the former figures. And alſo I vnderſtande by the former table, that thei be called *Surſolides*.

Likewaies I ſee in the ſirte place of the forſaied figures, *Cubike Cubes*, made by .6. multiplications. But commonly the nombers of thoſe quantities, be named *Squares of Cubes*. So that for their names, thus farre I am perfecte inough.

The

# The extraction of Rootes.

## Maister.



OWe will I shewe you, how you shal extract the roote out of any soche nomber. *The extraction of rootes*

And first I must admonishe you, that you shal alwaies vnderstande, soche a roote, as the nomber doeth admit. So that in a square nomber, you shall seke a *Square roote* onely, and no *Cubike roote*, nother any other kinde.

Likewaies a *Cubike nomber* hath no other roote, but a *Cubike roote*. Excepte the namebee compounde, as *Zenzicubike*, or *Squared Cube*. For in soche there are 2. sortes of rootes, accordyng to the 2. names that thei beare. That is bothe *Square* and *Cubike roote*: as I will anon shewe you. But firste I will beginne with Square nombers, and their rootes. And this generalle order muste you obserue, before all other: That you shall haue by harte, in readie memorie all soche nombers, whose rootes are digites. For as it is superfluous to seke rules for theim, so must thei helpe in all greater nombers, whose rootes are aboue 9. And for your ease in remembraunce, I haue here sette foorthe a table for square nombers. Where in the firste columpne, you se the rootes set, and in the seconde piller, right against eche roote, there is set his square. Touchyng whiche I nede to saie no more, but that you be not in any vncertaintie of them, whē

*The table of Squarerootes vncompounde.*

| Rootes. | Squares. |
|---|---|
| 1. | 1. |
| 2. | 4. |
| 3. | 9. |
| 4. | 16. |
| 5. | 25. |
| 6. | 36. |
| 7. | 49. |
| 8. | 64. |
| 9. | 81. |

you shall nede their aied, whiche shall be continually in vse of searchyng for other greater rootes.

Now for greater nombers, this is the order.

1. First set doune the nomber as it is. Then sette a pricke vnder euery odde place, I meane the firste, the thirde, the fifte, the seuenth, and so forthe: and so shall euery pricke haue. 2. nombers, excepte the laste, whiche some tymes hath but one.

2. Secondarily, marke the nombers that belong vnto the laste pricke, toward the lefte hande: And whether he haue belongyng to it one nomber, or twoo, looke what the roote maie be of that nomber, if it bee square. And that roote sette by a croked line, as you place the *quotiente* in diuision: & cancell all that square nomber, belongyng to that pricke.

3. But and if the nomber belongyng to that pricke, bee not a Square nomber, then take the roote of the greateste square, whiche is contained in it, and place the roote as I saied before. And the square of it shall you abate from the nomber, that belongeth to that laste pricke, and let the rest be set ouer those nombers cancelled, as you doe in diuision. And so haue you ended your worke for that pricke.

Scholar. This moche is easie inough, if I vnderstande you rightly.

Master. Then proue it in a nomber, or twoo. And first worke with this nomber. 5152900.

Scholar. I muste marke euery odde place with a pricke, thus.

5152900.

And here I perceiue that vnto the first pricke, there belongeth 2 Cyphers onely, and to eche of the other. 2. prickes folowyng, there are appointed. 2. figures. But the fourthe pricke hath but one nomber, and that is. 5.

Now according to the second rule, I seke the roote of 5. (for bicause there belongeth no more nombers to

that

that pricke) and I see, it is no square nomber. Wherfore accordyng to the thirde rule, I take the greatest square in it, whiche is. 4. and the roote of. 4. is. 2. Therfore I doe substracte. 4. out of. 5. and cancell that. 5. and the. 1. that remaineth, I set ouer. 5. as here you see.

| | |
|---|---|
| 1 | |
| 5 | 152900(2. |

And the roote. 2. I sette behinde the *quotiente* line, as you taught me, and then the nõbers stand, as you se.

Master. Pou haue doen wel. Proue again in this nomber. 18766224.

Scholar. First I set theim doune and pricke theim, as here doeth appeare. And now I see, that the laste pricke hath twoo nombers belongyng to it, that is. 18. with whiche I must begin. And seyng it is no square nomber, I find 16. to be the greatest square in it: wherfore I subtract 16. out of. 18. and set. 2. ouer the. 8. And the roote of. 16. whiche is. 4. I sette behinde the *quotiente* line, as here is seen.

| | |
|---|---|
| | 18766224. |
| 2 | |
| 18766224(4. | |

Master. This maie suffice for the first woorke.

Now to procede, you shall double your roote, and put that double vnder the nexte space, towarde the right hand, that is behinde the nexte pricke. Alwaies forseyng, that if the double doe contain more figures then one, that the first shall be sette vnder that place, and the seconde vnder the nexte figure, towarde the lefte hande. 4.

Then seke a *quotiente*, as you doe in diuision, whiche shall shewe how often that double nomber maie be found in that, that is ouer it, appertainyng to that place: whiche *quotiente*, you shall set before the firste roote, within the *quotiente* line. 5.

But this regarde muste you haue here specially, that you maie leaue ouer the nexte pricke, toward the right hande, as moche as the square of that *quotiente*,

with which you worke, for out of that reſt, the ſquare of that *quotiente* muſte bee abated. And then make bothe ſubtractions, and note the remainer, if any be, and place your *quotient*, and then haue you doen with that pricke alſo.

For the more plaines, I will giue you an example in your firſte nomber, whiche ſtoode thus, after your worke was ended.

Here I ſee ouer the laſte pricke ſaue one. 11 5. vnder the middell figure of whiche I muſt ſet the double of the former roote. 2. that is. 4. And then I ſeke how often. 4. is to bee founde in. 11. And I finde that I maie haue it twoo tymes, and. 3. remainyng. Whiche. 3. with. 5. ouer the nerte pricke, doe make. 35. and that is more then the ſquare of my *quotiente*. 2. Therfore am I bolde to ſette doune that *quotiente*: And accordyng to it, to abate twiſe. 4. (whiche is. 8.) out of 11. and there reſteth. 3. Therfore I cancell. 11. and ſette. 3. ouer it. Then doe I multiplie the laſte *quotiente* ſquarely: and it maketh. 4. whiche 4. I ſubtracte out of the nomber ouer the pricke, that is. 35. where. 5. maie ſuffice for this nomber. Therfore I abate. 4. out of. 5. and cancell that. 5. and ſet. 1. whiche remaineth, ouer the. 5: And then will the whole nomber ſtande thus.

This worke, whiche I haue wrought now, muſt be repeted as often as there bee any prickes, or pricked nombers remainyng. Wherby you maie eaſily geſſe, that it muſt bee twiſe more repeated in this example, bicauſe there reſteth yet. 2. prickes vntouched.

Scholar. Although I thinke, I could doe, as I haue marked you to doe, yet for more certaintie I praie

| | |
|---|---|
| 1 | |
| ⨍ 1 5 2 9 0 0 | (2. |

| | |
|---|---|
| 13 | |
| ⨍ 1 5 2 9 0 0 | (22. |
| 4 | |

| | |
|---|---|
| 131 | |
| ⨍ 1 5 2 9 0 0 | (22. |
| 4 | |

praie you worke out this example.

Maister. Then marke it well.

I shall begin againe with doublyng of all, that is within the *quotiente* line. And that double is 44. whiche I must set vnder. 312. that remaineth of the laste worke. And then will the nombers stande, as here you see.

Then I loke how often tymes maie I finde. 44. in. 312. And I see it will be abated 7 times, and 4 remain: whiche 4 with the. 9. ouer the nert pricke doeth make. 49. And that will suffice to extracte the square of my *quotiente*. 7. For. 7. tymes. 7. maketh iuste. 49. Thus seyng I maie take. 7. for my *quotiente*, I worke with it, as the rule teacheth: abating first. 7. times. 44. (that is. 308) out of. 312. and there resteth. 4. ouer the space before the nerte pricke. Whiche. 4. with. 9. ouer the pricke doe make. 49. out of whiche I abate the square of my *quotiente*. 7. (that is. 49.) and so resteth nothyng, but. 2. Cyphers. And the nomber standeth thus.

| |
|---|
| 131 |
| 5 1 5 2 9 0 0 (22. |
| 44 |

| |
|---|
| 1314 |
| 5 1 5 2 9 0 0 (227 |
| 44. |

And seyng there remaineth one pricke vntouched, I should repeate thesame order of worke againe, by doublyng all the *quotiente*, whiche would bee. 454. and settyng it so that. 4. whiche is in the firste place, should be sette vnder the Cypher, that is without the pricke, and the other figures in order, toward the left hand. But all this worke were in vaine, seyng there is nothyng lefte, to serue for the subtraction.

Yet bicause there is lefte one pricked place vntouched, I must set for it a Cypher in the *quotiente*.

For this rule is generalle: that how many prickes so euer your square nomber doeth containe, your *quotiente*, or roote shall haue so many nombers. Wherfore this roote must be made vp thus. 2270.

*The proofe.* And so it appeareth that your nomber. 5152900. is a iuste square nomber. Which you maie proue by the orderly proofe of extraction of rootes. That is to multiplie that *quotiente*, or roote (which you haue founde) by it self. And if it doe make the first nomber exactly, then haue you wrought well.

Scholar. That proofe is as certaine, as can be. And therfore I will proue, whether it will agree with this worke. Wherfore multiplyng 2270. by it self, I see that it yeldeth the firste somme. As here it doeth appeare. So is this worke approued good.

| |
|---|
| 2270. |
| 2270. |
| 158900. |
| 454 |
| 454 |
| 5152900 |

And now will I attempte the like worke in the seconde example. Which was. 18766224.

But after the firste worke was ended, and the greatest square subtracted out of 18. it did remain in this forme.

| |
|---|
| 2 |
| 1 8 7 6 6 2 2 4 (4. |

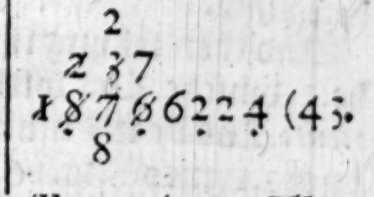
Now to continue the worke as you did, and as the rule doeth teache, I must double. 4. which is the roote, and standeth by the *quotiente* line: and must set it vnder. 7. that standeth in the space, betwene the laste pricke (whose worke is ended) and the nexte pricke towarde the right hande. And then will it stande thus as you see.

| |
|---|
| 2 |
| 1 8 7 6 6 2 2 4 (4. |
| 8 |

That doen, I must seke a *quotiente*, that maie declare how often 8. maie bee subtracted out of. 27. and that *quotiente* I finde to be. 3: bicause that after I haue taken. 3. tymes 8. (that is. 24. out of. 27. there will remain. 3. which 3. with. 6. that standeth ouer the pricke, doe make. 36. And I see that nomber to bee greate inough, for the abatemente of the square of my *quotiente*: which is but. 3. tymes. 3. that is. 9.

Wherfore

Wherfore I sette doune. 3. for my *quotiente*, before. 4. in the *quotiente* line. And multiplying 8. by that 3. there riseth 24. whiche I doe subtract out of. 27. that is ouer. 8. and there will remain. 3. That 3. with. 6. ouer the pricke, maketh. 36. out of whiche I must abate. 9: whiche is the square of my *quotient*. 3. and so will there reste. 27. ouer that pricke.

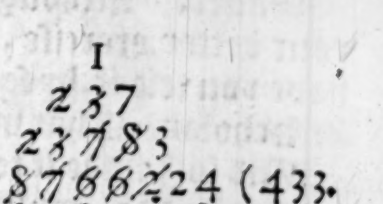


And thus haue I ended. 2. prickes, and yet. 2. more doe remain: in whiche bothe I must repeate the same forme of worke.

Therfore I double the whole *quotiente*, and it maketh. 86: whiche I set vnder. 276.

And then I seke the *quotiente*, declaryng how many tymes. 86. maie be abated out of. 276. whiche maie be. 3. tymes. And for that cause I set. 3. in the *quotiente* before the. 43.

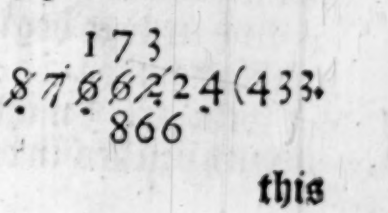
Then doe I firste multiplie. 86. by that. 3. saiyng. 3. tymes. 8. maketh. 24. which I abate out of. 27. and there resteth. 3. And again I saie, 3. tymes. 6. is. 18. whiche I abate out of. 36. and there doeth remain. 18.



That doen, I take the square of my *quotiente*, that is. 9. whiche I doe subtract out of. 12. (for the. 2. ouer the pricke must borowe. 1. of. 8.) and then will there remain ouer that pricke. 173.

And thus is that pricke ended.

Now, for the laste pricke in worke, though he be firste in place. The double of my *quotiente* is. 866. whiche I muste sette vnder 1732. As here is doen, where I leaue out many cancelled figures, as superfluouse in this



this place.

And then sekyng for a newe *quotiente*, I finde it to be. 2. whiche I set with the other nombers in the *quotiente*. And by it I multiplie and subtract the 866. saiyng: 2. tymes. 8. is. 16. whiche I abate out of. 17. and there resteth. 1. Again. 2. times 6 is. 12 that I subtract out of. 13. and there remaineth. 1. Thirdly, I saie. 2. times. 6. giueth. 12. whiche I abate from. 12. and there is left nothyng. Saue that ouer the pricke there standeth 4 whiche is equall with the square of my *quotiēt*.

| |
|---|
| 1 1 |
| 1 7 3 |
| 1 8 7 6 6 2 2 4 (4332. |
| 8 6 6 |

Wherfore abatyng the square of my *quotiente* out of it, there resteth nothyng at all.

And therby I see that. 18766224. is a iuste *square nomber*. And his roote is. 4332.

*The proofe.* Maister. Although I knowe it to bee so, yet for your better exercise, and full perswasion: I would haue you trie it, by square multiplication.

Scholar. That maie I sone doe.

And so I finde it to be true.

For. 4332. multiplied by it self, doeth make. 18766224. As this woorke here set, doeth shewe.

| |
|---|
| 4332. |
| 4332. |
| 8664. |
| 12996. |
| 12996. |
| 17328. |
| 18766224. |

Maister. Yet bicause some other small doubtes, maie happen in woorking, that maie trouble a yong practiser, I will propounde to you one or twoo examples more. Wherein you shall finde some varietie, as well in the nomber propounded, as also in the *quotiente*.

And firste to begin, I will you to extract the roote of this nomber. 22071204.

Scholar. I must set doune the nomber, and note it with prickes in euery odde place: For that rule I perceiue

perceiue neuer faileth.

Master. No moredoeth any of the other, although the woorke maie varie in some smalle poinctes: whiche yet maie be greate inough to trouble a young learner.

| 22.07.12.04.(

Scholar. Then accordyng to the firste rule, I seke out the greatest square in. 22. (for I see it is no square nomber it self) and it appereth to be 16. And his roote 4. wherfore I doe sette doune. 4. in the *quotiente*, and then I doe abate. 16. out of. 22. and the remainer is. 6. whiche I sette ouer the pricke, and cancell the. 22. as here is seen.

| 6
| 22.07.12.04.(4.

Now goyng on with the nexte pricke, I shall double the former roote in the *quotiente*, and sette it vnder the Cypher, betweene the. 2. prickes.

Then do I seke how oftē that 8 (whiche is the double of the *quotiente*) maie be found in 60 and I finde it to be 7 times, and 4 remainyng to be set ouer the Cypher. So that for the pricke there remaineth. 47. out of whiche I should abate the square of my *quotiēt*. But seing that. 49 (whiche is the square of 7) can not be taken out of. 47. there is a newe *quotiente* to be sought.

Therfore I take 6. And see that it will serue. So I set. 6. in the *quotiente*: and by it I multiplie 8 whereof commeth 48 That. 48. abated out of. 60. leaueth. 12. Therefore I cancell the 60. and set. 12. ouer it.

| 19
| 621
| 22.07.12.04.(46.
| 8

Then doe I multiplie the *quotiente*. 6. by it selfe: whereof riseth. 36. And that abated out of. 127. leaueth. 91. And so haue I ended the seconde woorke.

Now for the thirde woorke, I double. 46. and it doeth yelde. 92. to bee sette vnder. 911. as I haue put it here.

And then seking for a *quotient*: I se that I maie take

9. Wherfore I set that 9 in the *quotiente* with. 46. and by it I multiply 92 and subtract that, that riseth, in this forme.

Nine tymes. 9. maketh. 81. whiche I abate out of. 91. and there resteth 10. Then 9 tymes 2 giueth 18. whiche I must abate out of. 10. and there will remain. 83.

```
      7
    18
  1905
 62131
22071204 (469.
   92
```

And now muste I multiple that laste *quotiente*. 9. squarely, wherby will amounte. 81. that shall I subtract out of. 832. and there will remain. 751. and so that pricke with his woorke is ended.

Therefore procedyng to the fourthe pricke, I double all the *quotiente*, whiche will be 938. And I set it vnder 7510.

Then doe I seke a newe *quotiente*, whiche I finde to bee. 8. For 8. times. 9. giueth. 72. whiche I abate out of. 75. and there remaineth. 3. Again. 8. tymes. 3. is. 24. and that I deducte out of. 31. and so resteth. 7. Then saie I. 8. times. 8. is. 64. whiche beeyng subtracted from. 70. doeth leaue. 6. And that. 6. with the 4. ouer the pricke maketh. 64. out of whiche I muste withdrawe the square of. 8. that is my *quotient*, and it beyng 64. there resteth nothing. And the whole woorke standeth thus.

```
      751
22071204 (469
   938
```

```
      37
    751
  6
22071204 (4698
   938
```

*The proofe.*

Wherfore I saie that the first nōber 22071204. is a square nōber: and hath for his roote. 4698. As I maie proue also, by square multiplication. For, as in this example you see: 4698. multiplied by it self, doeth bryng forthe. 22071204.

```
   4698
   4698
  37584
  42282
  28188
  18792
22071204.
```

Master.

Maister. Yet one example more shall you proue: and that is this. 90174084 1. *Another example.*

Scholar. I set it doune, and pricke it accordyng to the rule: And then I see ouer the laste pricke, one onely nomber, that is. 9. whiche hath. 3. for his square roote. That. 3. I set within the *quotiente* line, and therfore I cancell. 9.

| 9 0 1 7 4 0 8 4 1 (3

After this I should proceade with doublynge the roote. 3. and that double should I set in the nert spare, ouer whiche remaineth no nomber, for. 9. beyng cancelled, the Cypher is nothyng. And so am I at a staie.

Maister. Seyng that you can not set the double of your *quotiente* doune there, where no nomber is (for if it so chaunce, as some times it doeth, that the nomber ouer it, is lesser then the double) then set a Cypher in the *quotiente*, and so haue you doen with that pricke. For in soche case there nedeth no multiplication, nor subtraction.

Scholar. Then am I instructed fully for that poincte: The worke is so easie. I must therfore set my nombers thus.

| 9 0 1 7 4 0 8 4 1 (30
| 6 0

Maister. And doe you not see, that the double of the *quotiente*, is greater then the nomber ouer it?

Scholar. I was so mindfull of the one halfe of the rule, that I forgate the other halfe.

But now I see, I must set an other Cypher yet in the *quotient*. And then shall I set the double of all that, in the thirde space, after this sorte.

And nowe proceadynge to searche for anewe *quotiente*, I see that. 2. shall serue me.

| 9 0 1 7 4 0 8 4 1 (300.
| 6 0 0

Wherfore I sette. 2. in the *quotiente* line, with. 300. And by it shall I multiplie the double aforesaied; saiyng. 2. tymes. 6. maketh. 12.

to bee abated out of. 17. and the remainer will bee. 5.

Then shall I ouerpasse the twoo Cyphers, bicause thei make nothing by multiplication: and so cõmyng to the pricke, I bate the square of my *quotiente*: whiche is 4 out of. 8. and there resteth. 4. Therfore I cancell. 8. and set doune. 4. and so haue I ended that pricke. And haue but one worke more behinde.

```
      5   4
9 0 1 7 4 0 8 4 1. (3002.
        6 0 0
```

Wherfore I set doune the nombers, with the double of al the *quotiente*, thus. And then I loke for a new *quotiente*, whiche I finde to be. 9. by it therfore I multiplie, first 6 and it maketh 54. that doeth abate the 54. ouer it. Then omit I the 2 Cyphers, and multiplie 4, by 9 whereof there commeth. 36. whiche I abate out of. 44. beyng ouer it, and there remaineth. 8. That. 8. with. 1. ouer the pricke maketh. 81. out of whiche I muste abate the square of. 9. beyng also. 81. And so is nothyng lefte, wherby it appeareth, that. 90174081. is a square nomber, and his roote is. 30029. The proofe of it doeth confirme thesame. For 30029 multiplied by it self, doeth bringe forthe. 90174081.

```
      5   4
9 0 1 7 4 0 8 4 1 (3002.
        6 0 0 4
```

```
  30029.
  30029.
-------
 270261.
  60058.
  90087.
-------
90174081.
```

*The nigheste roote of vnsquare nombers.*

Master. This shall suffice for soche nombers as bee fully square. Other nombers there bee infinite, whiche be not square, and therfore haue thei no square rootes. Yet often tymes it happeneth, that we shall bee occasioned to searche for the nigheste nomber, that maie resemble their rootes.

Wherfore in soche case, this shall you doe. Firste extracte

extract the roote, as if it wer a square nõber. And that roote wil serue for the greatest square, that is in your former nomber: and there will be a remainer besyde. Of whiche remainer with the *quotient*, you shal make a fraction, in this sorte.

Set the remainer ouer the line, for the numerator, and the double of the roote (that you haue founde) set vnder the line, for the denominator. And this shall be a sufficiente precisenesse in greate nombers, for any common woorke.

Scholar. I will by an example, taken by chaunce, proue this rule. For it semeth to haue no difficultie. Wherfore I take. 296882.

And this, I am assured, can be no square nomber. For, I remẽber you told me before, that no soche nomber might be a square, which had 2 for his first figure.

Then to searche his nigheste roote, I place it, and pricke it thus.

| 4<br>2 9 6 8 8 2 (5.<br>. . . . . |
|---|

And vnder .29. I finde the greateste roote to bee .5. whiche I set in the *quotiente* line, and cancell 29 settyng 4 ouer it. After that I double it, and there cometh 10. & that double I set in the nerte space vnder 46. Then finde I a newe *quotiente*, whiche is 4 and by it I multiplie. 10. whereof amounteth 40. to be abated out of 46. And so remaineth. 6. Again I multiplie. 4. by it self squarely, and there riseth. 16. whiche I abate frõ 18. (seyng. 8. is to small) and the remainer will be. 2. So standeth the whole nomber, as you se. Wherfore I double the *quotiente*, whiche is. 54. And it yeldeth. 108. that must be set vnder 528 as I haue here doen.

| 4 5 2<br>2 9 6 8 8 2 (5 4<br>1 ø . . |
|---|

Then I looke for a *quotiente*, how often I maie abate. 108. out of. 528. And I see it will be but. 4.

| 4 5 2<br>2 9 6 8 8 2 (5 4.<br>1 0 8 |
|---|

tymes. Wherfore I set. 4. in the *quotiente*, with the other nombers, and then doe I woorke with it: Firste multiplyng. 4. and. 1. together, whereof cõmeth but 4. whiche I abate out of. 5. And there remaineth. 1.

Again I multiplie. 8. by. 4. whereof commeth. 32. that doe I subtract out of. 128. and there will remain 96. Then shall I take the square of my *quotiente*. 4. whiche is 16. And that must I abate out of 962. And so remaineth. 946. of whiche nomber set as the numerator, with the double of the roote, set for the denominator, I shall make a fraction in this sorte. $\frac{946}{1088}$. whiche is almoste. $\frac{9}{10}$.

1 9 4
5 2 6
2 9 6 8 8 2 (5 4 4.
1 6 8

Master. You haue doen wel. And so you perceiue that the nigheste roote of your former nomber is $544\frac{473}{544}$. For those fractions are all one.

And hereby also you maie vnderstande, that if the remainer ouer your nomber bee euen, you maie take halfe of it for the numerator, and the whole *quotiente* for the denominator.

So maie you take the quarter of the remainer (if it will so bee parted) for the numerator, and the halfe of the roote for the denominator.

And in like maner generally, if the remainer and the roote in the *quotiente*, bee *nombers communicante*, diuide them so, that the diuisor of the remainer, be euer double to the diuisor of the *quotiente roote*. And so maie you easily reduce that fraction, to his least termes.

*The firste proofe.*

But now for proofe of this woorke, there be twoo waies: the one is certain, and the other but in a nerenesse. For as the roote of soche nombers, is not a precise roote: So if you multiplie that roote by it self, it will make a nomber, very nighe to that former nomber, but not exactly thesame.

Whiche faulte some men thinke to redresse, by adding

dyng of. 1. to the denominator: and yet that amendemente sometymes increaseth the erroure.

But bicause you shall not wante a sure proofe, doe thus: Multiplie the *quotiente*, or *Roote* of whole nombers by it self, and vnto the nomber that amounteth thereof, adde the whole remainer. And if then it make your firste nomber, your woorke was well doen: els haue you misled. *The seconde proofe.*

Scholar. That maie I proue here quickly. The *quotiente* in whole nombers was. 544. whiche beyng multiplied squarely, doeth yelde. 295936. vnto whiche nomber, if I doe adde. 946. that did remain, it will amounte to. 296882. and that was the nomber proponed to me: wherfore it appereth that the worke was well doen.

| |
|---|
| 544. |
| 544. |
| 2176. |
| 2176. |
| 2720. |
| 295936. |

Master. You shall neade no more examples, for this forme of woorke.

But one other waie wil I shewe you, how you shall gesse verie nighe vnto the roote. And you shall go as nighe as you will desire, in any practike woorke. If you desire to gesse within lesse then $\frac{1}{10}$. of one, then set before your nomber. 2. Cyphers. And if you would not erre $\frac{1}{100}$. then set doune 4. Cyphers: But and if you liste to sette doune. 6. Cyphers before your nomber, you shall not misse $\frac{1}{1000}$ of an vnitie frō the true roote. And if you list to go any higher in preciseneſſe of partes, adde still euen Cyphers. *An other waie to finde the nigheſte roote.*

Scholar. I would faine proue this forme, in the same example, whiche I wroughte laste: Bicause I would se the agremente betwene the bothe woorkes.

Master. Go to. Your consideration is reasonable And bicause the partes maie the better agree, sette doune. 6. Cyphers. And then shall your roote expresse thousande partes of the whole nomber.

Scholar. I sette doune the nomber, and pricke it thus.

thus. Whereby I perceiue that I shall haue thesame order of worke, and the selfe same nombers that I had before, till that I come to the Cyphers and their prickes.

| 29688200000(

Maister. Truthe it is. And therfore maie you in soche a case sette doune onely the remainer, with the Cyphers. Or els cancell all the nombers, saue the remainer, and the Cyphers: and set the former whole roote, without the fraction, in the *quotiente*.

Scholar. Then will it stande thus.

Now accordyng to the rule I will proceade: as if this whole nõber wer the first nomber proponed vnto me. And therfore I doe double al the *quotiente*, whiche maketh. 1088. and that doe I set vnder. 9460. And then shall I seke a *quotiente*, that maie declare how often tymes, that double is cõtained in the nomber ouer it. And I se it will bee. 8. wherfore I set doune. 8. in the *quotiente*, and by it I multiplie the double, and subtracte it, in this sorte: saiyng 8. tymes. 1. out of 9. leaueth. 1. remainyng. Again. 8. times. 8. (that is. 64.) out of 146 will leaue. 82. Then farther I abate. 8. tymes. 8. out of. 820. and there resteth. 756. And last of all, I take the square of the *quotiente*, whiche is also. 64. out of 7560. and there will remain. 7496. And so haue I doen with the firste pricke of the Cyphers.

| 946
| 29688200000(544.

| 4
| 78
| 1829
| 94666
| 29688200000(5448
| 1088

*A notable consideratiõ.*

Maister. Conside now that by those. 2. Cyphers you haue gotten 8 into the *quotient* more then you had before. And all your former nomber of the roote, remoued by it into one place higher, then it was before

So

So that, where by the first worke, your roote was 544. and almoste $\frac{9}{10}$: by this worke you haue founde it to bee $\frac{5448}{10}$, and $\frac{3748}{5448}$ of $\frac{1}{10}$: whiche is verie nighe the same nomber, that you had before.

Scholar. In deede, if I reduce the fractions, it wil bee. 544. $\frac{8}{10}$ and $\frac{937}{1362}$ of $\frac{1}{10}$: whiche is in one fraction, $\frac{11833}{13620}$ aboue. 544.

Maister. Marke this triall. And vse the like after euery twoo Cyphers are ended: And you shall see a goodly agremente of the woorkes together.

Scholar. In the meane tyme, to proceede with the former worke, I set doune the nomber with the remainer, and the doble of the *quotiente*, as here appeareth.

```
          7496
29688200000(5448.
          10896
```

And searchyng for a newe *quotiente*, I finde that it will be. 6.

Therfore I sette doune. 6. in the *quotiente* with the other nombers. And by that. 6. I doe multiplie the double of the whole *quotiente*, and subtract it orderly, saiyng: 6. times. 1. beyng abated out of. 7. leueth. 1.

Likewaies. 6. tymes. 8. maketh. 48, whiche I shall abate out of. 49. and so resteth. 1. Then 6. times. 9. (whiche is. 54.) must be subtracted out of. 1016. and there will remaine. 962. Againe I shall abate. 6. tymes. 6. (that is. 36.) out of 9620. and there is lefte. 9584. Then take I the square of my *quotiente*, whiche is also 6 times 6, 0r 36. and that I must abate out of. 40. and there resteth. 4. And thus is the seconde pricke of the Cyphers ended.

```
          5
          968
          10120
          749644
29688200000(54486
          10896
```

And now I finde in the *quotiente* not. $\frac{8}{10}$ as I did in

the laſte woorke before this. But I finde $\frac{86}{100}$: whiche goeth more nighe to $\frac{9}{10}$. For $\frac{90}{100}$ would be $\frac{9}{10}$: and $\frac{80}{100}$ is equalle with $\frac{8}{10}$. And I maie eaſily ſe, that $\frac{86}{100}$ is more nigher to $\frac{90}{100}$ then to $\frac{80}{100}$: beſide the remainer, whiche will make $\frac{47902}{544860}$ of $\frac{1}{100}$. or els $\frac{47902}{54486000}$ of one.

Maſter. I ſee, a well willyng mynde can marke diligently, and learne ſpedily: wherfore go forwarde with your woorke.

Scholar. I muſte ſette doune the double of all my *quotiente*, whiche will be. 108972. And it will ſtande thus.

Wherfore I doe ſeke for a newe *quotiente*, and I finde it to be. 8. whiche. 8. I ſet in the *quotiente*, with the other nombers, and by it I woorke after my rule, ſaiyng: 8. tyme. 1. is. 8. whiche I abate from. 9. and there reſteth. 1. Then take I. 8. tymes. 8. (that is. 64.) out of. 158. and the remainer will bee. 94.

95802
2988820000000(544868
108972

Again I ſubtract. 8. tymes. 9. (beeyng. 72.) from .940. and there is lefte. 868. Farthermore I take. 8. times. 7. (whiche is. 56) out of. 82. and there reſteth. 26. Then doe I withdrawe. 8. tymes. 2. or. 16. out of. 60. And there remaineth. 44.

8624
19486
958024
2988820000000(544868
108972

Laſt of al I take 8 times 8024 (whiche is the ſquare of my laſt *quotiente*) outof 862440 and the remainer will be. 862376. And ſo haue I ended all my woorke.

And nowe I haue for the roote $\frac{544868}{1000}$ that is. 544. and $\frac{868}{1000}$ beſide $\frac{431188}{544868}$ of $\frac{1}{1000}$ or in leſſer termes $\frac{107797}{136217}$ of $\frac{1}{1000}$ that is $\frac{107797}{136217000}$ of one: Whiche beyng reduced into one fraction with the $\frac{868}{1000}$ will make $\frac{118344153}{136217000}$.

Maſter. You haue doen well.

And

And here you see, that you drawe nigher & nigher still, to the very roote, if it might haue any. For $\frac{868}{1000}$ is a nigher nomber to $\frac{9}{10}$. then is $\frac{86}{100}$ as that was nigher then. $\frac{8}{10}$.

And if you would worke with more Cyphers, you should perceiue still, that it would drawe nigher and nigher. But this maie suffice for examples sake.

Scholar. Then I praie you tell me, what is the chief vse of this rule: and for what maters it serueth.

Master. One yere will not suffice, to expresse the commodities of it. It serueth so many waies, in building: in proiection of plattes, for measuring of groūd Timber, or stone: And also in warre, for framyng of battailes, for makyng of diuerse engines, and generally for all woorkes of *Geometrie* and *Astronomie*. But for to satisfie you partly, I will sette forthe twoo or three questions, that depende of this worke of extraction of square rootes.

And firste of a battaile: bicause it semeth to serue leaste for that purpose.

*A question of an armie.*

A capitaine generall hauyng three greate armies, would caste theim into three square battailes, but he knoweth not how many men, he shall set in the frōte of eche battaile.

The nombers of the three armies, are for the firste 5625: For the second 9216: And for the third 15129

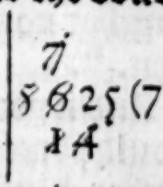
Scholar. I dooe perceiue easily, that for eche of these nombers, I muste searche out the square roote, and then haue I the fronte, or flanke. Sith bothe are equalle in a square battaile.

Wherfore I set doune the firste nomber thus, with his prickes. And then vnder the firste pricke towarde the lefte hande, I finde the greateste roote to bee. 7. seeyng the greateste square is. 49. That roote doe I set within the *quotiente* line: and his square doe I abate from. 56. and so

5625(

remaineth. 7.

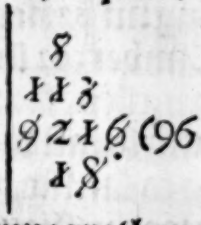
Then doe I double that roote, and sette the double vnder. 72. and see that the newe *quotient* will bee. 5. And there will remaine. 25. whiche is the iuste square of the laste *quotiente*.



Wherby it is euidente, that his first armie contained a square nomber, and the roote, or side of it is 75. And so many menne shall be in the fronte of the firste battaile, and as many in the flanke.

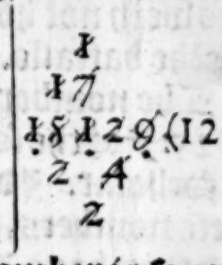
Now for the seconde battaile, I seke the square of 9216. and finde it to bee. 96. As in this example I haue wrought it.

For the firste nomber is. 9. seyng it is the greateste square roote, that can bee founde in. 92. And so is the double of it. 18. and the *quotiente* for it. 6. as it appeareth manifestly inough.



Wherfore I saie that the second battaile shal haue in euery ranke. 96. men.

And now for the thirde battaile, I sette doune the nomber, according to this rule: and I finde the firste roote to be. 1. bicause. 1. tymes. 1. maketh. 1. And his double is. 2. whiche I abate twise from the nomber ouer it: and after double those bothe nombers, whiche make. 24. And finde that to be abated. 3. tymes.



And so haue I gathered that the nomber is square and the roote 123. According to whiche nomber, that thirde battaile must be marshalled.

Maister. Seyng you are so redy in this poincte so sone. Tell me how many menne, shall be sette in the fronte, if all these. 3. armies be ioined into one square battaile.

Scholar. Firste I must adde all. 3. nombers toge-

ther.

ther. And so will thei make. 29960. as here by example doeth appere.

But this nomber can bee no square nomber, bicause it hath one odde Cypher in the firste place: for I remember your saiyng, that square nombers can not begin with odde Cyphers. Wherfore this nomber will not make a square battaile.

| |
|---|
| 5625 |
| 9216 |
| 15129 |
| 29960. |

Yet wil I proue, what maie be the frõt of the greatest square battaile, that maie be made of that nõber.

And for that purpose I pricke the nombers, and finde the greatest roote in. 2. to be. 1 and thesame nõber to bee the square also. Then double I that roote, and place his double vnder. 9. that is vnpricked: and serchyng for a *quotiente*, I finde it to be. 7. with whiche I woorke by the rule, and so doeth remain for the nexte pricke. 10.

| |
|---|
| 1 1 3 |
| 1 5 9 4 1 |
| 2 9 9 6 0 (173 |
| 2 3 4 |

Then doe I double that. 17. whereby commeth 34 whiche I set vnder. 106. And for it I finde. 3. to be the meteste *quotiente*: with whiche if I woorke accordyngly, there will remaine. 31. as the excesse aboue the greateste square.

Wherby it appeareth that. 29929. is a square nõber: and hath. 173. for his roote. And that should bee the fronte of this greate battaile.

Maister. Now will I proue you with an other question of like sorte.

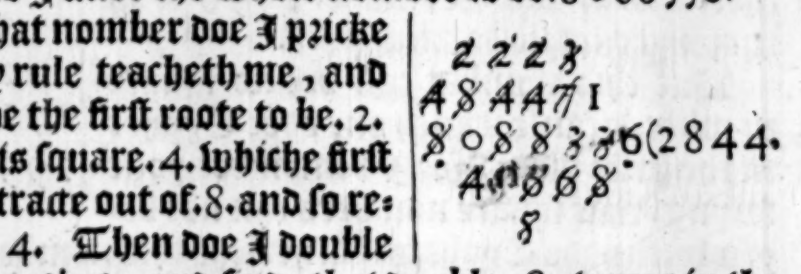
A Prince hath an armie verie greate. With whiche he passeth in a Vallie, so that in marchynge the fronte can be but. 18. menne. And by that meanes the flancke containeth. 449352. *The seconde question of an armie.*

After that the armie is passed that valie, the kyng mindyng to occupie all the beste grounde, willeth the battaile to be set square. How would you doe it?

Scholar. first I multiple the flancke, by the front.

And so I finde the whole nomber to be. 8088336.

That nomber doe I pricke as my rule teacheth me, and I finde the first roote to be. 2. and his square. 4. whiche first I subtracte out of. 8. and so resteth. 4. Then doe I double that *quotiente*, and finde that double. 8. tymes in the somme ouer it.



And so doe I procede till I haue founde out all the 4. figures, accordyng to the. 4. prickes vnder that nōber. And then the roote appeareth to be. 2844.

*The thirde question of an armie.*

Master. Yet one question more, for to exercise your penne, will I propounde of a like mater.

A generalle hath three armies, to the nomber of 28289. men: and none of those three armies is apte to make a square battaile, yet he is appointed by his soueraigne, to sette theim in three square battailes.

These be the. 3. nombers of the. 3. armies. In the firste there are. 10296. men: In the seconde. 9493: and in the third. 8500. Now let me see how you can cast them into three square battailes.

Scholar. I thinke it reasonable, to take the greateste squares of the first and second nombers, and the excesse of them bothe, to put to the thirde nomber.

Master. So are you not sure that the third nomber, will be a true square.

Scholar. Then knowe I not how to doe it.

Master. Take the greateste square in the thirde nomber also. And note those three excesses, and their rootes also.

Then put one to euery roote, and marke the squares that will rise of them.

Thirdly, subtract the firste. 3. nombers, out of those. 3. newe squares, and note the difference of eche of the firste nombers, from those squares: and so haue you. 3

nombers

nombers of excesse, and .3. other of wante.

Now compare those excesses and wantes well together: and you shall easily see from whiche you shall take any nomber, and to whiche you shall adde any.

Scholar. In the firste nōber the greatest square is 10201. and therby the excesse is. 95. and the roote 101.

In the second nomber the greatest square is. 9409 and his roote 97. So is the excesse. 84.

And in the thirde nomber, the greateste square is 8464: and the roote of it. 92. Wherfore the excesse appeareth to be. 36.

And thus haue I founde the. 3. excesses.

Now for to finde the 3 defaultes or wantes, I adde one to eche roote, and multiplie theim square: and so of. 102. I finde the square to bee. 10404. and if I subtracte the firste nomber, whiche is. 10296. out of it, there will remain. 108. for the firste wante.

Then for the seconde roote. 97. I take. 98. whose square will bee. 9604. out of whiche I abate the seconde nomber, whiche is. 9493. and there is left 111 as the wante of the seconde nomber.

Thirdly, I take 93 for the newe roote, nert aboue 92. and I finde his square to bee. 8649. from whiche when the thirde nomber. 8500. is abated, the defaulte appeareth to bee. 149. And thus haue I the. 3. defaultes or wantes, and also the. 3. excesses. Whiche for ease of comparyng, I set in order thus.

| | *A.* | *B.* | *C.* | *A. B.* and *C.* beto-ken the order of the 3 first nōbers. |
|---|---|---|---|---|
| *Excesses.* | 95. | 84. | 36. | |
| *Wantes.* | 108. | 111. | 149. | |

And here I compare the excesses with the wantes, to see if any. 2. excesses will make vp the others want And I see by a lighte proofe, it will not serue.

As for the wantes, I doe not compare theim to the excesses,

excesses, for I se that euery one want, is greater then any one excesse. And therefore .2. wantes are farre to greate aboue any one excesse. And so am I at a staie.

Master. Therfore although that rule bee generalle, yet where it faileth, this shall you doe.

Take the .2. wantes, of any .2. nombers, and adde theim firste together, and then abate theim from the thirde nomber: and if the remainer be a square nomber, then haue you gotten your purpose.

Scholar. That will I proue here. And first I take the wantes, of the .2. firste nombers, whiche make 219. And that doe I abate from the thirde nomber 8500. and there remaineth. 8281. whiche as I see, maie be a square nomber. And therfore I proue it, in my tables, and I finde it so to bee. And. 91. to bee the roote of it.

Wherfore I saie to the question, that these shall be the nombers of the 3 battailes, as here I haue set thē.

| | | |
|---|---|---|
| The firste battaile. | 10404. | and his fronte. 102. |
| The second battaile. | 9604. | and his fronte. 98. |
| The third battaile. | 8281. | and his fronte. 91. |
| The somme of all the .3. battailes. | 28289. | |

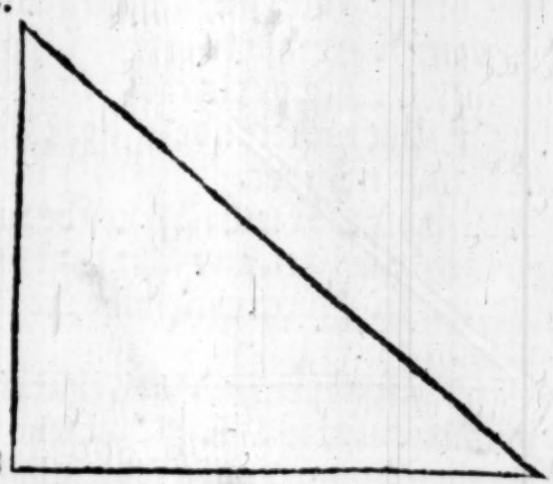
And bicause these nōbers are not onely square, but also their whole somme doeth agree, with the somme of the 3 seuerall armies, you maie be sure that thei are well parted, accordyng to the intente of the question.

But bicause soche questions, haue more difficultie then commoditie, to them that are not mete, to be trauelled in soche marshall affaires, I wil leaue that mater to marshall men, and will come to lower maters in warre.

*A question of scalyng.*

A citie should bee scaled, beyng double diched. And the inner diche .32. foote broade. And the walle. 21. foote high. The capitain commaundeth ladders to be made

made of that iuſte lengthe, that maie reche from the vtter b2ow of the inner diche, to the toppe of the wal: as in this figure is partly exp2eſſed.




Where the line *A.B.* ſtandeth fo2 the b2edth of the diche. And the line. *B. C.* fo2 the heighte of the walle. Nowe I demaunde, what ſhall be the lẽgth of the line. *A.C.* whiche here doeth rep2eſente the ladder?

Scholar. This figure doth occaſiõ me to remẽber the 33. theo2eme of the pathewaie, whiche ſaith thus.

*In all righte anguled triangles, the ſquare of that ſide, whiche lieth againſt the righte angle, is equalle to the twoo ſquares of bothe the other ſides.*

Wherby I vnderſtand, that I muſt multiply thoſe twoo ſides ſquarely, that is, eche of theim by it ſelfe. And then addyng thoſe. 2. ſquares together, I muſte extract the roote of that whole nomber: whiche roote ſhall be the true lengthe of the ſlope line.

Wherefo2e, firſte I multiplie. 32. by it ſelf, and there riſeth of it 1024.

| | |
|---|---|
| 32 | |
| 32 | |
| 64 | |
| 96 | |
| 1024 | |

Againe, I multiplie. 21. by it ſelf, and it yeldeth. 441. Theſe bothe ſommes, beyng added together, doe make. 1465. whiche nomber maie bee ſquare, bicauſe it begin-

| | |
|---|---|
| 21 | |
| 21 | |
| 21 | |
| 42 | |
| 441 | |

N.j. neth

neth with. 5.

Master. It is no square nomber, as it appeareth at the firste sighte. For although the firste nomber be 5. yet in sothe nombers it is requisite, that the seconde figure should be. 2. els can it not be square: and here, you see, that the seconde figure is. 6. so that it can not be a square nomber.

Wherefore you shall seke the nigheste roote, that you can finde in it, and take that for your purpose.

Scholar. Here is my woorke set forthe.

| 2 |
|---|
| 5 8 1 |
| 1 4 6 5 (38 $\frac{21}{76}$ |
| 6 |

And so it appeareth well that the nigheste roote is. 38. $\frac{11}{76}$, whiche is lesse then a quarter of a foote, aboue 38. foote and that must be the lengthe of the ladder.

Master. Yet one question more will I propound agreable to the firste forme.

*A questiō of encampyng.* A capitaine generalle hauynge three armies, in three seueralle battailes, in the firste. 4900. menne, in the seconde. 2401. And in the thirde. 2500. (so that the greateste armie, is as mochе as bothe the other, excepte one manne) is inforced to ioine all three battailes in one. But is in doubte, whether he maie haue good and conueniente grounde to encampe thē, in battaile forme. Wherefore consideryng, that all. 3. battailes together, are but double to the greateste of the. 3. alone. The capitaine desiryng a mete grounde for his armie, so ioined in one square battaile, is in doubte, what square of grounde will serue his purpose. But sure he is, that it muste bee double to the grounde, that the greateste armie of the 3. did occupie and that was square euery waies. 210. foote. Wherfore his demaunde is, how many foote square, shall the side of that grounde bee, that is double to the former square platte, whose side was. 210. foote euery waie?

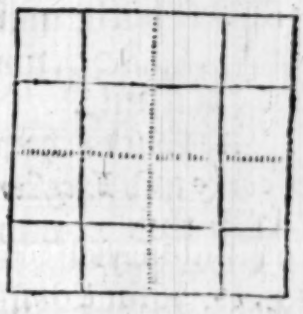
Scholar.

Scholar. Firſte I muſt multiplie. 210. by it ſelf, and ſo haue I the iuſt platte of grounde, of. 44100. foote, that muſt I double, and it will be. 88200. And out of this nomber, ſhall I ſeke the nigheſte ſquare roote. For a iuſte ſquare, I ſe, it is not: by reaſon that after the euen Cyphers, there foloweth. 2, whiche is one of thoſe figures, that can not beginne any ſquare nomber.

Wherefore, ſekyng for the nigheſte roote, I finde it to bee 296. $\frac{121}{148}$ that is almoſte. 297. foote euery waies ſquare. And ſo mochе muſte the ſquare ſide of that grounde bee, whiche ſhould ſerue for that whole armie.

```
 1 4
1 4 5 8
4 2 1 2 4
8 8 2 0 0 (296 121/256.
  4
    58
```

And hereby I doe perceiue, the ouerſighte of many men: whiche being required to double a ſquare platte do double the ſide of it, thinking the mater eaſily doen



But if thei marke it well, thei maie perceiue, that thei doe make, by that meanes, a ſquare fower times ſo bigge as their firſte ſquare was. As by this figure, any man maie ſee.

For if 2. be the ſide of the ſquare then is the ſquare 4. But if I double the ſide, and make it. 4. the ſquare therof will be 16. whiche is. 4. tymes. 4. and not onely double.

So that the roote of the double platte, ſhould bee the roote of. 8. whiche is ſomewhat leſſe then. 3. and therfore mochе leſſe then. 4.

Maſter. You maie perceiue theſame, with the reaſon of it, by the 18. propoſition of the. 8. booke of *Euclide*, as it is befoze alleged.

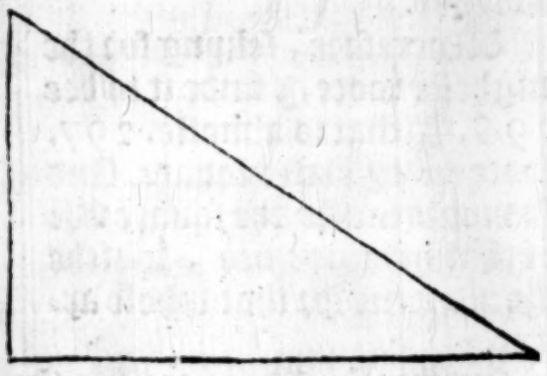
But now for to ſhewe the larger vſe of this rule,

*A question geographical*

I demaunde this question.

There be. 2. tounes, as *Chicheſter* and *Yorke*, whiche lye Southe and Northe, and betwene them. 220. miles. A thirde toune as *Exceſter*, lieth plaine Weſte frō *Chicheſter*. 120. miles. I deſire to knowe the iuſte diſtaunce of *Yorke* from *Exceſter*.

Scholar. I muſt ſet thoſe. 3. tounes, in forme of a Triangle, with their diſtaunces: As here is repreſented. Where *A*. ſtādeth for *Exceſter*, *B*. for *Chicheſter*, & *C*. for *Yorke*.




And then accordyng to the rule, I multiplie. 120. ſquarely: and it maketh. 14400. Likewaies I dooe multiplie. 220. and it yeldeth. 48400.

Theſe bothe nombers I ſhall ioyne in one, and ſo haue I. 62800. whoſe roote is very nigh. 250. miles and $\frac{3}{5}$ of a mile.

And that is the true diſtaunce of *Yorke* and *Exceſter*.

By this example I gather, that this rule doeth helpe to *Geographie*, for to drawe the true platte of any countrie.

| |
|---|
| 2 3 |
| 62800(250$\frac{3}{5}$. |
| 4 |

Maſter. If I ſhould ſtande in propoundyng exāples of this rule vnto you, vſyng but one for euery arte and ſcience, and for euery different kinde of commodiouſe practiſe: it would make a greate booke.

And therefore omittyng that, till occaſion ſerue otherwaies, I will proceade to the extraction of *Cubike rootes*.

## Of Cubike rootes.

When any *Cubike nomber* is propounded, whose roote you should extract After the nomber is written doune orderly: you shall set a pricke vnder the firste figure: and vnder the. 4. and so vnder euery third figure, omittyng still. 2. figures vnpricked. And looke how many prickes, your nomber hath, so many figures shall the roote of your nõber contain

Then to begin the searche, for the firste figure of the roote (in this order) you shall looke what maie be the roote of the nomber, belongyng to the last pricke toward the lefte hande. And that roote shall you sette by a *quotiente* line, as you did in square rootes. I.

And if the whole nomber ouer that pricke, be a *Cubike nomber*, you shall cancell it all. But if it bee no *Cubike nomber*, then subtracte out of it, the greateste *Cube* in it, and cancell the whole nomber, and set the reste ouer it: as you did in square rootes.

But consideryng, that you ought to haue in ready remembraunce, all those *Cubike rootes*, whiche be digites, with the *Cubes* that thei make: for without theim you can not procede in this woorke. I thinke it good to set forthe herein a table, all those rootes with their *Cubes*, that therby you maie be the more adsured in tyme of your woorke. For els a litle mistakyng, might be the occasion of a greate errour.

| | |
|---|---|
| 1. | 1. |
| 2. | 8. |
| 3. | 27. |
| 4. | 64. |
| 5. | 125. |
| 6. | 216. |
| 7. | 343. |
| 8. | 512. |
| 9. | 729. |

And now for this first rule I saie, as I saied of *Square rootes*, this shall be euer more the firste woorke, and shall not be repeted in any one *Cubike nõber*. Whereas all the other rules folowyng, shal be so often repeated, as there are prickes in

your nomber.

2. And of theim this is the firſte: that you ſhall triple the firſte roote. And that triple ſhall you ſet vnder the nexte nomber, toward the righte hande, beſoze that pzicke, whiche you did laſte ende.

3. Then multiplie that triple, by theſame *quotiente*. And ſet it doune vnder the firſt triple: and that nomber ſhall be called your diuiſoz.

4. Thirdly, loke out a *quotient*, that maie declare how often the diuiſoz is in the nomber ouer it.

In whiche doyng, you muſt haue this regard, that betweene that pzicke that is ended, and the nexte that ſtandeth toward the right hande, you muſt ſubtracte 2. other nombers. That is to ſaie, the ſquare of the laſte *quotiente*, multiplied by the fozmer triple. 10. tymes: and the *Cube* of theſame *quotiente*.

Scholar. This rule is very obſcure in wooꝛdes.

Maſter. Then will I terme it thus.

2.4.3. Take the ſquare of your whole *quotiente*, 300. tymes:

4. and that ſhal be your diuiſoz. Then ſeke a newe *quotiente*, declaryng how often that diuiſoz, maie bee founde in the nomber, that doeth belong to the nexte pzicke. But ſo that the ſquare of that newe *quotiente*, multiplied by the laſt *quotiente*. 30. tymes: and alſo the *Cube* of that newe *quotiente*, ioyned all in one ſomme, maie be taken out of theſame nomber. And if you vnderſtande this, there reſteth no moze difficultie.

Scholar. I truſt by exãple, to vnderſtand it better.

Maſter. Then take you this exãple. 26463592 whiche I ſhall ſet doune and pzicke, as I taught you beſoze: and as you maie here ſee. Where the. 3. pzickes declare vnto me, that the roote will haue. 3. figures,

26.463.592(

And then vnder the pzicke that is nexte the lefte hande, whoſe nomber is. 26. I finde the greateſte *Cubike nomber* to bee. 8. and his roote. 2.

Foz

For. 27. whiche is the nerte *Cube*, is to greate.

Therfore I set. 2. in the *quotiente*, and his *Cube*, beyng. 8. I doe abate out of 26. and so remaineth. 18.

That. 18. I doe sette ouer. 26. whiche I muste cancell: and then standeth the nomber, as here you doe see.

```
 18
26 463592(2
```

This is that firste woorke, whiche is not repeted.

Then to procede forward, I doe triple the *quotiente* 2, and so haue I. 6. whiche I shall set vnder. 4. beyng the nerte nomber, on the righte hande of the pricke that is ended.

And that triple must I multiplie, by the first *quotiente*, wherby amoũteth that nomber, that must be the diuisor: and it is in this woorke 12. whiche must be set vnder the same triple: as here I haue placed it.

```
 18
26 463592(2.
  6
 12
```

Then shall I seke for a newe *quotiente*, declaryng how often tymes. 12. maie be founde in the nomber ouer it, that is 184. And I see it maie be in appearaunce. 15. tymes, but more then. 9. you shall neuer take for a *quotiente*: wherefore it appeareth, that I maie boldly take. 9. whiche I shall sette in the *quotiente* with the firste. 2. And then shall I multiplie. 12. whiche is the diuisor, by. 9. and thereof commeth. 108. to bee sette vnder 184. benethe the line, whiche shall euermore be drawen vnder the diuisor.

Now muste I take the square of my laste *quotiente*. 9. (whiche is 81.) and multiplie it by the triple of the former *quotiente* (that is by 6.) and so haue I. 486. to be sette one place more toward the right hande.

```
  2
 18 07 4
26 463592(29
  6
 12
-----------
 108
  486
   729
-----------
 16389
```

Last

Laſte of all, I ſhall multiplie the laſte *quotiente Cubikely*: and that maketh. 729. whiche muſt be ſet, yet one place more toward the right hand, that is to ſaie, vnder the nerte pricke. And then ſhall I adde thoſe 3 ſommes into one: wherby will riſe. 16389. to be ſubtracted out of. 18463. and ſo will remaine ouer that pricke. 2074.

And the worke of that pricke is doen.

This order of worke, if you marke well, you haue learned the whole arte of extraction of *Cubike rootes*.

For how greate ſo euer your nomber be: you ſhall not haue any newe kinde of worke.

But yet bicauſe I did teache you before, theſame worke in other wordes, I will worke theſame example again, accordyng to theſe wordes.

And firſte, after that the nomber is ſet doune, and the firſt *Cubike roote* taken, and the *Cube* abated. Then take the ſquare of that roote. 300. tymes, that is in this example. 4. tymes. 300, whiche maketh. 1200. and that ſhall be your diuiſor. This nomber, and all other in this worke, ſhall you ſet doune ſo, that the firſte nomber, ſhall be vnder the nerte pricke, toward the righte hande.

Then ſeke your *quotiente*, with the former cautele, and it will bee. 9. Wherefore multiplyng. 1200. by. 9. there will amounte 10800. to be ſet vnder the line.

After this, I ſhall take the ſquare of. 9. (whiche is the newe *quotiente*) and multiplie it by. 2. (whiche was the laſte *quotiente* before) 30. tymes. So muſte I multiplie 81. by. 60. and it will make. 4860. whiche I place orderly.

| |
|---|
| 18 |
| 2 ℔ 463 592 (29 |
| 1200 |
| 10800 |
| 4860 |
| 729 |
| 16389. |

Then ſet I doune the *Cube* of the *quetiente*, whiche maketh

maketh.729. And so are the.3. nombers placed, and agree with the former woorke, in all thinges, saue in 2. poinctes. For here the triple of the *quotiente*, is not set doune, but kepte in memorie. And again, here are diuerse cyphers, whiche are not in the former woorke.

Scholar. Sir, I perceiue, that the Cyphers dooe nothyng els, but set the nombers in their due places. And the triple of the *quotiente*, is supplied in woorke by 2. multiplications. First by.300. and then by.30. So that it is all one in effecte.

And by the one woorke I vnderstande the other the better: when I compare theim bothe together. But yet I praie you, ende the woorke that you began.

Master. To continue that woorke, firste I must set doune the nombers, as thei should remaine, after 16389. is abated out of.18463. and then will thei stande thus.

```
  2074
2846.3592.(29
```

Then shall I repeate the former woorke, by settyng doune the triple of all the *quotiente*, whiche will be.87. and that must be placed vnder.45.

Marke that I shall multiplie that.87. by.29. and there will come.2523. whiche must be the diuisor.

Wherfore I seke for a new *quotiente*, that maie shewe me how often.2523. is contained in.20745. And it will bee.8. That 8 doe I set in the *quotient* and by it I multiplie.2523. and it giueth.20184 whiche I sette doune, as here you see.

Then doe I multiplie that *quotient* squarely, and that wil be 64. Whiche I shall multiplie by the triple, that is 87, and there will amounte.5568. to be set one place more toward the righte hande.

```
  2074
2846.3592.(298.
    87.
  2523
 ------
 20184
  5568
   512
 ------
 2074592.
```

D.j. Last

Last of all, I must take the *Cube* of. 8. that is. 512, and it shall bee sette yet one place moze towarde the righte hande.

And then by additiõ, I shall bryng thẽ all into one nomber: and it will bee. 2074592. whiche is equall with the whole nomber aboue, that is vncancelled. And therfoze if I abate the one out of the other, there will remain nothyng.

Wherefoze I see, that the firste nomber, is a iuste *Cubike* nomber. And his roote is. 298.

Scholar. I haue marked you so well, that I trust to doe the like, without erroure.

But I praie you wooꝛke this laste parte also, by your seconde rule, as you did wooꝛke the other: that I maie see the due agremente of theim bothe: and also perceiue the righte vse of this wooꝛke, the better by that other foꝛme.

*The seconde woorke.*

Maister. I must in that case sette doune the nombers, as thei were set in the other wooꝛke. And then I shall multiplie al the *quotiẽt*, whiche is. 29. by it self squarely, and it will make. 841. whiche must be multiplied by. 300. And so there amounteth. 252300. to be sette doune, as here you see.

| |
|---|
| 2074 |
| 2ϐ46ϐ592(29. |

Then I shall seke out a *quotiente*, declarynge how often 252300. maie bee founde in 2074592. And that *quotiente* will bee. 8: whiche I set in the *quotient* roome, with the other nombers.

| |
|---|
| 2074 |
| 2ϐ46ϐ592(298. |
| 252300 |
| 2018400 |
| 55680 |
| 512 |
| 2074592 |

And then I dooe multiplie the diuisoꝛ by the *quotiente*, and thereof riseth 2018400 whiche I set vnder a line, as you maie see.

Nexte that, I doe multiplie the newe *quotient*, by it

self,

selfe squarely, whereof commeth. 64. and that square of the last *quotient*, I shall multiplie by. 870. whiche is. 10. times the triple of the former *quotiente*. 29: and thereof commeth. 55680. whiche I set doune also orderly.

Laste of all, I multiplie. 8. (that is the laste *quotiente*, *Cubikely*, and it maketh. 512. whiche also I set doune in cōuenīēt order.

| | |
|---|---|
| | 8 |
| | 8 |
| | 64 |
| | 870 |
| | 4480 |
| 512 | |
| | 55680. |

And then shall I adde theim all together. And so haue I thesame somme, that I had before in the other former woorke, and it is. 2074592.

Scholar. I neade no more instruction for this: I thinke my selfe so cunnyng, by occasion of your examples, whiche you haue wrought so in double forme.

Maister. That maie you proue, by this nomber 47832147.

Scholar. Firste I shall pricke it, as you taughte me, omittyng still. 2. nombers. *An other example.*

And then out of the nomber ouer the laste pricke, I shal seke out the *Cubike roote*, and abate the *Cube* therof, out of thesame nomber, and set the remainer ouer it, cancellyng the reste.

And so in this nomber, I finde in. 47. the greateste *Cuke* to bee. 27. and the roote of it 3. Wherfore I abate. 27. out of. 47. and finde the reste to be. 20. therfore I cancell. 47. and set. 20. ouer it. And the. 3. whiche is the roote, I set in the *quotient*. And so is the first woorke eanded.

| | |
|---|---|
| | 20 |
| | 47832147 (3 |

Then doe I triple that *quotiente*, and it maketh. 9. whiche I set doune vnder. 8.

Again I multiplie that. 9. by. 3. and it yeldeth. 27. whiche I set vnder the triple, and take it for my diuisor.

Wherefore I shall now seke a *quotiente*, that maie

declare how often. 27. is in. 208 and I see, it will bee. 7. tymes. Therfore I sette doune. 7. in the *quotiente*: and by it I multiplie 27 and it maketh. 189. whiche I set vnder the line: and then I doe multiplie. 7. by it self, whiche maketh. 49. & that square doe I multiplie by the triple of the former *quotiente*, that is, by. 9. and it yeldeth. 441. whiche I set one place more toward the righte hande.

```
20
47832147(37.
   9
  27
 ----
 189
  441
   343
 -----
 23653
```

Last of all, I take the *Cube* of. 7. whiche is. 343. and that doe I sette doune, yet one place more toward the righte hande.

These. 3. sommes beyng added together, doe make 23653.

Master. That will be hardely abated out of a lesser somme.

Scholar. I see now my errour. I must take a lesse *quotient*: whiche thyng I might haue perceiued by the seconde nomber. For thei twoo wer to greate, before the thirde was added.

So that I should haue taken but. 6. for the *quotiente* And then would the firste nomber haue been but 162 and the seconde. 324. and the thirde. 216. but that their placyng would make theim to be of other values, saue the last of the.

```
  1
 2∅176
47832147(36.
   9
  27
 ----
 162
  324
   216
 -----
 19656
```

Therfore, I set euery one in his due roome: and adde theim together, and there amounteth 19656. to bee subtracted out of 20832. and the remainer will be 1176. And thus is that pricke with his worke eanded.

Then for the nexte pricke, I repeate thesame very

forme

forme of worke again. First settyng doune the triple of the whole *quotiente*, whiche is. 108. so that it shall stande vnder. 11761. or vnder. 761. accounptyng figure for figure.

That triple must I multiplie againe by the whole *quotiente*. 36. and it will make. 3888. whiche nomber I muste take for my diuisor.

Wherfore I seke how many times, I maie finde that diuisor in. 11761. and I see, it will bee. 3. tymes. Therfore I set. 3. as my *quotiente*, in his due place: and by that *quotient* I do multiplie. 3888. and so haue I for my firste nomber. 11664.

| | |
|---|---|
| 1 176 | |
| 4783 2147 | (363 |
| 108 | |
| 3888 | |
| 11664 | |
| 972 | |
| 27 | |
| 1 176147 | |

Againe I doe multiplie the laste *quotiente*. 3. squarely, and so haue I. 9. whiche I shall multiplie by the triple of the former *quotient*, and it yeldeth. 972. that shall be set more nigher the right hande, by one place.

Thirdly, I take the *Cube* of. 3. whiche is. 27. and that doe I set yet one place more towarde the righte hande.

Then doe I adde those 3 sommes into one, and thei make. 1176147. whiche is equalle somme, with all the nombers ouer it, that be vncancelled.

Wherfore I saie that. 47832147. is a *Cubike nomber*, and the *Cubike roote* of it is. 363.

Maister. Now doeth the order of teachynge require, that I should instructe you, how to extracte the nigheste *Cube roote*, out of any nomber, that is not a true *Cube*. As this nomber for example maie serue. 694582951.

*The nigheste roote in a nõber not Cubike.*

Where firste I muste extracte the nigheste roote, as I taughte you, for the nigheste *Square rootes*, in nõbers that are not square: and then shall I note the re-

mainer: whiche I shall set for the numerator. And his denominator shall be founde, as I will tell you anon. But firste doe you worke the example, to his nighestе roote in whole nombers.

Scholar. I set it doune, and pricke it, and finde the greatestе *Cube* ouer the lastе pricke to bee 512. and the roote of it is. 8.

| |
|---|
| 182 |
| 694582951 (8. |

Wherfore I set doune. 8. in the *quotiente*. And I abate. 512. out of. 694. and so resteth 182. and the former. 694. cancelled.

Then to procede, I must triple that roote. 8. and it maketh. 24. whiche. 24. I set vnder. 1825. And then I doe multiplie that again, by the *quotiente* or roote. 8 and it maketh 192. to be set vnder thesaied triple. 24: as the diuisor. For whiche I seke a newe *quotiente*, and it will be 8. That. 8. I set in the *quotiente* place, and by it I multiplie the diuisor. 192. and there riseth. 1536. to be set vnder the line, in conueniente order.

Nexte I multiplie the *quotiente* squarely: whiche yeldeth. 46. and that square I multiplie again by the triple, and so haue I. 1536. also. But this must stand more forwardly by one place.

Last of all I take the *Cube* of the *quotient*. 8. and that is. 512. whiche I set vnder the other twoo sommes, and that by one place more forwardly.

| |
|---|
| 13 |
| 182110 |
| 694582951 (88. |
| 24 |
| 192 |
| 1536 |
| 1536 |
| 512 |
| 169472 |

Now gatheryng all these. 3. sommes into one, thei will make 169472 whiche I shall abate out of. 182582. and so remaineth there. 13110. And that pricke with his worke eanded.

Therfore hauyng one other space to worke, I must repeate thesame order of worke again, by triplyng the whole

*quotiente*

*quotiente*. 88. and that will bee. 264. And againe I must multiplie that triplede nomber, by the saied *quotiente*, and it will make. 23232. whiche shall bee the deuisor.

Wherfore I seke a newe *quotiente*, whiche is easily perceiued to be. 5. That. 5. doe I set in the *quotiente*, and by it I dooe multiplie the deuisor 23232. and there amounteth 116160. as the firste nomber, to bee set vnder the line.

Againe I shall multiplie the *quotient* squarely, whiche giueth. 25. and that square shall I multiplie by the triple. 264. and so will there rise. 6600. to bee sette, as the seconde nomber vnder the line: and one place more forwardly, towarde the righte hande.

| |
|---|
| 1428 |
| 1311095 1 |
| 6945829 5 1 (885. |
| 264 |
| 23232. |
| 116160 |
| 6600 |
| 125 |
| 11682125 |

Last of all, I shall sette vnder them bothe, and one place more towarde the righte hande, the *Cube* of. 5. whiche is. 125.

And then shall I adde all those. 3. sommes together of whiche commeth. 11682125. to bee abated out of 131109511. and so the remainer will bee. 1428826. Whereby I see, that the firste nomber that was proponed, I meane 69458295 1 is no *Cubike nomber*, but the greateste *Cube* in it is. 693154125. and his roote is. 885.

And so, I see, all other nombers of like kinde must bee wroughte.

But now for the remainer, how shall I dooe to bryngе it vnto a fraction, that maie aptly expresse the nigheste roote in that sorte?

Master. There bee as many waies, as there bee writers almoste, for euery manne deuiseth, how to bryng

bryngе it moſte nigheſte to a true roote, if any ſoche were: whereof *Cardane* his rule is this.

*Cardane.*

*Multiplie the roote ſquarely, and againe by 3. and that nomber ſhall be the diuiſor vnto the remainer,*

Where he might haue vſed moze plaineſte in wozdes, if he had ſaied: and that nomber ſhal be the denominatoz, to the remainer. Wherefoze as here your roote is. 885 ſo is the ſquare of it 783225 and the triple of that is. 2349675. So would that fraction bee $\frac{1428826}{2349675}$.

But how nigh this doeth go to the truthe, I leaue it till an other tyme.

*Scheubell.*

*Scheubelius* doeth allege an other reaſon, and inſerteth an other ozder, diuerſe frō this, and ſoche as impugneth this, ſaiyng:

*Triple the roote, and the ſquare of it alſo, and adde bothe thoſe nombers together, and. 1. more: And ſo haue you a denominator for your numeratour.*

The numeratoz euermoze is vnderſtād to be the remainer. By whiche meanes the fractiō in this wozke would bee $\frac{1428826}{2312351}$: whiche is a leſſer fraction by a good deale, then is the fozmer fractiō, after *Cardanes* fozme.

But bicauſe at this preſente, I maie not ſpende ſo moche time, to ſcan their ſeueralle opinions, wherein eche of theim, pleaſeth hymſelf well: the one alleging demonſtration (whiche ſcarſely ſerueth) and the other nampnge it a ſecrete, as it is wozthie to bee: I will procede to a thirde waie, moze certain then ether of theſe bothe. And that is by addition of certain Cyphers, to the remainer, in ſoche fozte, that thei muſte all waies bee ternaries, as. 3. 6. 9. 02. 12. ꝛc. And then

ſearche

searche forward with the like order of worke, as you vsed before.

In this maner of practise, looke how many prickes your ciphers hath (or els how many ternaries of Ciphers, there bee set to your nomber) so many figures shall the numerator of your fraction contain. And the denominator shall euermore, contain 1. more. Whereof the laste onely shall bee an vnitie, and all the other shall bee Cyphers.

That is to saie, that if I adde but 3. Ciphers to the nomber, the fraction shall contain certain. 10. partes And if I adde. 6. Cyphers, it shall expresse. 100. partes. So. 9. Cyphers maketh the denominator to bee 1000. partes: And 12. Cyphers geueth 10000 partes.

For example. I will adde to our laste nomber that remained. 12. Cyphers. And then will the nomber be 1428826 000.000.000.000. vnto whiche I set no more prickes, then serueth for the ciphers, bicause I haue passed all the other prickes, in my former worke.

And now to continue my worke, I shall triple all the former *quotiente*, and it will be 2655. whiche nomber I shall place, as here you see it set. And then shall I multiplie that triple, by the former *quotiente*. 885. whiche will yelde. 2349675. to be set vnder thesaied triple: as I haue sette it here also. And this nomber shall be the diuisor.

1428826 000000000000(
2655
2349675

Then shall I seeke for a *quotiente*, whiche can bee none other then. 6: wherefore I sette. 6. in a *quotiente* line, and by that. 6. I dooe multiplie thesaied diuisor 2349675. and it giueth. 14098050. to be the firste nomber vnder the line.

After that, I take the square of thesaied *quotiente*, whiche is. 36. and by it I multiplie the triple. 2655.

wherby is made 95580. to be the seconde nomber vnder the line: & set, as it ought, one place moze toward ẽ righte hande.

```
 18064984
1428826000(8856.
    2655
 2349675
-------------
14098050
   95580
     216
-------------
1410761016
```

Last of all, foz the thirde nomber I take the *Cube* of the saied *quotiente* whiche is. 216. and place it as you see, with his firste figure vnder the pricke.

Then doe I adde those. 3. nombers into one, whiche maketh. 1410761016. And that beyng subtracted out of 1428826000. doeth leaue 18064984. And so is the wooꝛke of the firste pricke canded.

Whereby it appeareth, that the fraction is somewhat moze then $\frac{6}{10}$ oꝛ $\frac{3}{5}$: as it shall bee tried better, by the wooꝛkes that shall ensue.

Therefoze I pꝛocede to the nexte pricke. And firste I triple that whole *quotiente*, whiche yeldeth. 26568. to bee set, as it is often befoze repeated, and therefoze nedeth not hereafter to bee tediously rehearsed.

That triple shall I multiplie again, by the whole *quotiente* (as here I haue sette it in wooꝛke, bicause the nomber is greate, and not easily wꝛoughte by memoꝛie) and it doe I set in his due place, as you see.

```
18064984000000000(88560
    26568
235286208
```

```
   26568
    8856
---------
  159408
  132840
  212544
+ 212544
---------
235286208
```

But then seeyng that diuisoꝛ is greater then all the nomber ouer it, I shall set a Cypher in the *quotiente*: in token that the diuisoꝛ, can not be abated ones out of the nomber ouer it.

it. And so is the worke of that pricke eanded, without any more trauell.

Wherfore to go forward, I triple all that *quotiente* and set it downe, as the rule would, as here is seen.

| | |
|---|---|
| 1594819256457 | |
| 18064984000000 | (885607. |
| 265680 | |
| 2352862080 | |
| 1647003456 | |
| 1301832 | |
| 343 | |
| 1647016474343 | |

Then doe I multiplie that triple, by the whole *quotiente*, whereof cōmeth. 2352862080. and that shall bee the diuisor. And the *quotiente* for it will be. 7.

So then if I multiplie that diuisor by. 7. there will amounte. 1647003456. for the first nomber to be set vnder the line.

And for the nexte worke, I shall multiplie. 49. (whiche is the square of the newe *quotiente*) with the triple of the former *quotiente*, and it will bring forthe. 1301832. whiche shall bee the seconde nomber, to bee set vnder the line.

The thirde nomber shall bee the *Cube* of. 7. whiche is. 343.

| | |
|---|---|
| 265680 | |
| 49 | |
| 2391120 | |
| 106272 | |
| 1301832 | |

And those. 3. sommes added together, will make 1647016474343. whiche is to bee abated out of 18064984000000. and then shall there remain 1594819256457. And so haue I eanded. 3. prickes of the Cyphers. And thereby maie saie, that the fraction is $\frac{607}{1000}$ and somewhat more: That is somewhat more then $\frac{1}{2}$.

Scholar. I see by the fraction, that it is $\frac{6}{10}$ and $\frac{7}{100}$,

beside the quantitie of the remainer. But I praie you eande the woorke of that other pricke, whiche dooeth remaine.

Maister. I muste triple all the *quotiente*: whereby will rise. 2656821. whiche muste bee multiplied by the saied *quotiente*: and thereof will procede the diuisor, beyng 235289927534 7. And his *quotiente* will bee. 6.

| |
|---|
| 2656821 |
| 885607 |
| 18597747 |
| 15940926 0 |
| 13284105 |
| 2125456 8 |
| 2125456 8 |
| 235289927534 7 |

Wherefore firste I set. 6. in in *quotiente* line, with the other nombers: and then doe I multiplie the diuisor by that *quotiente*, and it bryngeth foorthe 14117395652082. For the firste nomber to be sette vnder the line.

| |
|---|
| 1830787347930 24 |
| 1594819256457000 (8856076. |
| 2656821 |
| 235289927534 7 |
| 14117395652082 |
| 95645556 |
| 216 |
| 14117405216639 76 |

And again the square of 6. beyng multiplied by the triple, will yelde. 95645556: whiche shall bee the seconde nomber vnder the line.

| |
|---|
| 2656821 |
| 36 |
| 15940926 |
| 7970463 |
| 95645556. |

The thirde nomber shall be. 216. bicause it is the *Cube* of. 6. And those 3. nombers beeyng added together, doe make. 14117405216639 76. to be abated out of. 1594819256457000. And so dooeth there remaine. 1830787347930 24.

Wherefore

Wherefore it doeth appeare, that beside the first. 3 nombers of the roote, that is. 885. the reste (that is 6076.) standeth for the numerator of a fraction, and the denominator vnto it is. 10000.

So that the nigheste roote is. $885\frac{6076}{10000}$. beside the fraction that doeth remaine: whiche would make but $\frac{1}{8}$ of $\frac{1}{10000}$.

Scholar. This is a sufficiente precisenes. And so I iudge it sufficiently taughte.

Wherefore I praie you propounde some questiõs, that doe require this arte, for their solution.

Master. I am contente. And let this be the firste.

*A question of doublyng a Cube.*

The Grecians giuen to idle banketting, and soche like wantonnesse, did procure thereby soche mortalle sickenesses: that the quicke were scarse hable to burie the dedde. Wherefore consultynge with their Goddes, for redresse thereof, thei receiued aunswere, that when thei would double the Altare, whiche was of *Cubike* forme, thei should bee deliuered from that plague. Meanynge that learnyng is a due meane, to deliuer realmes from plagues and enormities. But to the question, what saie you? If the side of a *Cube* be. 3. foote (as that altare might bee) how many foote shall the side be of that *Cube*, whiche must be double vnto it.

Scholar. This I cõsider. That firste I must finde the quantitie of the *Cube*, that is proponed. And then shall I double that quantitie. Thirdly, I muste extracte the *Cubike roote*, of that double nomber.

So in this question, the side of the knowen *Cube* is 3. and therfore the whole *Cube* is. 27. whose double is 54. And the *Cubike roote* is. 3. and $\frac{27}{37}$ by *Cardanes* rule: That is. 4. whiche is plainly false, for. 4. is the roote of. 64. and not of. 54. But by *Scheubelius* rule, it wil be. $3\frac{27}{39}$ that is. $3\frac{3}{4}$ almoste: whiche is moche nigher the truthe. For. $3\frac{3}{4}$ multiplied *Cubikely*, doeth make. 52. $\frac{27}{64}$. whiche is to litle by a good deale, that is by. $1\frac{37}{64}$.

whereas. $3\frac{27}{37}$ doeth make a lesser somme: that is to say but $51\frac{25959}{50653}$. and so wanteth. $2\frac{24694}{50653}$. And although bothe these sommes goe nigher to the truthe, then *Cardanes* rule, whiche misseth. 10. wholy: Yet maie it be eaſily seen, that *Scheubelius* rule is not so good, as he would it were. And the woꝛse here, foꝛ the addynge of that one moꝛe.

Maſter. Pou are lepte verie sodenly from a scholar, to a cõptroller. And yet I can not but pꝛaise your diligente obseruyng of soche thynges.

Pꝛoue nowe by the Cyphers, howe it will frame.

Scholar. I sette doune the nomber with. 6. Cyphers, and pꝛicke them thus.

Then dooe I take the greateſte *Cubike nomber* in. 54. whiche is. 27 and that I doe abate from 54. and so reſteth. 27. the roote of the *Cube* is. 3. whiche I sette in the *quotiente* line.

```
27
54.000000(3
```

And then I triple. 3. whiche maketh. 9. that muſte be multiplied by the *quotiente* againe, and so commeth 27. to be the diuiſoꝛ. And his *quotiente* semeth to be. 9.

Wherfoꝛe wooꝛkyng with it, the firſte nomber is. 243. and the seconde is. 729. that is. 81. multiplied by 9. whiche is the triple.

Againe, the *Cube* of. 9. is. 729. And all thei together, dooe make 32319 whiche sõme is to greate, and therfoꝛe I muſt take a lesser *quotiente*. As I mighte haue perceiued well inough by the second nõber, if I had marked it in time.

```
27
54.000000(39
  9
 27
---------
243
729
729
---------
32319
```

But nowe amendyng my ouer sighte, I take. 8. foꝛ the *quotiente*. And wooꝛkyng with it I see, the firſte nomber vnder the line, will be

```
27
54.000000(38.
  9
 27
---------
216
576
```

bee. 216. and the seconde. 576. And here all ready I espie my ouersighte again.

Wherfore I take. 7. to be the *quotiente*. And by it I multiplie the diuisor, and so haue I. 189. for the firste nomber.

And for the seconde nomber, I doe worke with. 49. whiche is the square of the *quotiente*, multiplied by. 9. that is the triple: and it yeldeth. 441.

Thirdly, I take the *Cube* of. 7. whiche is. 343. And then addynge al. 3. nombers together, I finde the somme to bee. 23653. whiche is to bee abated out of 27000. and so resteth 3347. Wherby I see, that. $3\frac{7}{10}$ with somewhat more is the roote that I should finde.

```
  3
27347
54000.000(37.
  9
 27
-----
189
441
  343
-----
23653
```

But for farther triall, I triple all the *quotiente*, and finde thereby. 111. whiche I multiplie by thesame *quotiente* again, and so commeth 4107. to bee the diuisor. And his *quotiente* will bee 8. as it semeth: and so the first nomber will bee. 32856. And the seconde shall bee. 7104. but those. 2. are to greate, as it is manifeste all readie.

```
33347000.(378.
  111
 4107
-------
32856
 7104
```

Wherfore I take 7 for the *quotiente*. And by it multiplyng the diuisor, there riseth 28749.

And for the seconde somme, there is founde. 5439.

And for the thirde some. 343.

All whiche. 3. sommes ioined in one, doe make. 2929633. And that beeyng abated out of the higher somme. 3347000. doth leaue. 417367.

```
 417367
33347000.(377.
  111
 4107
-------
28749
 5439
  343
-------
2929633
```

Wherefore

Wherefore I maie boldly saie, that the fraction is $\frac{77}{100}$ and more, by the portion of the remainer, whiche is nighe $\frac{1}{700}$.

And it is sone seen that $\frac{75}{100}$ are equalle to $\frac{3}{4}$: wherefore $\frac{77}{100}$ shall be more then $\frac{3}{4}$.

And so dooeth *Scheubelius* rule erre more, then I thought before.

So is your question aunswered, that the side of the double *Cube*, shall be. 3. foote and $\frac{77}{100}$ and $\frac{1}{7}$ of $\frac{1}{100}$.

*Of the rootes of fractions.*

Master. For the rootes of fractions, I shall nede to saie no more but this: that if the numerator and denominator bothe be *Squares*, or *Cubes*, &c. then maie you finde in that fractiō the like roote. But if any of bothe dee swarue from that name, then hath that fraction no sothe roote.

As $\frac{16}{27}$ is nother *Cubike* nor *Square*, bicause his partes dooe not agree in *Square* name, nor in *Cubike* name: although the numerator bee a *Square*, and the denominator a *Cube*.

Scholar. That dooeth appeare reasonable, at the the firste sighte.

Master. Then seeyng you are so readie in learnyng: aunswere me to this question.

*A question of a Gonne.*

A Gonne of fire inches diameter in the mouthe, dooeth shotte a bollet of twentiepound weighte: what weighte shall that bollette haue, that serueth for a gonne of. 14. inches in the mouthe?

But to helpe you in this question, and in all sothe like, you shall marke well *Euclide* his saiyng, in the 18 proposition of his. 12. booke, whiche is this.

*All Globes bee together triple that proportion, that their diameters doe.*

So in this example, the proportion of the *diameters* beyng as. 14. to. 6. Or as. 7. to. 3. I shall triple it, and then haue I the proportion of their Globes.

Wherefore

Wherefore I sette the.3. fractions thus. $\frac{7}{1}\ \frac{7}{1}\ \frac{7}{1}$ and thei make $\frac{343}{27}$. that is.12. $\frac{19}{27}$. And so is the proportion of the Globes, as well in weighte, as in bignesse.

Therfore I must multiplie. 20. that is the weight of the lesser bollette, by the numerator of the proportion, and diuide it by the denominator.

And so shall I haue. $254\frac{2}{27}$ for the weighte of the greater bollete.

| | |
|---|---|
| 343 | 113. |
| 20 | 2412 |
| 6860 | 6860(254 $\frac{2}{27}$ |
| | 2777 |
| | 22 |

Now proove you the like woorke. Remembryng that *Cubes* also, as well as Globes, doe beare triple proportion, in comparison of their sides. As you learned before by the.19. proposition, of the.8. booke of *Euclide*.

A *Cube* of Braſſe of.4. inches square, doeth weighe 7. pounde weighte, what shall a *Cube* of Braſſe of. 9. inches square, waie? *A queſtion of.2. Cubes.*

Scholar. The proportion of the sides is as $\frac{9}{4}$ whiche I must set doune thrise, and multiplie them together, as fractions should bee. And so will it bee thus. $\frac{9}{4}\ \frac{9}{4}\ \frac{9}{4}$. that maketh. $\frac{729}{64}$.

Wherefore I multiplie the weighte of the lesser *Cube*, beyng.7. by.729. and it maketh.5103. and that doe I diuide by.64. and so finde I.79. $\frac{47}{64}$, whereby I maie knowe, that the weighte of the greater *Cube*, is 79. pounde weighte, and very nighe $\frac{3}{4}$.

Maister. These.2. questions dooe teache you, rather the proportion of *Cubes*, then the vse of the rule: wherfore to make the questiõs more agreable to this rule, I propounde them thus, in backer order.

A bollette of yron of.7. inches *diameter*, doeth waie 27. pounde weighte: what shall be the *diameter* to that bollette that shall waie.125. pounde weighte?

Scholar. I praie you aunswer to it your self, that I maie see the apte forme of applyng soche questions

to this rule.

Maister. As the *Cubes* are in triple proportion to the sides, so are the proportions of the sides, to bee founde by triple diuision: that is to saie, by seking the *Cubike rootes*, of the 2. termes of the proportion.

Wherefore I doe firste set doune the termes of the proportion of the bollettes, thus: $\frac{125}{27}$. And I see, that the *Cubike roote* of. 125. is. 5. and the like roote of. 27. is. 3. whiche nombers I shall set in the roome of the. 2 others, thus: $\frac{5}{3}$ And thei declare the proportion, betwene the *diameters* of the. 2. bollettes. Wherof one that is the lesser, is knowen to be. 7. Therfore I multiplie that. 7. by. 5. whereof commeth. 35. and that. 35. doe I diuide by. 3. whiche giueth. $11\frac{2}{3}$.

Wherfore I saie, that if. 7. inches bee the *diameter* to a bollette of. 27. pounde weighte, then. 11. inches and $\frac{2}{3}$ shall be the *diameter* to the bollete of. 125. poũde weighte.

Scholar. The proofe of this had nede bee certain, seeyng the woorke is obscure, to the common iudgemente.

*The proofe.* Maister. You saie well. And this is the very order of proofe for it. Multiplie bothe these rootes *Cubikely*. And if their *Cubes* be in soche proportiõ as their waightes bee (that is to saie in this exãple as $\frac{125}{27}$) then is the woorke good: els not.

Scholar. That must neades bee so. And therefore will I prowe it so in these nombers.

And for that eande, firste I multiplie. 7. *Cubikely*, and it giueth. 343. Then I multiplie. $11\frac{2}{3}$. *Cubikely*, and it maketh $4218\frac{75}{27}$. But now seyng the one nomber is a fraction, I will for ease tourne the other into a fraction of thesame denomination: and it will bee $9261\frac{27}{27}$ in whiche. 2. fractions, the proportion muste cõsist betwene the numeratours. So that thei bothe beeyng diuided by one common nomber, muste come to this

fraction

fraction $\frac{125}{27}$.

And so I see it will be: for the lesser beyng diuided by. 343. will yelde 27. And the greater diuided by the same. 343. will giue. 125. So that by triall, that worke is approued good.

Master. I will now proue your cunnynge, in a newe question, whiche Brasiers often tymes, haue occasion to vse: as thus.

I haue a dice of Brasse of. 64. vnces of Troye weighte, whose side is. 3. inches and $\frac{1}{2}$ and would haue an other dice of the same mettall of. 18. pounde weighte. *A question of weightes.*

My demaunde is: what shall be the side of the dice?

Scholar. This question must firste bee reduced to one kinde of denomination in the weightes, and then will it be more apte to be aunswered.

Wherefore I shall tourne. 18. pounde into vnces, multiplyng it by. 12. and it will be. 216.

And then I consider the proportion, that is betweene those. 2. nombers of weighte. 64. and. 216. and it is certainly. $3\frac{3}{8}$, or $\frac{27}{8}$ out of whiche proportion, I must extracte the *Cubike roote*, as I maie easily doo, seyng bothe the numerator and the denominator, are *Cubike nombers*.

And so is their roote $\frac{3}{2}$: whiche is the proportion of the sides of the twoo dice.

And seyng the side of the lesser die, is knowen to be 3. inches and $\frac{1}{2}$, the other his side must be in *Sesquialter* proportion to it, that is. $5\frac{1}{4}$: whiche is wrought also thus. I multiplie. $3\frac{1}{2}$ by. 3. and it maketh. $10\frac{1}{2}$ whiche I shall diuide by. 2. and there commeth. $5\frac{1}{4}$.

Master. Yet one question more I will propounde to giue you occasion, to vnderstande the apte conference of masses, of diuerse stuffe.

And for that purpose, I suppose this proportion in weighte, to bee betwene masses of one biggenesse.

*Examples of rates for weightes.*

That if I compare Wodde and stone of one quantitie together, the stone shall weighe more then the wodde by $\frac{2}{3}$.

Likewaies yron to be heuier then stone by $\frac{1}{2}$.

And Brasse to bee heuier then yron by $\frac{1}{3}$.

Ledde to be heuier then Brasse by $\frac{2}{5}$.

| Stoffe. | Weighte. | | | |
|---|---|---|---|---|
| Wodde. | 60. | 1 | | |
| Stone. | 100 | $\frac{3}{5}$ | 1 | |
| Yron. | 150 | $\frac{2}{5}$ | $\frac{2}{3}$ | 1 |
| Brasse. | 200 | $\frac{3}{10}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
| Ledde. | 280 | $\frac{3}{14}$ | $\frac{5}{14}$ | $\frac{15}{28}$ | $\frac{5}{7}$ |

All whiche rates, although thei be taken for examples, and not of truthe, yet thereby maie you learne, how to woorke with true rates, set in a like table.

And now for the vse of this table, take this questiō.

*A question of weighte.*

I would haue .5. weightes of *Cubike* forme, made of these .5. stuffes.

The weighte of the wodde shall be. 28. pounde.

The stone. 56. pounde.

The yron. 112. pounde.

The Brasse. 224. pounde.

And the Ledde. 448. pounde.

Of all these I haue but the yron weighte: whose side, or *Cubike roote* is. 12. inches $\frac{2}{3}$.

And my desire is to knowe, of what quantitie the sides of all the other weightes shall bee.

Scholar. The question is pleasaunt: and yet somewhat harder then the other.

Master. The table will helpe you fully, so that you cōferre it well, with that you haue learned before

But bicause I haue litle leiser, to spende moche tyme with you (saue that zeale to your furtheraunce doeth make me partly to forgette my owne businesse) therefore will I leaue this question to your self, to be aunswered at your laisure.

And so in all the rest, I must passe it ouer: and giue an eye to soche maters, that touche me more nighe:

and

and weighe moze heuily, then all ſoche weightes, by 20. folde.

Wherfoze, touchyng all the rootes of compounde nombers, you ſhall at my hand now, haue no priuate declaration. But ſoche as you haue learned all redie.

## Of compounde rootes.

IF the nomber bee compounde, other of *Square nombers*, oz of *Cubike nombers*, then accozdyngly as the cõpoſition is, ſo ſhal you dzaw the roote: and without one of theſe two there can bee no compoſition.

Wherefoze to begin with the ſmalleſt compounde nomber in that ſoꝛte, whiche is a *Square of ſquares*, you ſhall firſte extracte the ſquare roote, as you haue learned befoze. And out of that roote (whiche muſt nedes bee a *Square nomber*) you ſhall extracte his ſquare roote alſo: and that roote is the *zenzizenzike roote*, of the firſte *Square of ſquares* oz *zenzizenzike nomber*. *Squares of Squares.*

Foz erample take. 14641. whoſe *Square roote* is 121. and that ſame roote is it ſelf, a *Square nomber*: and hath foz his roote. 11.

```
  2
1 46 41 (2916
  2 24
```

Wherfoze I maie ſaie, that. 11. is the *Squared ſquare roote*, oz the *zenzizenzike roote* of 14641.

Again 8503056. is a *Square of ſquares*, and therfoze a *Square nomber*. And his *Square roote* is 2916. whiche is a *Square nomber* alſo, and hath. 54. foz

```
   4
2916 (54
 16
```

```
      121
4 9 9 8 9 3
8 8 5 3 0 5 6 (2916.
  4 8 8 2
      8
```

M.iij. his

his roote.

So that. 54. maie well bee called the *zenzizenzike roote* of. 8503056.

And so shall you woorke, with all of that name.

*Zenzizenzizenzikes*

But and if the nomber be compounde, of. 3. *zenzikes*, or. 3. *Squares*, as a *Square of squared squares*, or a *zenzizenzizenzike* (whiche some men for shortnesse, call *zenzizenzenzike*). Then shall you drawe firste the *Square roote*, and then the *Square roote* of that roote, and thirdly the *Square roote* of that laste roote.

As for example. 6561. is a *Square of squared squares*. And his firste roote is. 81. whiche is also a *Square nomber*, and hath 9. for his roote. That. 9. likewaies is a *Square nomber*, and hath. 3. for his roote.

| | |
|---|---|
| I | |
| 6561 | (81. |
| 16 | |

So that the *zenzizenzizenzike roote* of. 6561. is. 3.

And for these formes of nombers, I shall not nede to staie for any more explication, or examples: seeyng the mater is plaine.

Now for compounde *Cubike nombers*, you shall understande the like forme.

*Cubes of cubes.*

If the nomber bee a *Cube of Cubes*, you shall firste extraite the *Cubike roote*. And bicause that roote is a *Cubike nomber* also, therfore shall you seke the *Cubike roote* of it. And that seconde roote shall bee the *Cubicubike roote* of the firste nomber.

As for example. 512. is a *Cubike nomber*, or a *Cube of Cubes*. And his *Cubike roote* is. 8. whiche. 8. againe is a *Cubike nomber*, and hath. 2. for his roote.

So that. 2. is the *Cubicubike roote* of. 512.

Likewaies. 100776 96. is a *Cubicubike nōber*, and his firste *Cubike roote* is. 216. as you maie easily perceiue by these woorkes: where I haue sette forthe the order of extraction of his *Cubike roote*, whiche is. 216. And that. 216. is a *Cubike nomber*, you neade not to doubte,

```
  8 1 6
1ø ø 7 7 6 9 6 (216.
    6 3
  1 3 2 3
  -------
    7 9 3 8
      2 2 6 8
        2 1 6
    -------
    8 1 6 6 9 6
```

doubte, for that it is one of the, which you haue, I darc saie, in perfecte memorie: Bicause his roote is a digite, and that is. 6.

```
  2 8 1 6
1ø ø 7 7 6 9 6 (21
    6
    x z
  -------
  1 2 6 1
  -------
  1 2 6 1
```

By this you maie iudge of *Cubicubikes Cubikely*, or *Cubes of Cubicubes*. that in theim you shall firste seke their *Cubike roote*: And then the *Cubike roote* of that roote. And thirdly the *Cubike roote* of that roote againe. And so haue you the *Cubicubicubike* roote of that firste nomber. *Cubicubes Cubikely.*

The thirde waie of composition is, when *Squares* and *Cubes* be compounde together: as *Zenzicubes, Zenzizenzicubes, Zenzicubicubes*, or soche like, as it happeneth diuersely. *The thirde composition.*

In all these you shall as often abate the *Zenzike roote*, as that name is in the composition, and so likewaies of the *Cubike roote*.

So that in a *Zenzicubike*, you shall extracte firste the *Square roote*: and out of that *Square roote*, you shall extracte the *Cubike roote*. *Zẽzicubike.*

As. 6 4. is a *Zenzicubike nomber*, whose *Square roote* is 8. and that. 8. is a *Cubike nomber*, and hath. 2. for his roote.

So. 5 3 1 4 4 1. is a *Zẽzizenzicube*: whose firste *Square roote* is. 7 2 9. whiche nomber is a *Zenzicube*, & hath for his *Square roote*. 2 7. And that nõber is a *Cube*, and hath for his roote. 3. wherefore I maie iustly saie, that. 3. is the *Zenzizenzicubike roote* of. 5 3 1 4 4 1. *Zenzizenzicube.*

```
  3 4
7 2 9 (27
  4
```

```
    1 4 4
    4 3 0
5 3 1 4 4 1 (729
  4 4 4
    1
```

But as I saied before, that I might not staie long at

at this presente, so the vse of these greate nombers is rare in practise: and therefore I will ouerpasse them, for this tyme.

And yet for your aied in the meane season, I haue here drawen a table, whiche maie bee called the table of ease: in whiche you haue greate plentie of these nombers, with their rootes in diuerse kindes.

The table it self is so manifeste, that it needeth no declaration: if you haue not forgotten, what you learned before.

And if you lifte to enlarge this table, you maie easily doe it, multiplyng the nombers still by their rootes, whiche bee set ouer theim, in the hedde of the table. And so maie you make it to extende infinitely: whiche shall ease you wonderfully, in the extraction of any kinde of rootes. For which at some other time if my leisure serue me better, with quietnesse, I will giue you more specialle rules.

And also I councell you, well to examine this table, and trust not to my castynge. For haste and other troubles, maie often times cause errour in supputation.

The

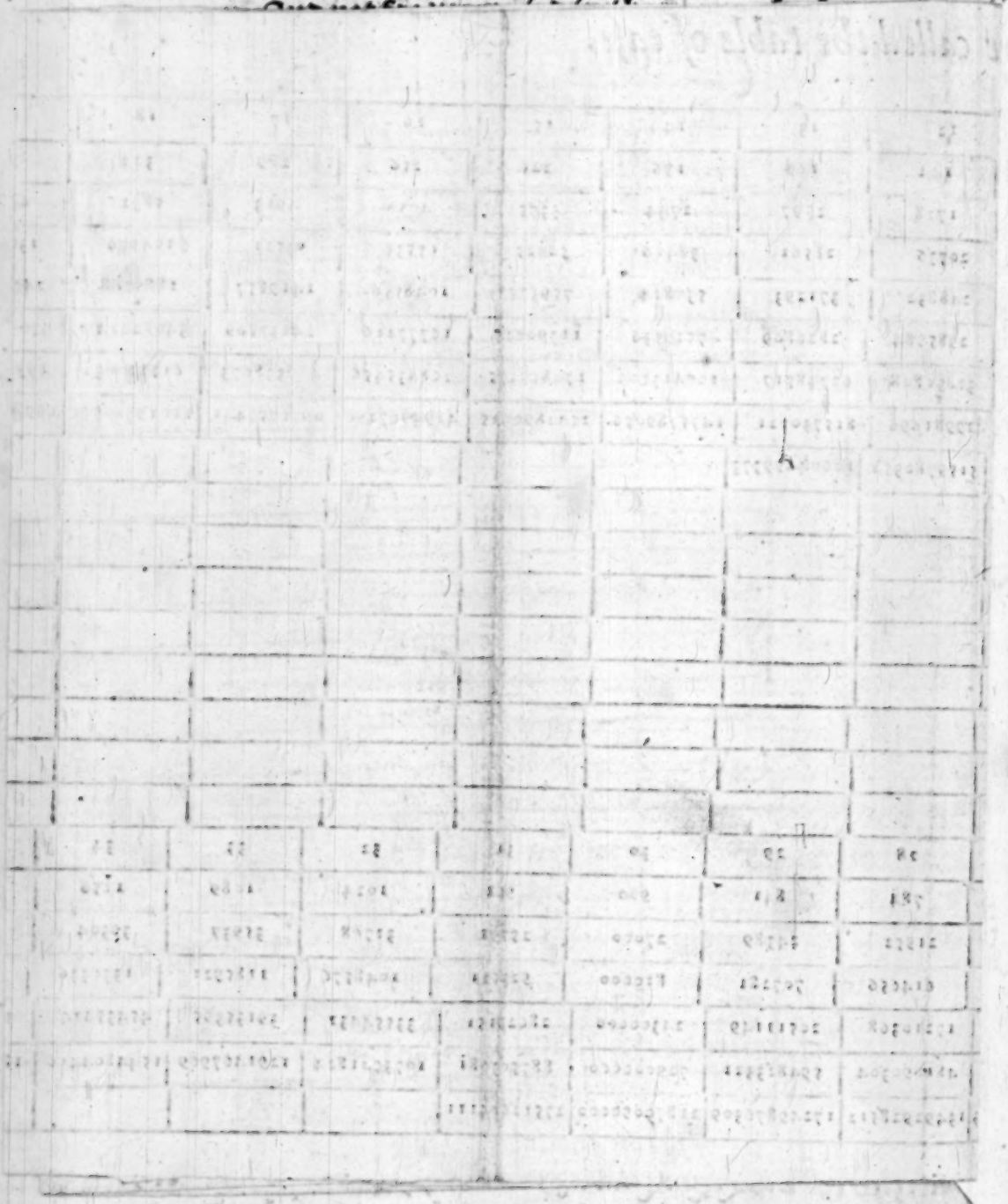
*of Coſsike nombers.*

## *Of nombers denominate.*

Thus haue I lightly ouer run the moſte *Nombers*

I make, in teachyng: And therfore will begynne with
*Coſsike* nombers firſt.

at this presente, so the vse of these greate nombers is rare in practise: and therefore I will ouerpasse them, for this tyme.



## Of nombers denominate.

THus haue I lightly ouer run the moſte common kindes of nombers *Abſtracte*. And now reſteth the treatice of nombers *Contracte*, or *Denominate*. Of whiche kinde there bee ſome called *nombers denominate vulgarely*: and other bee called nombers *denominate Coſsikely*. And a thirde ſorte there is of nombers *radicalle*, whiche commonly bee called *nombers irrationalle*: bicauſe many of theim are ſoche, as can not bee expreſſed, by common nombers *Abſtracte*, nother by any certain rationalle nomber. Other men call them more aptly *Surde nombers*. *Nombers contracte.*

And although many menne would not accoumpte them, with nombers *denominate*, yet I maie iuſtly doe it, for that thei require a reduction to one denomination, if thei haue ſeueralle ſignes of quãtities, as you ſhall heare hereafter. And thoſe nombers neuer goe alone, without ſome other ſigne, and name of rooted quantitie, annered to theim.

Of the firſt kinde of nombers denominate, whiche are vulgarely denominate, as. 10. ſhillinges. 10. men 20. ſhippes, 100. ſhepe. 1000. yeres, and ſoche like, I will ſpeake nothyng in this treatice. But of the other twoo kindes I will ſomewhat write, for youre learnyng and contentation.

Scholar. Sir, I am moche bounde vnto you: And therefore remit all to your owne diſcretion and good will. Truſtynge ſo to applie my ſtudie, and emploie my knowlege, that it ſhall neuer repente you of your curteſie in this behalfe.

Maſter. Then marke well my wordes, and you ſhall perceiue, that I will vſe as moche plaineſſe, as I maie, in teachyng: And therfore will beginne with *Coſsike* nombers firſt.

## Of Coßike nombers.



Nombers *Coßike*, are ſoche as bee contracte vnto a denomination of ſome *Coßike* ſigne as 1. nomber, 1. roote. 1. ſquare 1. Cube. ꝛc.

But as for cõpendiouſneſſe in the vſe of theim, there bee certain figures ſet for to ſignifie them: ſo I thinke it good to erpreſſe vnto you thoſe figures, before wee enter any farther, to thintente we maie proceede alwaies in certentie, and knowe the thynges that wee intermedle withall: for thei are the ſignes of all the arte, that followeth here to be taught.

And although there be many kindes of irrationall nombers, yet thoſe figures that ſerue in *Coßike nõbers*, bee the figures alſo of all irrrtionalle nombers, and therfore being ones well knowen, thei ſerue in bothe places commodiouſly.

Theſe therfore be their ſignes, and ſignifications briefly touched: for their nature is partly declared before.

9. Betokeneth nomber abſolute: as if it had no ſigne.

2. Signifieth the roote of any nomber.

z. Repreſenteth a ſquare nomber.

c. Erpreſſeth a Cubike nomber.

z z. Is the ſigne of a ſquare of ſquares, or Zenzizenzike.

ſz. Standeth for a Surſolide.

z c. Doeth ſignifie a Zenzicubike, or a ſquare of Cubes.

bſz. Doeth betoken a ſeconde Surſolide.

z z z. Doeth repreſent a ſquare of ſquares ſquaredly,

ly, or a zenzizenzizenzike.

ȝ ȝ. Signifieth a Cube of Cubes.

ȝ ſȝ. Expreſſeth a Square of Surſolides.

ȝ ſȝ. Betokeneth a thirde Surſolide.

ȝ ȝ ȝ. Repreſenteth a Square of Squared Cubes: or a Zenzizenzicubike.

ȝ ſȝ. Standeth for a fourthe Surſolide.

ȝ ſb ȝ. Is the ſigne of a ſquare of ſeconde Surſolides

ȝ ſȝ. Signifieth a Cube of Surſolides.

ȝ ȝ ȝ ȝ. Betokeneth a Square of ſquares, ſquaredly ſquared.

Eſȝ. Is the firſte Surſolide.

ȝ ȝ. Expreſſeth a ſquare of Cubike Cubes.

Fſȝ. Is the ſirte Surſolide.

ȝ ȝ ſȝ. Doeth repreſente a ſquare of ſquared ſurſolides.

ȝ bſȝ. Standeth for a Cube of ſeconde Surſolides.

ȝ cſȝ. Is a ſquare of thirde Surſolides.

gſȝ. Doeth betoken the ſeuenthe Surſolide.

ȝ ȝ ȝ ȝ. Signifieth a ſquare of ſquares, of ſquared Cubes.

And though I maie proceade infinitely in this ſorte, yet I thinke it ſhall be a rare chaunce, that you ſhall nede this moche: and therfore this maie ſuffice. Notwithſtandynge, I will anon tell you, how you maie cõtinue theſe nombers, by progreſſion, as farre as you liſte.

And farther you ſhal vnderſtande, that many men doe euer more call ſquare nombers zenzikes, as a ſhorter and apter name, other men call thoſe ſquares the *firſte quantities*, and the *cubes* thei call *ſeconde quantities*: ſquares of ſquares thei call *thirde quantities*, and ſurſolides *fourthe quantities*. And ſo namyng them all quantities (excepte nombers and rootes) thei dooe adde to them for a difference, an ordinall name of nomber, as thei doe goe in order ſucceſſiuely.

As here folo weth in example.

| | | |
|---|---|---|
| 𝆑. | Firſte. | And ſo forthe, of as many as maie bee reckened. |
| 𝆑𝆑. | Seconde. | |
| 𝆑𝆑𝆑. | Thirde. | |
| 𝆑𝆑𝆑𝆑. | Fourthe. | But althoughe ſomemen accompte this the more eaſie waie: bicauſe the other names be comberouſe, yet thoſe other names befoꝛe, do erpꝛeſſe the qualitie of the nomber, better then theſe later names doe. |
| 𝆑𝆑𝆑𝆑𝆑. | Fifte. | |
| 𝆑𝆑𝆑𝆑𝆑𝆑. | Sixte | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Seuenthe. | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Eighte. | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Nineth. | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Tenthe. | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Eleuenthe | |
| 𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑𝆑. | Twelfthe. | |

*Quantities.*

Scholar. I thanke you double, ſith you are contente to teache me double names: foꝛ ſo ſhall I be acquainted with bothe foꝛmes, as I ſhall chaunce on them in other mennes bookes.

Therfoꝛe now you maie pꝛoceade to numeration: whiche I thinke it nexte.

Maiſter. There be other. 2. ſignes in often vſe, of whiche the firſte is made thus —|— and betokeneth moꝛe: the other is thus made —— and betokeneth leſſe.

And where thei come in any nomber *Coſſike*, oꝛ other, that nomber is called a compounde nomber, bicauſe it conſiſteth of. 2. nombers. And where neither of theim is, the nomber is called vncompounde, although the ſigne be compounde. Foꝛ the compounde ſigne, maketh not a compounde nomber. And now I will pꝛocede to numeration.

## of Coſsike nombers.

## Of Numeration in nombers Coſsike, vncompounde.

Maſter.

NOmbers *Coßike* vncompounde, haue no difficultie in their numeration: for euer more the nõber repreſenteth, ſo many of that *Coßike* denominatiõ (be thei nõbers, rootes, ſquares, Cubes, ſquares of ſquares, or any other like) as ther be vnities in that nõber So. 6. ☉. is. 6. nombers: And. 6. ☍. is. 6. rootes: 20. ☽. is. 20. ſquares: 30. ☌. betokeneth. 30. Cubes. *Numeration*

Scholar. I ſee it well. For by this nõber. 20. ☽. is not appointed any nõber abſolute, of one certaintie, but onely ſo many *quãtities* of that kinde: whiche maie bee. 80. if. 4. be one ſquare. And if. 9. bee one ſquare, then 20. ſquares make 180. And if. 25. be one of thoſe ſquares thereby repreſented, then. 20. ſquares make 500. And as for the ſignes, you taught me the before.

## Of Addition.

Maſter.

THis numeration is ſo plaine, that wee maie paſſe from it vnto addition: whiche is as eaſie alſo, if the quantities be of one denomination. For then nedeth no more, but to adde the nombers together, and to put that ſame common *Coßike* denomination, to the totall thereof. *Addition of like ſignes.*

Scholar. I take it thus, 20. ☍. added to. 30. ☍. will make. 50. ☍. And. 12. ☽. added to. 16. ☽. bringeth forthe. 28 ☽.

Maſter. As you doe eaſily ſee al the matter of this addition, ſo maie you as eaſily conceiue, all the worke of ſubtractiõ. For it is wrought as in vulgare nõbers *Subtraction of like ſignes*

Scholar. Then if I abate .6. ℭ. out of .10. ℭ. there will reste 4. ℭ. And so .9. ℨ ℨ. out of .25. ℨ ℨ doeth leaue .16. ℨ ℨ.

Master. This is all for nombers of like signes *Cossike*.

Scholar. What then if I would adde .10. ℨ. to 6. ℨ: where the signes bee vnlike: maie it be doen: seyng thei be not of one denominatiõ, nor signe *Cossike*.

*Addition of vnlike signes*

Master. As well as shillynges maie bee added with poundes, or penies: and in like sorme.

For thei shall stand still as thei wer, with the signe of addition, whiche is this. —+—. + betokeneth more.

So that .10. ℨ. put to .6. ℨ. maketh .6. ℨ. —+— 10. ℨ. that is .6. ℨ. more .10. ℨ. or .6. ℨ. and .10. ℨ.

Scholar. And why not .10. ℨ —+— 6. ℨ?

Master. Bicause it is moste orderly, to sette the greateste signe *Cossike*, formoste in order.

As you saie .20. shillynges, and .6. pennies: rather then .6. pennies and .20. shillynges.

Scholar. Then I se, if .15. ℭ. be added to .18. ℨ ℨ. it will make .18. ℨ ℨ. —+— .15. ℭ. An so .12. ʃℨ. ioyned with .20. ℨ ℭ. dooe make .20. ℨ ℭ. —+— 12. ʃℨ.

## *Of Subtraction.*

Master.

*Subtractiõ of vnlike signes*

*Vbtraction* is as easie: for it doeth depend onely of the signe of abatemente, which is this. ———. and signifieth lesse, or abatyng. And therefore if I would abate 6. ℨ. out of .10. ℨ. I must sette it thus 10. ℨ. ———. 6. ℨ: that is to saie .10. ℨ. lesse .6. ℨ. or abatyng .6. ℨ.

Scholar. Then if I haue 30 ℭ. and would abate out of thẽ .12. ♀. I must set it thus .30. ℭ. ———. 12 ♀. that is .30. *cubes* saue .12. nombers. And if multiplica-

tion

tion and diuiſion, bee as eaſie, thei ſhall neade no greate ſtudie.

## Of Multiplication.

Maſter.

SOmewhat moze labourc is there in multiplication and diuiſion, to finde out the newe ſignes as I wil tell you anon. But foz findyng of the nombers, the common multiplication and diuiſion doeth ſerue. So that when. 12. ℨ. is multiplied by. 6. ℨ. it maketh. 72. ☌. And if. 24. ☌. bee multiplied by. 5. ℨ. there riſeth. 120. ſℨ. *Multiplication.*

Scholar. This paſſeth my cunnynge, foz the findyng of the newe ſigne: although the multiplication of the nombers, be as eaſie as can be.

Maſter. If you did well remẽber, what you haue learned befoze: the mater would not ſeme ſo harde.

Doe not you knowe, that a roote multiplied by a roote, doeth make a ſquare? And a ſquare multiplied by his roote, doeth bzyng fozthe a cube?

Scholar. That I knowe right well: and therfoze a *Square of Squares* multiplied by his roote, will yelde a *Surſolide*.

Maſter. Then by like reaſon, a *Cube* multiplied by a *Square*, ſhall make a *Surſolide*.

Scholar. In deede it is all one, to multiplie a *cube* by a *Square*, and a *Square of Squares* by a roote.

Maſter. Then foz a generalle rule, I will ſette fozthe here a pzeſidente foz you: whereby you maie knowe the newe ſigne, in all multiplication oz diuiſion: not onely by ſight very ſpedily, but that you maie alſo commit it aptly to memozie.

Wherfoze marke wel this table folowing: where you ſee in the higher rowe, a line of nombers, ſet in naturall

naturall progression: and vnder them you see the signes of *coßike nombers*.

## *The table of Coßike signes, and their peculier nombers.*

| 0. | 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|---|
| ꝙ. | ⅏. | ⅋. | ce. | ⅋⅋. | ſ⅋. | ⅋ce |
| 7. | 8. | 9. | 10. | 11. | 12. | 13. |
| $^{b}$ſ⅋. | ⅋⅋⅋ | ce ce. | ⅋ſ⅋. | $^{c}$ſ⅋. | ⅋⅋ce | $^{d}$ſ⅋. |

This table is largely set forthe, in the title of progression, whereunto you maie haue recourse, if your nomber be to greate for this table.

By this table maie you easily knowe, the signe that shall serue for your newe somme, in multiplication.

As for example, if I dooe multiple squares by rootes: I looke in the table, what nombers stande ouer them bothe, and puttyng those .2. nombers together, I seke the totall in thesame line, and vnder it I finde the newe denomination *coßike*, whiche I should haue

Scholar. I perceiue ouer .⅏. the nomber of 1. and ouer .⅋. the nomber .2. whiche bothe added together make .3. And bicause vnder .3. I find the figure or signe of .ce. I muste take that for the newe denomination.

Master. You saie truthe.

Scholar. Then if I multiplie .12. ⅋ce. by .8. ce. the somme will be .96. ce ce. For ouer. ce. I finde 3. and ouer .⅋ce. standeth .6. whiche bothe together doe make .9. and vnder .9. I see .ce ce. whiche I take for the denominator.

And if thesame rule bee generall, I am cunnynge inougbe

inoughe in it.

Maſter. It is generall, for multiplication in this kinde.

## Of Diuiſion.

BVt for diuiſion, you muſte abate the one nomber out of the other, to finde a newe denomination. *Diuiſion.*

Therfore if you would diuide 96.ȼ ȼ by.8.ȼ. the *quotiente* will be 12.ʒ ȼ. bicauſe that ouer the ſigne of your diuidende, ſtandeth 9. And ouer the diuiſors ſigne is ſet 3. Wherfore abatyng.3.from.9. there reſteth. 6. vnder whiche is the ſigne.ʒ ȼ.that I muſt take, to put to my *quotiente*.

Scholar. Then for an other triall, if I would diuide.260.ęſʒ. by.5.ſʒ. the *quotiēt* will be 52.ʒʒ ȼ For bicauſe that ouer.ęſʒ. I finde.17.and ouer.ſʒ. ſtandeth.5.then ſubtractyng.5.frō.17.there reſteth.12 vnder whiche in the table I finde. ʒʒ ȼ.

So diuidyng.20.ȼ. by.4.9. the *quotiente* will bee 5.ȼ: and ſo of other.

Maſter. But and if you would diuide.12.ȼ. by 5.ʒ. that muſt be ſet in forme of fraction, thus. $\frac{12 ȼ}{5 ʒ}$.

So.18.ʒ. by.7.ʒo. maketh. $\frac{18 ʒ}{7 ʒo}$ and 6.ʒ. by.2.ȼ. yeldeth. $\frac{6 ʒ}{2 ȼ}$. of whiche fractions, wee will ſpeake emongeſte the fractions of *Coſsikes* compoūde. For thei degenerate out of this kinde.

Wherefore this maie ſuffice briefly, for the cuſtomable woorkes of whole *Coſsike nombers*.

## Of Fractions in Coſsike nombers.

AND as for fractions, the woorkyng is like in euery poincte, vnto the woorke of nombers *Abſtracte*: remembryng onely that as thoſe broken nombers, haue a *Coſsike* denomination annexed with them, ſo muſt *Offractions in nombers Coſsike.*

that denomination followe the rules, now lafte declared.

Wherefore I shall not nede to doe any more, but to set forthe onely certain examples, of euery kinde of woorke in them.

## Examples of Numeration.

$\frac{2}{3}$ ☉. Signifieth $\frac{2}{3}$ of a *Roote*.

$\frac{8}{9}$ ☐. Betokeneth $\frac{8}{9}$ of a *Square*.

$\frac{12}{17}$ ☇. Representeth $\frac{12}{17}$ of a *Cube*.

And so of all other formes of *Cossike* signes: whereby is intended, that the *Cossike quantitie*, is diuided into so many partes, as the denominator containeth, and there is here represented onely so many of them, as the numerator doeth importe.

Scholar. Hereby I dooe perceiue, that a fraction *Cossike*, maie signifie a nomber, and not onely a parte of an vnitie, as it did in nombers *Abstracte*.

For when I saie $\frac{2}{3}$ ☐. if that *Square* be. 9. then that fraction signifieth. 6. But if the *Square* be. 4. then that fraction doeth represente. $2\frac{2}{3}$.

Likewaies $\frac{3}{4}$ ☇. if the *Cube* be. 8. then that fraction doeth signifie. 6. But if the *Cube* be. 27. then that fraction is equalle to. $2\,0.\frac{1}{4}$.

Master. You doe consider it well.

## Of Addition.

*Addition.* Now for addition, take these examples.

$\frac{2}{3}$ ☐. added to $\frac{3}{4}$ ☐. doe make $\frac{17}{12}$ ☐. or. $1$ ☐. $\frac{5}{12}$.

$\frac{5}{7}$ ☇ ioined with $\frac{7}{8}$ ☇. doe make $\frac{89}{56}$ ☇. or $1.$ ☇ $\frac{33}{56}$.

And in vnlike signes.

$\frac{3}{4}$ ☐. added to $\frac{4}{5}$ ☇. doe make $\frac{4}{5}$ ☇. $+$ $\frac{3}{4}$ ☐. or els thus by one common denominator.

$$\frac{16.\text{☇} + 15.\text{☐}.}{20.}$$

Of

Of whiche I will speake moze in the *Binomialles*, and therefoze will omitte it, till we come to them.

Scholar. As foz the reste, I see it well: Foz the woozke is all one with fractions *Abstracte*.

And here the denominatiõ oz *Coßike* signe is not varied, although here be vsed diuerse multiplications.

Master. And good reason: foz the whole *quotiente* whiche is repzesented by that *Coßike* signe, is not multiplied, but certaine partes of it: and therefoze ought that *Coßike* signe, to stand vnaltered, as the quantitie repzesented by it, is not multiplied noz altered.

## *Examples of Subtraction.*

$\frac{2}{3}$ ☌. abated out of $\frac{3}{4}$ ☌. doe leaue $\frac{1}{12}$ ☌.

$\frac{5}{9}$ ☽. out of $\frac{7}{8}$ ☽. there resteth $\frac{23}{72}$ ☽.

$\frac{9}{21}$ ☽☽. subtracted frõ $\frac{4}{5}$ ☽☽ doe leaue $\frac{20}{35}$ ☽☽, oz $\frac{4}{35}$ ☽☽

And in vnlike signes.

$\frac{1}{8}$ ☽☌ abated frõ $\frac{6}{11}$ ☌☌ doe leue $\frac{6}{11}$ ☌☌ —— $\frac{3}{8}$ ☽☌

$\frac{1}{4}$ ☽ taken out of $\frac{9}{13}$ ☌. the reste is $\frac{9}{13}$ ☌ —— $\frac{1}{4}$ ☽.

Likewaies as in additiõ, so in this sozte of subtraction, there maie be an other kinde of woozke, whiche I will remit to the treatice of *Binomialles*.

## *Examples of Multiplication.*

$\frac{1}{3}$ ☽ multiplied by $\frac{3}{8}$ ☽. doe make $\frac{9}{40}$ ☽☽.

$\frac{5}{7}$ ☾. multiplied by $\frac{8}{9}$ ☾. bzyngeth fozthe. $\frac{40}{63}$ ☽.

$\frac{11}{19}$ ☽. multiplieb by $\frac{3}{5}$ ☽. doe yelde $\frac{24}{57}$ ☽☽, oz $\frac{8}{19}$ ☽☽.

Here the signes doe alter, as in the multiplication of whole *Coßike* nombers.

Scholar. This doeth somewhat trouble me: that the *Coßike* signes should chaunge here, rather then in addition, or subtraction: Seyng there was as moche multiplication, in any of them bothe, as there is here.

Master. Marke the mater well, and you shall bee sone satisfied.

For in addition and subtraction, the multiplicatiō serueth onely for the reduction, of the. 2. fractions, vnto one denomination: And therefore in them, you neuer multiplie the numerators together: but you multiplie crosse waies, the numerator of the one, by the denominator of the other, where as in multiplicatiō, you vse no reduction, but doe make a plaine multiplication.

And so likewaies in diuisiō, there is vsed no meane of reduction: and therefore in it the signes must aiter, as before is declared.

## *Examples of Diuision.*

$\frac{6}{7}$ ☉ ☉. diuided by $\frac{6}{11}$ ☉. doe make in the *quotiente* $\frac{66}{42}$ ☉. or $\frac{11}{7}$ ☉.

$\frac{7}{9}$ ☾. diuided by $\frac{8}{15}$ ☾. doeth yelde $\frac{105}{72}$ ☿. or els $\frac{35}{24}$.

For seyng I shall diuide. ☾. by. ☾. I must therefore abate. ☽. from. ☽. and so resteth nothing, whiche is signified by this Cipher. ○. and that standeth ouer the signe of nomber: therefore the fraction, that is as the *quotiente*, must be taken as a nomber *Abstracte*.

Likewaies $\frac{8}{3}$ ☉ ☉. diuided by $\frac{8}{9}$ ☉ ☉. doeth make $\frac{72}{24}$ ☿. that is to saie. ☽. And so $\frac{8}{15}$ ☾ ☾. diuided by $\frac{9}{10}$ ☾ doeth bryng forthe $\frac{80}{135}$ ☉ ☾. or $\frac{16}{27}$ ☉ ☾.

Scholar. This is sufficiente for diuision. Now if you thinke good to speake of progression, I can not but remember you of your promise.

Of

## Of Reduction.

Master.

ALthough *Reduction* should go in order before *Progression*, yet seeyng this *Reduction*, consisteth in the onely nombers, and not in the signes: and therefore agreeth with vulgare reduction of fractions (as here you maie see before in diuerse examples) therfore wil we omitte it, and go in hande with *Progression*: whiche is more straunge. *Reduction.*

Scholar. I praie you so: For I see this reduction, is but to reduce the greater fraction, to a lesser in nōber: as I learned long a gone by your other booke.

## Of Progression in Cossike signes.

Master.

PRogression is thus wrought: Firste sette doune as many vulgare nōbers, in their naturall progression, as you liste to haue *Cossike* signes, that by them you maie the better know, the true places of the *Cossike* signes: so that you set in the firste place a Cipher, and vnder it. ☉. And then vnder. 1. set. ☽. vnder. 2. put ☿. and vnder. 3. write ♂. As you see in the table folowyng. And by these shall you set, as many as you liste.

For all the vulgare nombers, whiche you haue set in the higher rewe, be other compounde nombers, or els vncompounde: and if the place, where you would set any *Cossike* signe, be noted with a nomber vncompounde, then must there be set one of the *Sursolides*.

For vnder the firste nōber vncompounde, you must set the firste *Sursolide*, and the second vnder the second nomber vncōpounde: and the thirde vnder the thirde,

and so forthe.

The nombers vncompoũde, are these in their progression.

5. 7. 11. 13. 17. 19. 23. 29. 31. 37. 41. 43. 47. 53. 59. 61. 67. &c.

Vnder nethe. 5. must you set. ſ3. and vnder. 7. b ſ3. vnder. 11. c ſ3. and vnder. 13. d ſ3. and so forthe, til you come to. 67. vnder whiche you must set. r ſ3. and vnder 71 you must set s ſ3. and so as farre as you list.

But for any other place, bicause the vulgare nomber is compounde, that is set (as the peculiare nomber, in the higher rewe) therefore the *Cossike* signe must nedes be compounde, other of. 2. or of. 3. or els of bothe. And if it be cõpounde of. 2. then set doune. 3. so often tymes, as. 2. is in the cõposition of that nomber.

As for example: 16. is cõpounde of. 2. fower tymes (not by addition, but by multiplication, as in saiyng, twise. 2. twoo tymes, twise.

Scholar. I perceiue twise. 2. to bee. 4. and twise that to be. 8. and twise that to make. 16.

Master. So maie you worke backewarde, in saiyng. 16. diuided by. 2. maketh. 8. that is ones: then. 8. by. 2. yeldeth. 4. that is twise. Again. 4. by. 2. maketh 2. that is thrise: and. 2. for himself, is the fourth: wherfore vnder. 16. I must set doune. 3 3 3 3.

And so vnder. 32. I muste sette. 5. 3. in one thus. 3 3 3 3 3.

And vnder. 64. I shall sette it. 6. tymes, thus. 3 3 3 3 3 3. Bicause. 64. is made of. 6. multiplications by. 2.

Scholar. Here by I see, that vnder. 8. I muste put 3. tymes that signe: and vnder. 4. twise thesame.

Master. So must you in deede.

And now for other places, if their nombers bee cõ-

pounde

pounde of. 3. onely, then muſt you ſet doune the ſigne of *Cube*, as oftentymes as. 3. is multiplied, to make that nomber.

As for example. 27. is compounde onely of. 3. and not of. 2. (for of all other compounde nombers herein then of ſoche as be cõpounde of. 2. or. 3. we take no regarde.) And. 3. multiplied thrise, doeth make. 27. in ſaiyng. 3. tymes. 3. thrise. And therefore vnder. 27. I ſhall ſet this ſigne of. ☌. three times, thus. ☌☌☌. whiche betokeneth a *Cube of Cubes Cubikely*.

But and if the nomber bee compounde, bothe of. 2. and. 3. then for euery tyme that. 2. is multiplied, to that compoſition, I ſhall ſette. ℨ. and for euery tyme that. 3. is multiplied, I ſhall ſet. ☌. remembryng ſtill to ſet. ℨ. before. ☌. and not after hym.

As for example. Vnder. 24. I ſhall ſet. ℨ ℨ ℨ ☌. bicauſe that. 2. 2. 2. 3. that is to ſaie. 2. tymes. 2. twiſe thrise, doeth make. 24. Or by reſolution, thus. 24. diuided by. 2. giueth. 12. For that firſte. 2. ſet. ℨ. Again 12. diuided by. 2. yeldeth. 6: for this ſeconde. 2. ſet. ℨ. alſo. Then diuide. 6. by. 2. and it maketh. 3. For the. 2. I muſt ſet. ℨ. and for. 3. I muſt put. ☌. and ſo all together maketh. ℨ ℨ ℨ ☌. in the. 24. place.

Likewaies vnder. 36. I muſt ſette. ℨ ℨ ☌ ☌. bicauſe that. 2. 2. 3. 3. doeth make it, that is. 2. tymes. 2. thrise, thrise. And by reſolution, thus. 36. diuided by 2. giueth. 18. For that. 2. I ſet. ℨ. Againe. 18. diuided by. 2. maketh. 9. For that. 2. I ſette doune againe. ℨ. Thirdly, for bicauſe. 9. can not bee diuided by. 2. but by. 3. 3. tymes: therefore I muſte ſette doune, for thoſe twoo. 3. twiſe. ☌. & ſo the whole ſigne is. ℨ ℨ ☌ ☌.

Now if the nomber of the place, or peculiare nomber, bee compounde of one of theim twoo, with ſome other nomber vncõpounde, then muſt we ioyne their ſignes together.

As. 10. is compounde of. 2. and. 5. therefore muſt I

ſet

set vnder. 10. the signe that is in the fifth place, whiche is ſz. and before it I muste set the signe of. z. for 2. So must that signe be. z ſz.

Likewaies, bicause. 15. is compounde of. 3. and. 5. I shall ioine together the signe of c. and of. ſz. and make it. c ſz.

Scholar. So I vnderstande it now, that I cannot misse it. Saue that for lacke of vse, and throughe forgetfulnesse, when I heare the name of composition in nombers, I doe mistake it sometimes for addition, els here can be no errour. For when I doe consider, that. 20. is compounde of. 2. 2. 5. that is twise. 2. and. 5 (sith. 2. tymes. 2. maketh. 4: and. 5. tymes. 4. maketh 20.) I maie sone consider, to set. z. twise before. ſz. and then it will be. z z ſz. to be put in the. 20. place.

Likewaies in the. 21. place, I set. c bſz. seyng 21 is compounde of. 3. and. 7. and. c. is the signe to the thirde place, as bſz. serueth for the. 7. place.

Master. What shall you set in the. 84. place?

Scholar. 84. is compounde of. 2. 2. 3. 7. therefore his signe must be. z z c bſz.

Master. Now I see, you are cunnyng inough in this, and therefore take here this table, for a patrone: and then will we procede to the worke of *Cossike nombers* compounde.

## *The table for progression Cossike,*
### *whiche maie increase it self infinitely, without any difficultie.*

| 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 𝄞. | 𝅝. | 𝅗𝅥. | 𝅘𝅥. | 𝅘𝅥𝅘𝅥. | 𝄽𝅗𝅥. | 𝅗𝅥𝅘𝅥. | 𝅘𝅥𝅘𝅥. | 𝅗𝅥𝅗𝅥. | 𝅘𝅥𝅘𝅥. | 𝅗𝅥𝄽𝅗𝅥. | 𝄽𝅗𝅥𝄽𝅗𝅥. |

| 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. |
|---|---|---|---|---|---|---|---|---|
| 𝅗𝅥𝅗𝅥. | D𝄽𝅗𝅥. | 𝅗𝅥E♭. | 𝅘𝅥𝅘𝅥. | 𝅗𝅥𝅗𝅥. | E𝄽𝅗𝅥. | 𝅗𝅥𝅘𝅥. | F𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. |

| 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. |
|---|---|---|---|---|---|---|---|
| 𝅘𝅥B♭. | 𝅗𝅥C𝄽𝅗𝅥. | G𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | 𝄽𝅗𝅥. | D𝄽𝅗𝅥. | 𝅘𝅥. | 𝅗𝅥B♭. |

| 29. | 30. | 31. | 32. | 33. | 34. | 35. | 36. |
|---|---|---|---|---|---|---|---|
| H𝄽𝅗𝅥. | 𝅗𝅥𝅘𝅥. | I𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | 𝅘𝅥C𝄽𝅗𝅥. | 𝅗𝅥E𝄽𝅗𝅥. | 𝄽𝅗𝅥B♭. | 𝅗𝅥𝅘𝅥. |

| 37. | 38. | 39. | 40. | 41. | 42. | 43. | 44. |
|---|---|---|---|---|---|---|---|
| K𝄽𝅗𝅥. | 𝅗𝅥F𝄽𝅗𝅥. | 𝅘𝅥D𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | L𝄽𝅗𝅥. | 𝅗𝅥𝅘𝅥B♭. | M𝄽𝅗𝅥. | 𝅗𝅥C𝄽𝅗𝅥. |

| 45. | 46. | 47. | 48. | 49. | 50. | 51. |
|---|---|---|---|---|---|---|
| 𝅘𝅥𝅘𝅥. | 𝅗𝅥G𝄽𝅗𝅥. | N𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | B♭𝄽𝅗𝅥. | B♭𝄽𝅗𝅥. | 𝅗𝅥𝄽𝅗𝅥. | 𝅘𝅥E𝄽𝅗𝅥. |

| 52. | 53. | 54. | 55. | 56. | 57. | 58. |
|---|---|---|---|---|---|---|
| 𝅗𝅥𝅗𝅥. | D𝄽𝅗𝅥. | O𝄽𝅗𝅥. | 𝅗𝅥𝅘𝅥. | 𝅘𝅥C𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | 𝅗𝅥B♭. | 𝅘𝅥F𝄽𝅗𝅥. | 𝅗𝅥H𝄽𝅗𝅥. |

| 59. | 60. | 61. | 62. | 63. | 64. | 65. |
|---|---|---|---|---|---|---|
| P𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | Q𝄽𝅗𝅥. | 𝅗𝅥I𝄽𝅗𝅥. | 𝅘𝅥𝅘𝅥B♭. | 𝅗𝅥𝅗𝅥. | 𝄽𝅗𝅥D𝄽𝅗𝅥. |

| 66. | 67. | 68. | 69. | 70. | 71. | 72. | 73. |
|---|---|---|---|---|---|---|---|
| 𝅗𝅥𝅘𝅥. | C𝄽𝅗𝅥. | R𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥E♭. | S𝄽𝅗𝅥. | 𝅗𝅥B♭. | T𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | V𝄽𝅗𝅥. |

| 74. | 75. | 76. | 77. | 78. | 79. | 80. |
|---|---|---|---|---|---|---|
| 𝅗𝅥K𝄽𝅗𝅥. | 𝅘𝅥𝅘𝅥. | 𝅗𝅥𝅗𝅥F𝄽𝅗𝅥. | B♭𝄽𝅗𝅥. | C𝄽𝅗𝅥. | 𝅗𝅥D𝄽𝅗𝅥. | X𝄽𝅗𝅥. | 𝅗𝅥𝅗𝅥. | 𝄽𝅗𝅥. |

In this table. 𝅗𝅥. 𝅘𝅥. and 𝄽𝅗𝅥. are the grounds: of all the reste aboue them. For of these thre, all those other bee made.

## Of Cossike nombers compounde.

*Cossike nombers* compounde, are made by addition of. 2. or more simple *Cossike* nombers together:

As. 6. ze. —+— .5. z. or.

12. ce. —+— 4. ze. —+— 3. 9. and so foorthe in diuerse formes, whiche be infinite. Howbeit for briefnesse, we maie comprehende, vnder thesame name (bicause of the like worke) all other *residualles Cossike*, whiche be made by subtraction: as. 3. ce. ——— .4. z. And also those that bee made by addition and subtraction, bothe together: As. 9. zz. —+— .4. ze. ——— 6. z. In whose numeration is no hardnesse.

Scholar. Then your rules maie be the shorter.

## Of Numeration.

Maister.

This *Numeration* is easily vnderstãde by addition of simple *Cossikes*. For this is the forme. 6. z —+— 10. 9. that is. 6. *Squares*, more. 10. nõbers. Likewaies. 8. ce. —+— .11. ze. is 8. *Cubes* and. 11. ze.

Now for *residualles*, take these examples. 9. zz. ——— .12. ce, whiche is. 9. *Squares of Squares*, saue. 12. *Cubes*. Also. 4. fz. ——— .15. z. that is. 4. *surfolides*, abatyng. 15. *squares*.

And for bothe together, this is the forme.

10. zz —+— .6. ce. ——— 30. ze. whiche signifieth 10. *Squares of Squares*, and. 6. *Cubes*, abatyng. 30. *rootes*.

Scholar. This is plaine. For so maie I vnderstande of all other As. 9. zce. ——— .3. z. —+— 8. 9 that is, 9. *Squares of Cubes*, lesse 3. *Squares*, more. 8. *nõbers*.

Maister.

Master. It were more orderly, to kepe the signes of more and lesse in order, then to followe the order of the *cossike* signes: bicause that addition, is orderly placed before subtraction. So were it better to set theim thus 9.z.ℭ.—|—8.ꝙ.————.3.z. Nobeit in deede all is one in these kinde of nombers, but not so in other *Surde nombers*, where the order folowth of necessitie, as shall be declared in their place more largely.

## Of Addition.

IN addition, you must haue consideration of the *Cossike* signes: for noe other nomber, maie bee added into one, then suche as appertain to one signe *Cossike*.

As in vulgare denominations, you doe not adde the nõbers of shillynges, to the nombers of pennies: but you ioine shillynges to shillinges, and pennies to pennies: t poundes to poundes, so in *Cossike* nombers, *Cubes* muste bee ioined to *Cubes*, *Squares* to *Squares*, and generally, like to like.

Scholar. If this be al, I cã marke it well inough.

Master. There is somewhat more to be considered, that if there bee any signe in the one nomber, whiche is not in the other, that seueralle signe with his nomber, muste bee sette doune with his figure of —|—.02.————.as it standeth there.

And farther, touchyng those twoo signes.—|—.————. whiche bee the figures of more and lesse, you must giue regarde, whether thei bee like or vnlike, in those nombers that must be added: For if thei be like in nombers, of one denomination, then muste thei so remain as ther be. But if thei be vnlike, euermore abate the smaller nomber of theim, that followe those

vnlike signes, out of the greater: and sette doune the reste, with the signe of the greater nomber.

Scholar. By examples, I shall better conceiue those rules.

Master. Take these examples.

| | |
|---|---|
| 10.z.—+—.12.q. | 10.z.———.12.q. |
| 4.z.—+—.8.q. | 4.z.———.8.q. |
| 14.z.—+—.20.q. | 14z.———.20q. |

| | |
|---|---|
| 10.z.———.8.q. | 10.z.—+—.8.q. |
| 4.z.———.12.q. | 4.z.—+—.12.q. |
| 14.z.———.20.q. | 14.z.—+—.20.q. |

| | |
|---|---|
| 10.z.—+—.12.q. | 10.z.———.12.q. |
| 4.z.———.8.q. | 4.z.—+—.8.q. |
| 14.z.—+—.4.q. | 14.z.———.4.q. |

| | |
|---|---|
| 10.z.—+—.8.q. | 10.z.———.8.q. |
| 4.z.———.12.q. | 4.z.—+—.12.q. |
| 14.z.———.4.q. | 14.z.—+—.4.q. |

Here haue I varied one example diuersly, to the intente you male marke the vse of your rules in theim.

And for the reason of those rules, you shall marke those

thoſe eramples well.

Foꝛwhere in the firſte erample, bothe ſignes are —|—, it muſt nedes be, that after the addition of the firſte nombers, the ſeconde muſte bee added with the ſigne. —|—.

In the ſeconde erample, where bothe the ſignes be ——. bicauſe there wanteth. 21. ♀. of the firſt. 10. ☽. Therfoꝛe is it reaſon, that bothe thoſe wantes ſhould be ſette doune with the ſigne of. ——: and ſo in the thirde and fourthe eramples.

In the fifth erample, the ſeconde ſomme is not fully. 4. ☽. but there wanteth of it. 8. ♀. and therefoꝛe if you put donne the. 4. ☽. fully, you muſt abate. 8. out of the. 12. ♀. in the higher ſomme: and ſo of the other eramples.

But foꝛ moꝛe practiſe, and better declaration of the vſe of them, here are other eräples, of moꝛe varietie.

| | | | |
|---|---|---|---|
| 20. ☽ ♂. | —\|— | 9. ☽. | —— .120. ☿. |
| 15. ☽ ♂. | —\|— | 5. ☽. | —\|— .16. ☿. |
| 35. ☽ ♂. | —\|— | 14. ☽. | —— .104. ☿. |

| | | | |
|---|---|---|---|
| 16. ☽ ☽. | —\|— | 28. ♀. | —— .16. ☽. |
| 12. ☽ ☽. | —\|— | 12. ☽. | —— .19. ♀. |
| 28. ☽ ☽. | —\|— | 9. ♀. | —— .4. ☽. |

In the firſte erample of theſe. 2. you ſee. 120. ☿. with the ſigne of leſſe, to bee added with. 16. ☿. with the ſigne of moꝛe: and therefoꝛe, ſeeyng the ſignes of one *Coſsike* denomination diſagree, I dooe ſubtracte the leſſer, out of the greater: and that. 104. whiche remaineth, I doe ſet doune with the ſigne of

lesse, bicause the remainer is of that uomber, that bare that signe.

And in the seconde exãple, the placyng of the signe —+— before —— maketh nombers to bee sette before squares: and so the like denominations, dooe not stande one ouer an other. Yet is the wooꝛke dooen as if thei did stande eche ouer his like.

Scholar. I praie you lette me trie my cunnynge, with an exampleꝛ oꝛ twoo.

17. ʒ ʒ. —+—. 10. ☍. ——. 2. ☉.
16. ʒ ☍. —+—. 12. ʒ. ——. 6. ☉.

16. ʒ ☍. —+— 17. ʒ ʒ. —+— 10. ☍.
—+— 12. ʒ. ——. 8. ☉.

I set the exampl, as nombers came to my mynde: but I had almoste set my selfe on grounde: saue that I called to remembꝛaunce, the comparison that you made, to vulgare denominations of poundes, shillinges, and pennies: and so was instructed to place euery seueralle denomination seuerally. And to sette the greateste denominatiõ first, & eche other in his oꝛder.

Now will I pꝛoue an other exampl, oꝛ twoo.

3. ſʒ. —+—. 4. ☍. ——. 20. ☬.
20. ☍. ——. 8. ʒ. ——. 16. ☬.

3. ſʒ. —+—. 24. ☍. ——. 8. ʒ. ——. 36. ☬.

13. ʒ ☍. —+—. 8. ☍. ——. 4. ☉.
7. ʒ ☍. ——. 6. ☍. ——. 7. ☬.

20. ʒ ☍. —+—. 2. ☍. ——. 4. ☉. ——. 7. ☬
6. ʒ.

6.ʒ. — + — .10.ʒʒ. —— .8.ʃ.
4.ʒ. — + — .17.ʃ. —— .7.ʒʒ.

10. — + — .3.ʒʒ. — + — .9.ʃ.

4.ʒ.ʒʒ. — + — .5.ʒ. — + — .6.ʒʒ.
8.ʒʒ. —— .8.ʒ. —— 10ʒʒ.

4.ʒ.ʒʒ. — + — .8.ʒʒ. —— .3.ʒ. —— .4.ʒʒ.

Maſter. You haue doen well: And for proofe of your worke, you maie in this arte not onely proue it, by the contrary kynde, as you did in nõbers *Abſtracte*, but alſo by the reſolution of all thoſe *Coſsike* nombers into nõbers *Abſtracte*, takyng any nomber for a roote and then the *Squares* and *Cubes*. &c. accordingly. As here in this table, you maie briefly ſee, but more largely in the table at the eande of nombers figuralle.

## *A table for trialle by reſolution, of any woorke in this arte.*

| ʒʒ | ʒ | ʒʒ.ʒʒ | ʃʒ | ʒ.ʒʒ | bſʒ |
|---|---|---|---|---|---|
| 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 46384 |

| ʒ.ʒ.ʒ | ʒʒ.ʒʒ | ʒ.ſʒ | |
|---|---|---|---|
| 256 | 121 | 1024 | |
| 6561 | 19683 | 59049 | |
| 65536 | 262144 | 1048576 | |

And if this table in any parte, ſeme to ſhorte or to litle:

litle: you maie haue recourse to the table, at the ende of figuralle nombers, whiche therfore is made large and generalle: so that it maie well be called the frutefull table, or table of ease.

But now for triall of the laste example: firste there is. 4. ☞ &: for whose roote I take 2. and therefore those. 4. ☞ &. make. 256. Whiche I sette doune in nomber *Abstracte*. Nexte is. 5. squares, whiche accordyng to that roote, must nedes be. 20. and that. 20. I sette doune also: and then. 6. rootes, whiche make 12. And all thei yelde. 288. and that is all the firste somme.

256.
20.
12.
288.

Then for the seconde somme, I see firste. 8. *Cubes*, whiche make. 64. to bee added. Then foloweth. 8. squares lesse, that is. 32. to bee abated, and also. 6. rootes lesse, that is. 20. also to bee abated: So must I abate. 52. (for theim bothe) out of. 64. and then there resteth but 12. whiche added vnto 288. of the firste somme doe yelde. 300.

32.
20.
52.
64.
52.
12.
288
12.
300

Now if the totall agree with this, then is the woorke good.

For triall whereof, I resolue. 4. ☞ &. into nomber *Abstracte*, and thei will make. 256. then. 8. &. maketh. 64: whiche bothe yelde 320. Then foloweth in thesame somme. 3. ☞ and. 4. ☜. to be abated. The. 3. ☞. make. 12. and the 4. rootes yelde. 8. whiche together do amounte to. 20. and that must bee abated frō thesaid somme of 320 and then there remaineth onely 300. agreable to the former somme aboue the line.

256
64
320.

Scholar. This proofe I like well: And I perceiue that if I would woorke the like, takyng for the roote 3, or any other nomber, the proofe will succede a like.

Master. Now to make an ende of Addition, bicause

cause you shall the better remember the rules of it, I will giue you them in this brife forme.

*In greatenesse like and signes also,*
*Adde like to like there nedes no mo:*
*And where the greatenesse disagree,*
*Place eche by other severally.*
*With signe of eche, as doeth require,*
*But if the signes vnlike appere:*
*Then from the more abate the lesse,*
*The greater his signe with the excesse.*
*Will make the somme,*
*Of that addition.*
*The proofe is by resoluyng,*
*Eche nomber into his reckenyng.*

This lesson doeth containe the former rules onely in brief, and therefore needeth no declaration: but the greatenesse doeth betoken the *Cossike* denomination, and signes betoken specially, —|— and . ——— . the signes of more and lesse, and no other signes.

Scholar. This brief lesson will helpe memorie moche: and shall suffice for the rules of Addition.

## *Of Subtraction.*

Master.

When for subtraction, this shall you marke in especiall: that when your nombers are sette doune, after the cõmon maner, firste the totall, and then the deduction: you shall consider well, whether the signes bee —|— or ———. For in the deduction, if you haue —|— then must that be subtracted from the like aboue. 1. *Rule.*

And if that somme in the deduction, that hath the 2. *Rule.*

signe —|—, bee greater then the nomber of the like quantitie ouer hym, with the like signe —|—. then abate the higher out of the lougher, and write the reste with this signe ———.

3. Rule. But if the like quantitie in the totall, haue the signe ———, then adde bothe nombers together and set them vnder the line with that signe ———.

4. Rule. And if the seconde somme (that is the deduction or abatement) with any nomber, haue this signe of lesse ———, it must be accoumpted for more, and must be added to the like nomber ouer it, excepte the ouer nõber haue the signe of lesse also: For then must you abate the lesser, out of the greater, and sette doune the reste, with the signe of the greater nomber: whiche thei haue at this conferẽce: I meane to regarde what the signe of the seconde somme is by estimation, and not by writyng, for thei are contrary.

Scholar. I see good reason in this: For in any abatemente, the more is abated, the lesse by so moche shall remain: and the lesse is abated, the more doeth remain by so moche.

5. Rule. Master. Yet one thyng more is to bee marked, that if there be some denommations, in the one sõme that are not in the other, you shall marke in whiche somme thei bee. For if thei bee in the firste, then shall thei kepe still their owne signe. And if thei bee in the seconde somme, whiche is the deduction, then shall thei chaunge their signe to the contrary: But where soeuer thei be, thei must be set in the remainer.

Scholar. I can better vnderstande you, then remembre those rules.

Master. Then take this brief lesson, apter to bee remembred, then to bee vnderstande, but by the letter before, and by the examples folowyng. But memorie liketh well soche aide.

## A brief rule of Subtraction.

1. *When ſignes and greateneſſe bothe agree,*
*Your woorke procedeth forthe commonly.*
2. *But if thabatemente greater bee,*
*Thexceſſe ſhall chaunge his ſigne therby.*

3. *And where the ſignes doe diſſagree,*
*The higher ſigne muſt reſt duely:*
*And though the batemente be the greater,*
*The reſte ſtill ioyneth boibe ſommes together.*

4. *If quantities doe diſſagree,*
*Place them with ſignes all ſeuerallie:*
*The totall kepeth the ſigne he had,*
*The batemente ſtill, to chaunge is glad.*

**Scholar.** Now ſome examples, will lighten theſe rules well.

**Maſter.** I will propounde the like, as I did in addition, to the intēte you maie iudge the likeneſſe, and diuerſities in bothe woorkes.

| | |
|---|---|
| 10.☉.—+—.12.☾. | 10.☉.—+—8.☾. |
| 4.☉.—+—.8.☾. | 4.☉.—+—12.☾. |
| 6.☉.—+—.4.☾. | 6.☉.——4.☾. |

| | |
|---|---|
| 10.☉.——.12.☾. | 10.☉.——.8.☾. |
| 4.☉.——.8.☾. | 4.☉.——.12.☾. |
| 6.☉.——.4. | 6,☉.—+—.4. |

| | |
|---|---|
| 10.z. — + — .12.9. | 10.z. — + — .8.9. |
| 4.z. ——— .8.9. | 4.z. ——— .12.9. |
| 6.z. — + — .20.9. | 6.z. — + — .20.9. |

| | |
|---|---|
| 10.z. ——— .12.9. | 10.z. ——— .8.9. |
| 4.z. — + — .8.9. | 4.z. — + — .12.9. |
| 6.z. ——— .20.9. | 6.z. ——— .20.9. |

The firste and thirde examples be very plaine: and in the seconde where .12. should bee abated out of .8. there is .4. to fewe: and therefore I abate the higher, out of the lougher, and I set doune .4. with the signe of wantyng, or abatemente.

In the fourthe example: bicause the higher nomber is the lesser, I doe subtracte him out of the nether, and sette doune the reste .4. with a contrary signe of — + —.

But in the .4. later examples, where the signes do disagree, the nõbers that followe the signes, are not subtracted one from an other, but are added together: and thei take still the higher signe. Bicause in value, the signe of abatemente is contrary, to that it appeareth to bee.

And for your exercise, to make you full prompte in this arte, I haue set forthe more examples.

| | |
|---|---|
| 6.ce. — + — .120.9. | 8.z.ce. |
| 9.ce. ——— .40.9. | 9.z.ce. ——— 89.9. |
| 160.9. ——— .3.ce. | 89.9. ——— .1.z.ce. |

3. ☉. —+— 18. ☽.
12 ☽ ——— . 3. ☉.
6. ☉. —+— . 6. ☽.

18. ☽. —+— . 3. ☉.
12. ☽. ——— 3. ☉.
6. ☽. —+— . 6. ☉.

3. ☉. —+— . 18. ☽. ——— . 10. ♀.
12. ☽. —+— . 8. ♀.
3. ☉. —+— . 6. ☽. ——— . 18. ♀.

4. ♂☉. —+— . 10. ♌. ——— . 6. ☉.
5. ☉☉. —+— . 12. ☉. —+— . 3. ♀.
4. ♂☉ —+— 10. ♌. ——— 5. ☉☉. ——— 18. ☉. ——— 3 ♀.

Here in the firſte example, where I would abate 9 ♌. out of. 6. ♌. I maie eaſily perceiue, that there are 3. ♌. to fewe. And therefore doe I ſette doune. 3. ♌. with this ſigne ———, whiche ſignifieth wante or abatemente: and the. 2. nombers that followe the vnlike ſignes, I ſet doune bothe added into one: and put therto the ſigne of the totall or ouermoſte ſomme.

In the ſeconde example, there is the like wooꝛke: For in abatyng. 9. out of. 8. I finde. 1. to fewe: that. 1. doe I ſet doune with his denomination of. ☉ ♌: and the ſigne. ———.

And the nomber 89 that folowcth the ſigne ——— in the ſeconde ſomme, ſtandeth in force as —+—, for the leſſe is abated, the more muſt remain: therfore in the remainer, I ſet not the ſigne of more, before that nomber of. 89. but I put it in the firſte place of the ſomme: whiche place of it ſelf, ſignifieth ſtill more.

And bicause ouer that nomber 89. there are no nũbers in the totall, therefore I muste putte doune that somme as it is, without addyng to it, or abatyng frõ it, in it self.

Scholar Those. 2. examples might be set thus, as I thinke, bicause the places doe so require.

6.ȼ.—+—.120.ϥ.
9.ȼ.———.40.ϥ.
———.3.ȼ.—+—.160.ϥ.

8.ʒ.ȼ.
9.ʒ.ȼ.———.8.ϥ.
———.1.ʒ.ȼ.—+—.8.ϥ.

Master. Remember your self well, and marke the remainer how it is written.

Scholar. I see my owne ouersighte: For no nomber maie begin, with signe of lesse: and therfore must their places be altered of necessitie, and set in order as thei were before.

Master. Then for all the reste of the examples, or any other like, I shall not neade to giue you any farther instruction: sith that by these former, you maie iudge of all other.

*Proofe.* And for the examination of your worke, the trialle by resolution doeth serue here, as well as els where: remembryng onely (as the order of subtraction maie admonishe you) that the somme of the totalle, whiche is the firste somme, must counteruaile the other bothe sommes: that is of the deduction, and of the remainer.

So to trie the firste example, takyng. 3. for a roote; 6.ȼ. make. 162. whiche I put to. 120. and it yeldeth 282. Then in the seconde somme. 9.ȼ. are. 243. whereof. 40. must bee abated for the signe ———, so is

is that ſomme. 203. Again in the remainer. 3. &. are 81. whiche muſt bee abated out of. 160. and ſo reſteth 79. whiche with. 203. doe make. 282. agreable with the firſte ſomme.

Scholar. This doe I well vnderſtande, and praie you to proceede to multiplication.

## Of Multiplication.

Maſter.

IN multiplication, there is no difficultie, ſo that you dooe well marke the ſignes —+— and ———, whiche beyng bothe like, will haue the ſigne —+— ſette in the totalle: and beyng vnlike, thei will haue in the totalle the ſigne ———.

And likewaies in diuiſion —+—. diuided by ——— or cõtrary waies ——— by —+—. will alwaies haue in the totalle ———: but —+— diuided by —+—, or ——— by ———, will make alwaie —+—.

Whiche rule for ready remembraunce, I haue giuen you here in meter.

*Who that will multiplie,*
*Or yet diuide trulie:*
*Shall like ſtill to haue more,*
*And miſlike leſſe in ſtore.*
*Their quantities doe kepe ſoche rate,*
*That. M. doeth adde: and. D. abate.*

Scholar. So meane you, that like ſignes multiplied together, doe make more, or —+—: And vnlike ſines multiplied together, doe yelde leſſe, or ———.

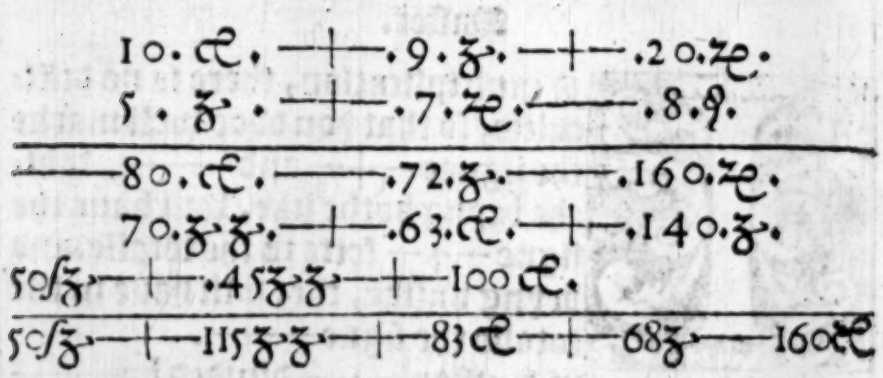
Maſter. So is the rule. But to go forward now: of the nexte difficultie, as touchyng Coſsike quantities that chaunge their denomination, here is no more to

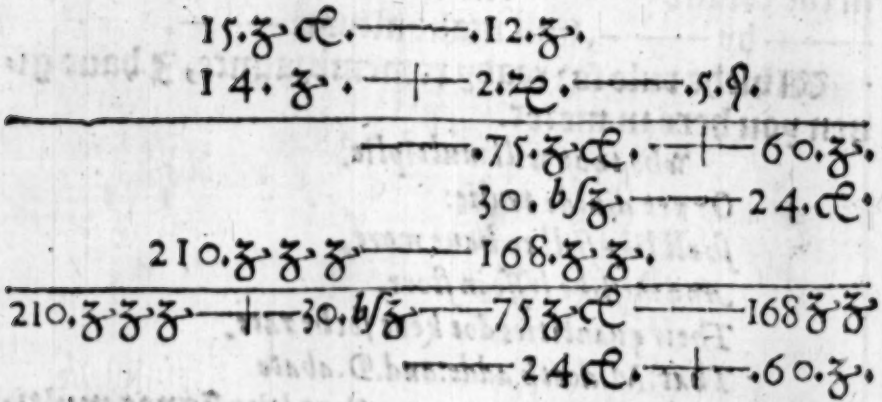
bee

bee saied, then was taught in multiplication of nombers *Cossike* vncompounde, and in the table set for the for the chaunge of their names.

Scholar. I vnderstande, that in multiplication (that is. M.) their figures must bee added. And in. D. (or diuision) thei muste bee abated. Therefore a fewe examples shall suffice for the reste.

Master. Take these for a presidente, of all that woorke: by whiche you maie iudge of all other like.




Scholar. I perceiue, that these woorkes doe appere more hard, then thei bee in deede, and that because of their straunge formes: but by vse I truste to bee acquainted with them well inough. and therefore I will begin with more easie examples. As these bee, that folowe

followe here.

18. ʒ. — + — .20 ϙ.
15. ze. ——— .4. ϙ.

——— 72. ʒ. ——— .80. ϙ.
270. ce. — + — 300 ze.

270. ce. — + — 300. ze. ——— 72. ʒ. ——— .80. ϙ.

16. ʒ. — + — .14. ze.
8. ce. ——— .7. ϙ.

——— 112. ʒ. ——— 98. ze.
128. ſʒ. — + — 112 ʒ. ʒ.

128. ſʒ — + — 112. ʒ. ʒ ——— 112. ʒ. ——— 98. ze.

And this I ſee farther now, that theſe wooꝛkes ſeme moꝛe difficulte to looke on, then thei be in pꝛactiſe, if a manne giue good hede to the ſignes, and the quantities.

Maſter. Beſoꝛe we go any farther, I will ſhewe you ſomewhat of the reaſon, why the ſignes ought to chaunge. And that by twoo plaine wooꝛkes, in nombers *Abſtracte*. As here foloweth.

Where you ſee that when I had multiplied. 16 — + — 12 by 20 it made. 320 — + — 240 that is in all. 560.

But bicauſe the multipliare ought not to be ſo moche by 4 theꝛfoꝛe it is reaſon, that I ſhall multiplie the higher ſomme by .4. and abate that out of the foꝛmer totall.

16. — + — .12.
20 ——— .4.

——— 64 ——— .48.
320 — + — .240

560 ——— 121
that is. 448.

Whiche thyng you see here doen by. ——. 64. ——. 48. whiche bothe make. 112. to bee deducted out of 560. and so remaineth 448. The iuste somme that commeth of that multiplication.

Scholar. This I vnderstande well: and maie proue it in this sorte. 16. —+—. 12. maketh. 28: and. 20. ——. 4. is. 16. Then if I multipie. 28. by. 16. it will yelde 448. as the woorke here declareth.

| |
|---|
| 28. |
| 16. |
| 168. |
| 28. |
| 442 |

And hereby maie I iudge, of *Cossike* nombers likewaies.

Master. Yet one example more will I propound bicause I would put you out of all doubte. Wherfore marke this forme of woorke.

Here you maie see, that if the firste somme of 24 ——3 wer multiplied by 15 it would make. 360. ——. 45. that is 315. But it ought not to bee so moche, but lesse by. 2. tymes 24 ——3. that is. 48 ——6: bicause the multiplier doeth wante. 2. of. 15.

| | |
|---|---|
| 24. ——. | 3. |
| 15. ——. | 2. |
| —— 48. —+—. | 6. |
| —— 360 ——. | 45. |
| 366 —— | 93. |
| that is. | 273. |

And so abatyng. 42. or. 48. ——. 6. out of. 315. there resteth. 273. whiche is the iuste totall, when. 21 is multiplied by. 13. wherby the multiplication is declared to bee good.

And for bicause that ——multiplied with—— doeth make —+—: marke here, that you maie not abate fully. 48. but 48. ——6,

Then seeyng in abatemente, the signes in figure are contrary to their owne estimation and force: therfore that. 4. 8. must be made ——. and the ——before. 6. tourned into —+—.

Scholar. I see it well, it must nedes be so.

For if thei were set, to bee subtracted, then should thei stande so. 48. ——. 6: whiche declareth that 42 should

ſhould bee abated.

But when theſame nombers, are ſet emongeſte o-ther to be added: as it is here in woꝛkyng of multipli-cation, then muſt thei be wꝛitten thus. —— 48 —+— declaryng that if you abate. 48. you muſte adde. 6. a-gain, bicauſe you abated. 6. moꝛe then you ought.

Maſter. Jou vnderſtand it well. Wherfoꝛe here will wee make an eande of multiplication: ſith there reſteth nothyng but the pꝛoofe of it: whiche maie bee wꝛought by reſolution, of all the *Coßike* nombers, in-to nombers *Abſtracte*, as in other kindes befoꝛe. One-ly conſideryng that the reſolutions of the firſt and ſe-conde ſommes, muſt be added togetheꝛ.

*The proſe of multiplica-tion.*

And therfoꝛe if you liſte to pꝛoue the firſte erample takyng. 2. foꝛ the roote, you ſhall finde the firſte ſome 80. —+— 36 —+— 40. that is. 156. And the ſeconde ſomme is, 20. —+—. 14. ——. 8. that is. 26. The thirde ſomme is. 1600. —+—. 1840. ——. 664. —+— 272. —+—. 320. whiche maketh. 4056. And ſo doeth. 156. multiplied by. 26.

Scholar. This maie J pꝛoue at any tyme; ſo that you ſhall not nede to ſtaie aboute it.

## *Of Diuiſion.*

Maſter.

DJuiſion is nerte in oꝛder, and agree-able in the generall rules: and hath noe moꝛe ſpeciall, then the very na-ture of the woꝛke dooeth require. Foꝛ as concernynge the ſignes of —+— and ——. theſame oꝛder is here, as is in multiplication. And touchyng the *Coßike* ſignes, it is all one with that J ſaied in diuiſion of nombers *Coßike* vncompounde.

Scholar. Then a felwe eramples maie ſupplie the

declaration of the vse of the rules, with the practike worke.

Master. Take these for your purpose.

An example of the firste worke.

60.
12.z z — 78.ce. — 80.z. (2.z.
6. z — 8. ze.

The remouyng of the diuisor, for the seconde worke.

60.
12.z z — 78.ce. — 80.z. (2z — 10ze
6. z. — 8. ze.

The proofe in nombers *Abstracte*, accoumptyng. 2. for roote.

3 480.
192. — 608. — 320. (8.
24. — 16.

480
192 — 608 — 320. (8 — 20.
24. — 16.

Thesame worke in vulgare forme.

z
1120 (28.
440

Here I haue not onely parted the worke, for your ease in vnderstanding: but I haue also put against it, the declaration of the same, by resoluyng the *Cossike* nõbers, into nombers *Abstracte*.

And finally, I haue putte one example of thesame nombers,

nombers, after the vulgare forme. all whiche. 3. agree together: and vouche one an other.

Scholar. Yet I praie you worke, one example more.

Maſter. Here is an other.

¶ The firſte extraction of the diuiſor.

$$\begin{array}{l} A\emptyset.z\mathfrak{C}.-+-A8.z\mathfrak{z}-+-20\mathfrak{C}-+-24.z\mathfrak{c}. \quad (8.z\mathfrak{z}. \\ 5.z.-+-6.9. \end{array}$$

¶ The remouynge forward of the diuiſor.

$$\begin{array}{l} A\emptyset z\mathfrak{C}-+-A8z\mathfrak{z}-+-20\mathfrak{C}-+-24\mathfrak{C} \quad (8z\mathfrak{z}-+-4z\mathfrak{c} \\ \qquad\qquad\qquad\qquad\qquad\qquad 5.z-+-.9. \end{array}$$

¶ The compꝛobation of theſame by reſolution, accoumptyng ſtill. 2. foꝛ a roote.

$$\begin{array}{l} z\mathfrak{z}6\emptyset\text{———}7\mathfrak{c}68-+-160-+-48. \\ 2\emptyset-+-6. \qquad\qquad\qquad\qquad (128. \end{array}$$

¶ The ſettyng forward of the diuiſor.

$$\begin{array}{l} z\mathfrak{z}8\emptyset-+-7\mathfrak{c}68-+-160-+-48. \quad (128-+-8. \\ \qquad\qquad\qquad\qquad 2\emptyset-+-.6. \end{array}$$

Scholar. Yet ones again, I praie you worke the like.

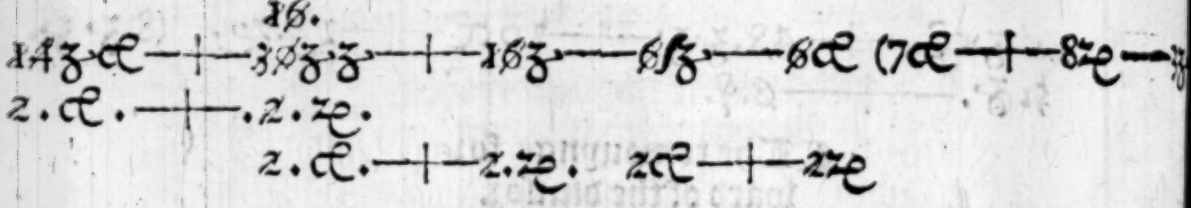
For although I perſwade my ſelf, that I perceiue the worke: yet would I ſee more confirmation of it, before I would be to conſtante in my perſuaſion.

Maſter. Good aduiſemẽte is euer ſure: but if you doubte, your councelloure is not farre abſente.

Scholar. I maie iuſtly reioice thereof: But foꝛ euery mater to require aied, and neuer to trauell my owne witte, it might ſeme mere daſtardlineſſe. And

so were it plaine babisheneſſe, to couet euery morſell, to be chawed before hande, and put into my mouthe.

Maſter. Then take this other example, in one platte complete: But with a caueat, to beware of to moche confidence, while you ſeme to flee doubtefulle daſterdlineſſe.



Scholar. Now haue I, that I looked for.

Maſter. Softe, lette vs trie this woorke, as wee haue doen the other: before we goe from it.

Scholar. I praie you let me doe it.

Maſter. With a good will.

| | |
|---|---|
| 64 | 16 |
| 14 | 30 |
| 256 | 480 |
| 64 | |
| 896 | |

Scholar. I kepe ſtill the old roote 2. Then is the. z ce. 64: whiche being multipled by. 14. maketh. 896. And ſo. 30. z z. doe yelde. 480. And 16. ſquares make. 64. All thei together yelde. 1440.

The reſte of the nombers, muſt be abated, bicauſe of the ſignes. ——. and thei make

| | |
|---|---|
| 32 | 8 |
| 6 | 6 |
| 192. | 48 |

240. For euery. fz. is. 32. and then. 6. times that, that maketh 192. whereunto I put. 48. for 6. *Cubes*: and ſo haue I. 240. to be abated out of. 1440. and then remaineth. 1200. for the diuidende. The diuiſor is but. 20. ſith. 2. ce. are. 16. and. 2. rootes make. 4,

896
480
64
1440

1440
240
1200

If I diuide now. 1200. by. 20. the *quotiente* will be. 60. agreably to the former *quotiente*. For 7. ce. make. 56

1200
2 0 (60

And

And. 8. rootes yelde. 16. that is. 72. From whiche I muſt abate. 3. ☉. that is. 12. And then it is iuſte. 60.

Maſter. This is well doen.

Scholar. Yea ſure, I am perfecte inough, in this ſcate of diuiſion, I trowe.

Maſter. You doe well to doubt.

Scholar. I thinke my ſelf ſure without doubte: As by one or twoo eramples, I will declare.

And firſt I take this nōber 322 ♄☉ — + — 115 ☉ ☾ —— 42. ☾ — + — 69. ☉. — + — 30 ☾. to be diuided by. 14. ☉. — + — 5. ☾. wherefore I ſette them doune thus.

| | |
|---|---|
| 322 ♄☉ — + — 115 ☉ ☾ —— 42 ☾ — + — 69 ☉ — + — 30. ☾ | (23 ſ☉ — + — 3 ☾ |
| 14. ☉. — + — 5. ☾. | 14 ☉ — + — 5. ☾. |
| 322 ♄☉ — + — 115. ☉ ☾. | 42. ☾ — + — 15 ☉. |

And finde the firſte *quotiente* to bee. 23. ſ☉. by whiche I multiplie the diuiſor, and it taketh awaie all the nombers ouer it: Wherefore I ſet the diuiſor forward, & finde 3 ☾. for the *quotiente*, whiche I multiplie into the diuiſor, & it maketh 42 ☾ — + — 15 ☉. wherby I am at a ſtaie. For although I ſee in the diuidende, the like nombers, yet the ſigne of —— declareth, that it is not poſſible, to abate this newe nōber thens: ſeyng —— 42. ☾. is leſſe then naughte.

Maſter. Therefore conſider it, in choſyng your *quotiente*: and giue your *quotiente* the like ſigne.

Scholar. But then riſeth an other doubte. For there will be ——. 15. ☉. whiche diſagreeth in ſigne from the nomber ouer it.

Maſter. Yet maie you ſubtracte it well inoughe, if you haue not forgotten, your rules of ſubtraction.

Scholar. Now I dooe better remember my ſelf: that by good reaſon, I muſt leaue as a remainer, not onely the whole nomber ouer it, whiche is. 69. ☉.

but

but I muſt adde therto. 15. ☉. more.

So ſhall I cancell the. 69. and ſet ouer it. 84. And then doe I remoue the diuiſor forward, ſettyng 14 ☉. vnder. 84. ☉. and the reſte in order, whereby I perceiue, that the newe *quotiente* will be. —|—. 6. ♀.

322. b☉ — 115☉ ☾ — 42☾ — 68☉ — 30. ☾ (23☉ — 3☾ +6
14. ☉. — 5. ☾ | — 14☉. — 5. ☾.
322 b☉ — 115☉ ☾ | — 42☾ — 15☉.
84. ☉. — 30☾.
14. ☉. — 5. ☾.

Whiche *quotiente* I doe multiplie into the diuiſor, and it doeth make. 84. ☉. — 30. ☾. agreable to the ſomme ouer it. And ſo there remaineth nothyng.

Maſter. You haue dooen well. But in chooſynge your diuidende, and the diuiſor, your lucke was better then your cunnyng.

Scholar. That ſhall I proue againe, by an other example, takyn alſo at all aduentures.

I would diuide this ſomme.
16. ☉ ☾. — 20. ☉. — 12. ☾. — 8. ♀. by
4. ☉. — 2. ☾. And therfore I ſet theim doune in order thus.

16. ☉ ☾ — 20 ☉ — 12 ☾ — 8 ♀. (4. ☉ ☉.
4. ☉. — 2. ☾.

And firſte I ſee, that. 4. is contained in. 16. fower tymes: and ſo maie I finde. 2. in any other nombers there. 4. tymes. Wherfore I ſet. 4. in the *quotiente*.

And bicauſe the. 4. in the diuiſor are. ☉. and the 16 to bee diuided, are. ☉ ☾. accordyng to the former rules, I finde the newe denomination *Coſike* to be. ☉ ☉

whiche

whiche I ſet in the *quotient* with 4 and ſo is it. 4.ʒ.ʒ.

Then ſaie I. 4.ʒ.ʒ. multiplied by. 4.ʒ. do make 16ʒ.ce. and therfore cleareth and conſumeth al that ſome ouer it. Then farther ſaie I 4.ʒ.ʒ. multiplied by. 2. ze. doe yelde. 8. ſʒ: But I ſee noe ſoche denomination in the diuidende.

Maſter. Then maie you perceiue, that you haue miſſed.

Scholar. Why ſir, I thinke I ought to doe as you did: that is to multiplie the *quotiente* into euery parte of the diuiſor.

Maſter. That is true: but I wil detecte the faute vnto you. And that is this.

That all nombers *Coßikes* compounde, can not bee diuided orderly, by diuiſors compounde. And thoſe that can bee diuided, will not receiue any other diuiſor of theſame kinde, but one of. 2. nombers, by multiplication of whiche, it was made: and ſo the other of thoſe. 2. ſhall be the *quotiente*: As it came to paſſe in all thoſe. 3. eramples, which I ſet forthe. And therefore it is loſte laboure, to goe aboute to diuide theim in that ſorte.

Scholar. Then are there but fewe nombers of *Coßikes* compounde, that maie be diuided.

Maſter. So many men ſaie. But I ſaie thereto, that though many of them can not be diuided, by like nombers *Coßikes* cõpounde, yet are there many thouſandes, that maie be ſo diuided.

And again I ſaie, that all ſortes of theim, maie bee diuided, by an *Abſtract* nomber. And alſo any of them maie be diuided, by conuerſion into a fraction: And ſo maie your erample be ſet thus.

$$\frac{16.ʒ.ce.\text{—}+\text{—}.20.ʒ.\text{—}++12.ze.\text{———}.8.ſ.}{4.ʒ.\text{—}+\text{—}.2.ze.}$$

And in all other caſes like, ſette the diuidende ouer a line, and the diuiſor vnder theſame line, and ſo is your diuiſion ended: and this is the reddieſte waie, and the moſte indifferente, in all ſoche nombers.

Scholar. That is ſone learned. And therfore needeth no moare examples.

It is like in nombers *Abſtracte*, when the greater nomber, doeth diuide the leſſer. As. 6. diuided by. 11. maketh $\frac{6}{11}$.

Maſter. Somewhat like it is. Howbeit here is a woorke moare like therevnto, as when we ſhould diuide the leſſer *Coſsike* nomber, by the greater, for then we muſt ſet them in that forme. So. 6. ☉. diuided by 7. ☌. ſhall be ſet thus: $\frac{6 ☉}{7 ☌}$. And. 20. ☌. diuided by 5. ſ☉. muſt ſtande in this maner: $\frac{20 ☌}{5 ſ☉}$.

Scholar. Why: 20. maie be diuided by. 5.

Maſter. But. ☌. can not be diuided by ſ☉. And in *Coſsike* nombers, the chief regard is to be had, to the *Coſsike* ſignes.

Scholar. Then, as for any other forme, of regulare diuiſion, here is none.

Maſter. Noe, excepte your diuiſor, bee a nomber *Abſtracte*: Or at the leaſte, if it haue one onely *Coſsike* ſigne, and be vncompounde, that ſigne muſt be other equalle, or leſſer then the leaſte *Coſsike* ſigne, in the diuidende.

For ſo. 60. ☉ ☌. — + — 48 ☌. — + — 18. ☉. maie bee diuided by any nomber, hauyng one of theſe. 3. ſignes *Coſsike*. ☉. ☌. ♀.

Scholar. I vnderſtand it well. For. ☉. is the laſte ſigne in the diuidende: And. ☌. and. ♀. are not onely leſſe then it, but alſo. ♀. leaueth the nomber, as if it were a nomber *Abſtracte*.

So if I would diuide your nomber, aſſigned by 40. ☉. the *quotiente* would bee thus.

60. ☉.

60.☉ ⊕ 48 ⊕ 18☉ (1½☉ ☉ ⊕ 1½☉ ⊕ 2♀.
40☉. 40☉. 40☉.

Maſter. Before we eande this worke of diuiſion, I will admoniſhe you, of one eaſie aied, in the diuiſiō of diuerſe nombers. And that is, to conſider, whether your diuidende, doe omit any *Coßike* denominations, betwene them, whiche it hath. For if it doe, you muſt yet ſupplie their roomes, with ſignes and Ciphers. As by erample, you ſhall vnderſtande.

I require to haue this nomber. 8.⊕. ⊕ 64.♀. diuided by. 2☉. ⊕. 4.♀.

Scholar. That will I doe quickely. For I ſee. 4. will be the firſte *quotiente* and his denomination will be. ☉. ſith. ⊕. diuided by. ☉. doe make. ☉.

But firſte I ſette doune the nombers orderly. And then I multiplie the diuiſor by the *quotiente*, & there riſeth. 8.⊕. ⊕ 16.☉.

8.⊕ ⊕ 64.♀. (4.☉.
2.☉ ⊕ .4.♀.
8.⊕ ⊕ 16.☉.

Maſter. Stande you now amaſed, for all your greate confidence? You ſee that you can not finde any ☉. in the diuidende. Therfore ſet doune the nomber as I told you before, in this ſorte.

⊕ 16☉.
8.⊕ ⊕ .0.☉. ⊕ .0.☉ ⊕ 46.♀. (4.☉.
2.☉ ⊕ .4.♀.
8.⊕ ⊕ .16.♀.

And then I take theſame *quotient* that you did, and I finde the remainer to be. ⊕. 16.☉. Wherefore I doe again ſette forward the diuiſor: And finde the *quotiente* to bee ⊕ 8.☉. by whiche I multiple the

diuisor, and it maketh. 16. z. ——— .32. ze so that abatyng the. 16. z. the reste, that is, ——— . 32. ze . shall be the remainer with the signe —+— by the rule of subtraction.

——— 16z —+— 32ze.
8ce —+— øz —+— øze —+— 64q (4z ——— 8ze —+— 16q.
2ze —+— 4.q.
2ze —+— 4.q.

Then vnder that remainer, I remoue the diuisor, and finde the newe *quotiente* to bee —+— 16.q. And so is the nomber clerely consumed.

Scholar. If I forgette any parte of this, I am deceiued to foule.

Master. Then haue you learned this parte, well inough, for this tyme. And therfore will we go forth vnto fractions, whiche partly were omitted before, and partly are compounde of them self.

## *Of fractions, and their numeration.*

FRactions of this kinde appere simple: and yet are scante so to bee iudged: as $\frac{4z}{3ce}$ betokeneth 4. z. to bee diuided by. 3. ce. Likewaies this fractiõ $\frac{12zz}{5zce}$ doeth import that 12 zz muste bee diuided by. 5. z ce. But $\frac{10z}{19}$ betokeneth. 10. z. to bee parted into. 19. portions.

And here shall you note, the doubtfull forme, that many menne in this arte vse, whiche write that laste fraction thus. $\frac{10}{19}$ z. where as this fractiõ doeth represent $\frac{10}{19}$ of a square: and not 10. z to be diuided by. 19.

Scholar. Bicause you sate, that some doe so vse it, and

and I would gladly excuſe all good writers: I maie ſaie for them that as in vulgare nombers, when. 10. ſhould be diuided by 19. And is ſet thus $\frac{10}{19}$ it doeth impoꝛte bothe that. 10. is diuided into. 19. and alſo that euery poꝛtion of thoſe. 19. is $\frac{10}{19}$ of an vnitie: ſo that if 10. ℓ. ſhould be parted emongeſt. 19. men, euery man ſhould haue $\frac{10}{19}$ of. 1. ℓ.

Maſter. Your woꝛdes haue ſo moche appeꝛaũce that thei maie perſuade hym, that is not very pꝛeciſe in termes, eſpecially ſeyng there is no other *quotiente* there, but thaſame nomber. But as the ſomme of 10. ℓ. beyng diuided by. 19. is farre moꝛe then $\frac{10}{19}$ of an vnitie: So. 10. ☉. to bee diuided by. 19. differ moche from $\frac{10}{19}$ of a ſquare. Foꝛ the one is 19. tymes ſo moche as the other. And therfoꝛe oughte to haue a diſtincte foꝛme in wꝛityng.

Scholar. Then you would haue me to wꝛite thẽ ſo, that $\frac{10}{19}$ of a Square, ſhould haue the ſigne againſt the line, as here is ſet $\frac{10}{19}$ ☉: and when I would repꝛeſent. 10. ☉. diuided by. 19. I ſhall wꝛite it thus. $\frac{10\text{☉}}{19}$. with the ſigne aboue the liue.

Maſter. You maie ſee their agremente, and their difference by reſolution, in this maner $\frac{10\text{☉}}{19}$ will make $\frac{20}{19}$ accoumptynge. 2. foꝛ a roote, and $\frac{10}{19}$ ☉. maketh $\frac{10}{19}$ of 4. oꝛ $\frac{40}{19}$ of. 1.

Again, accoumptyng. 3. foꝛ the roote, then $\frac{10\text{☉}}{19}$ yeldeth $\frac{90}{19}$: and $\frac{10}{19}$ ☉ maketh $\frac{90}{19}$ of an vnitie: ſo thei appere to bee equall in valewe by reduction.

But now maie you ſee, that the one doeth betoken the firſte nõber, whiche is to be diuided: and the other doeth ſignifie the *quotiente* of the diuiſion: and ſo are thei diſtincte in office and nature. But bicauſe by reſolutiõ, the one tourneth into the other, therfoꝛe many men accoumpt them as one. Howbeit, we ſtand to longe aboute this, conſideryng the erroure, is not alwaies daungerous.

But their ouersighte is more daungerous, whiche misplace the signe, when it should bee sette vnder the line: as a greate clerke doeth (except I shall for his excuse, impute the faulte to the printer) for he meaning to diuide .3. by .7. ☉ ☉. writeth it thus. $\frac{1}{7}$☉ ☉. where he should write it thus. $\frac{3}{7\text{☉}\text{☉}}$: and againe, myndyng to diuide .7. by .3. ☉ ☉. he writeth it thus $\frac{7}{3}$☉ ☉. where he should write. $\frac{7}{3\text{☉}\text{☉}}$.

Scholar. This faulte is manifeste, and detecteth the firste negligence: For $\frac{7}{3}$☉ ☉. doeth make in nomber, after the former resolution. $\frac{111}{3}$ and. $\frac{7}{3\text{☉}\text{☉}}$ dooeth make. $\frac{7}{48}$.

Master. Well, seyng you perceiue the faulte, we will stande no longer aboute it. Therfore to procede distinctly and certainly, whether that fraction be compounde, or simple, where the numerator is a *Cossike* nomber, and the denominator, a nomber absolute, yet maie you boldly thinke, that fraction to bee compoande, whose numerator is a nomber *Cossike* and the denominator an other *Cossike* of vnlike signe: as. $\frac{3\text{☉}}{3\text{☉}}$ and $\frac{3\text{☉}\text{☉}}{12\text{☉}}$.

Yet as in nombers Abstracte, it maie seme moste aptly to bee called a fraction, when the numerator is lesser, then the denominator, so in nombers *Cossike*, moste aptly the signe of the denominator, should bee the greater. Yet bothe formes come in vse.

And forbicause easinesse in worckyng, doeth oftentimes bring certaintie with it, before we take in hãde the addition of fractions, I thinke it good to speake somewhat of Reduction, to an other denomination. So that you forgette not, that any .2. nombers *Cossike* compounde, with a line betwene them, maie be called a fraction. As thus. $\frac{10\text{☉}\text{☉} - 1 - 32\text{☉} - 64}{3\text{☉} - 1 - 12}$ | that is, 5. ☉ ☉. — 1 — . 8. ☉. ——— . 6. ☉. to bee diuided by 3. ☉.

3.ℨ. — + — .12.9. and so of other like.

## *Of Reduction of fractions.*

Fractions *Cossike*, not onely in their nombers, but also in their signes maie be reduced to other valutions, and namely to their leaste termes, and yet continue still in one proporcion, betweene the numerator, and denominator.

So $\frac{12\mathcal{C}\ 3}{3\mathcal{C}}$ maie bee reduced to $\frac{4\mathcal{C}}{1}$: for so high as. $\mathcal{C}$ is aboue .9. that is in the thirde place from it: So is 3.3. in the thirde place aboue. 2.

Againe. $\frac{27\ 3}{3\mathcal{C}\ 3}$. by reduction doeth make $\frac{9\mathcal{C}}{1\ 3}$: And so $\frac{21\mathcal{C} + 11\ 3 - \mathcal{C}}{21\ 3\ 3 + \mathcal{C}}$ will bee by reduction. $\frac{4\mathcal{C} + 3 - 3\mathcal{C}}{7\mathcal{C} + 2\mathcal{C}}$

And so in all other fractions, where the nombers bee commensurable.

But if any one nomber, bee incōmensurable with the other, then can there be made no reduction in the nombers. yet in the signes *Cossike*, there maie be a reduction, other to greater, or to smaller signes: For those signes be euer commensurable.

And there is no exception, but thei maie bee reduced to smaller quantities, excepte any one quantitie of theim bee. 9. that is a nomber. For that can bee no smaller. And therfore none other maie be altered, sith euery one must be abated alike.

And looke how moche, the smalleste quantitie of that fraction, is aboue a number, so moche maie thei all bee abated: for thei are neuer reduced to the smalleste, till one of them be a nomber.

Scholar. And why maie not this reduction, serue for whole *Cossike* nombers?

Maister. Bicause the whole nomber, doeth not cō

sist

sist of a proporcion, as the fraction doeth, and so maie bee expressed in diuerse termes: but it importeth one somme certaine, whiche maie nother bee increased, nor decreased, but it will chaunge his valewe, and alter his office.

And if I saie: a foote is $\frac{1}{3}$ of a yarde, I maie saie as truely, increasyng bothe nombers, in the like proporcion, a foote is $\frac{4}{12}$ of a yarde: or in lesser termes: a foote is $\frac{1}{3}$ of a yarde.

But when I saie in whole nomber, a yarde is .3. foote, or a foote is. 12. ynches, I saie truely: and if I doe increase or abate any of those nombers, my wordes will be false.

So although in this nomber. 8.ſz.——.6.ce.——10.z. by reason of bothe nombers and signes, there might bee a reduction, yet bicause it is a whole nober, it should therby bee abated moche: as here you maie see. 4.ce.——3.zz.——.5.q. whiche by resolution into vulgare nombers, 2. beyng sette as the roote, doeth make. 32——6.——.5. that is. 33. and the other nomber before, doeth yelde by the like resolution. 256——48.——.40. that is. 264. and is 8. tymes so moche as the other.

Scholar. I perceiue now good reason, why reduction serueth for fractions onely. And if there bee noe more difficultie in it, then you haue declared. I can worke it easily.

*Reduction in signes onely* For other the reduction cõsisteth in the signes *Cossike* onely, as $\frac{10.ſz}{13.ce}$ where the nombers bee vncommensurable, and therfore can not bee altered to any lesser termes. But the signes *Cossike* maie bee abated by. 3. denominations: seyng the smalleste of them, is so many in order aboue. 9. And therfore it maie be reduced to $\frac{10.ſz}{13.q}$.

*Reduction in nõbers onely* Other els secondarily, the reduction consisteth in the nombers onely, when the nombers be communicante.

cante. And the signes *Cossike* bee all redy at the leaste: as when one of theim is. ♀. So $\frac{16\text{☌}}{12\text{♀}}$ will bee reduced to. $\frac{4\text{☌}}{3\text{♀}}$.

Or els thirdly, the reduction maie bee wrought, bothe in signes, and also in nombers. When all the signes be aboue. ♀. and the nombers be communicant *Reduction in signes and nomberses.*

So $\frac{50\sqrt{\text{☿}}}{35\text{☿}\text{☌}}$ maie be reduced well vnto. $\frac{10\text{♀}}{7\text{☌}}$.

Master. Yet one forme of reduction more, I will showe you, where not onely the like woorke maie be, but also the nomber maie be brought from his composition, to a more simplicitie, by abatyng some of his partes. *An other reduction.*

As this nomber $\frac{6\text{☌}\,\text{—}+\text{—}\,18\text{☿}}{8\text{☿}\,\text{☌}\,\text{—}+\text{—}\,14\sqrt{\text{☿}}}$ maie bee reduced, firste by his nombers to $\frac{3\text{☌}\,\text{—}+\text{—}\,9\text{☿}}{4\text{☿}\,\text{☌}\,\text{—}+\text{—}\,12\sqrt{\text{☿}}}$.

Secondarily, by his signes it maie be altered thus. $\frac{3\text{☿}\,\text{—}+\text{—}\,9\text{♀}}{4\text{☿}\,\text{☿}\,\text{—}+\text{—}\,12\text{☌}}$.

Thirdely, by abatynge the nombers, that followe signe of compositiõ (that is —+—) it maie be brought to. $\frac{3\text{☿}\text{☌}}{4\text{☿}\,\text{☿}}$. or. $\frac{3\text{♀}}{4\text{☌}}$. whiche fractions, kepe the self same proporcion, that the firste fraction did.

Likewaies with the signe of ———. nombers residualles, maie bee reduced. As. $\frac{6\text{☌}\,\text{———}\,18\text{☿}}{8\text{☿}\,\text{☌}\,\text{———}\,14\sqrt{\text{☿}}}$. will bee reduced, as the other was to $\frac{3\text{♀}}{4\text{☌}}$.

Scholar. This is vnto me a marueilouse mater, that those. 2. contrary nombers, should be reduced to one fraction.

Master. The like happeneth in vulgare nombers. For. $\frac{18\,\text{—}+\text{—}\,6}{24\,\text{—}+\text{—}\,8}$. will bee reduced to $\frac{3}{4}$. For firste it maketh $\frac{24}{32}$ and then $\frac{3}{4}$. So likewaies $\frac{18\,\text{———}\,6}{24\,\text{———}\,8}$ will make firste $\frac{12}{16}$ and then $\frac{3}{4}$.

And the reason of it, doeth depende of the. 19. proposition, of the fifth booke of *Euclide*, where it is written thus.

*If the proportion of the abatemente vnto abatemente be, as the whole is in proportion to the whole. Then shall the residue bee in like proportion to the residue, as the whole is to the whole.*

That is in the laste example. As. 18. is vnto. 24. so is 6 vnto 8. Therfore shall 12 be to 16. as 18. is to 24.

And for to exercise you the better, loe, here are one or twoo examples more, of the like reduction. $\frac{7c - 14}{8z - 16}$ maketh $\frac{7c}{8z}$ or $\frac{7}{8}$. Again $\frac{192 - 48z}{28z - 7c}$ yeldeth $\frac{192}{28z}$ or $\frac{48}{7z}$.

But this muste you farther marke, that in *Cossike* nombers, not onely the nombers, but also the *Cossike* signes must bee, accordyng to *Euclides* proposition.

Scholar. That doe I see.

For in the laste example: As. 9. is to. z. so. zc. is to. c.

And in the nexte example before: As. c. is to. 8z. so is. z. to. zz.

Likewaies in the other examples, as c is to zc so is. z. to. 8z.

Al this is good and reasonable.

Master. Now doe you see, bothe the maner of reduction, and also some reason for it. Therfore I will proceede, to declare the woorke of Addition.

## *Of Addition and Subtraction.*

IN Addition there is nothynge moare, then you haue learned before: For as for the multiplications of the denominators together, and then crosse waies with the numerator of thother, is iuste agreable with the reductions of Abstracte fractions, to bryng theim to one common denominator

nominator.

And then the numerators added together, doo make the newe numerator in addition.

And likewaies the lesser numerator, subtracted frō the other, doeth make the numerator in subtraction: wherfore a fewe examples maie suffice.

## Examples of Addition.

54.z. — + — .28.ce.

$\frac{6}{7}$.z. to $\frac{4}{9}$.ce.

63.

40.z. — + — .42.ze.

$\frac{5}{6}$.z. to $\frac{7}{8}$.ze.

48.

That is in smaller termes.

20.z. — + — .21.ze.

24.

Here you see how the. 2. fractions be sette betwene 2. lines: and vnder the nethermoste line, is sette the newe denominator: and ouer the higher line, are set the. 2. newe numerators ioyned in one.

The firste of them, can not be reduced to any smaller termes, bicause the nombers be not all. 3. commēsurable: & the denominator, also is a nomber *Abstracte*.

The seconde hath also a nomber *Abstracte* for his denominator, and therfore there can be noe reduction in signes: but the nombers all. 3. beyng commensurable, & diuisible by. 2. maie be reduced, as there you see.

## More examples of Addition.

16.sz. — + — .4.ce.

12.sz. — + — .9.ce. to 4.sz. — — — .5.ce.

20.z.ce. 20.z.ce.

20.z.ce.

That is in smaller termes. $\frac{4.z.\text{—}+\text{—}.1.9.}{5.ce.}$

Here is noe multiplication wrought, bicause the denominators are like.

## Another Example of Addition.

$$\frac{5.z.ce.\text{—}+\text{—}.20.ce.\text{———}.3.sz.}{\frac{5.z.ce.\text{—}+\text{—}.3.sz.}{6.ce.ce.} \text{ to } \frac{20.ce.\text{———}6.sz.}{6.ce.ce.}}$$

$$\frac{}{6.ce.ce.}$$

That is in lesser termes. $\frac{5.ce.\text{—}+\text{—}.20.9.\text{———}.3.z.}{6.z.ce.}$

Here is noe multiplication, nor reduction to one common denominator: sith thei bee one all ready: nother can the nombers be reduced, to any other lesser: but the quantities onely be reduced as you see.

Scholar. I praie you let me proue.

## Another Example.

$$\frac{80.bsz.\text{—}+\text{—}90.z.ce.\text{—}+\text{—}60.z.ce.\text{———}30.sz.}{\frac{8.ce.\text{—}+\text{—}9.z.}{10.ce.} \text{ to } \frac{6.ce.\text{———}.3.z.}{10.z.z.}}$$

$$\frac{}{110.bsz.}$$

That is

Maister. Marke your worke well, before you reduce it.

Scholar. I see my faulte: I haue sette. 2. nombers seuerally, with one signe *Cossike*: by reason I did not foresee, that. ce. multiplied with. ce. doeth make the like

like quantitie, as. z z. multiplied by. z. Therefore it should be thus.

$$\frac{80.bfz. \text{ — } + \text{ — } .150.z\,ce. \text{ ——— } .30.fz.}{110.bfz.}$$

Whiche maie bee reduced, by meane of the nombers, to this somme.

$$\frac{8.bfz. \text{ — } + \text{ — } .15.z\,ce. \text{ ——— } .3.fz.}{11.bfz.}$$

And now consideryng the *Cossike* signes, and workyng as I haue marked you to dooe: That is to abate the leaste signe, out of theim all: bicause. fz. is here the leaste, I abate it out of. bfz. and there resteth. z. and so doing with the other signe. z ce. there remaineth. ze & then fz out of fz dooth leaue. 9. or nōber: So will the fraction bee thus: $\frac{8z \text{ — } + \text{ — } 15ze \text{ ——— } 9}{11z}$ by reduction in signes and nombers also.

Master. Seyng you haue so well marked the reduction of the signes (whiche followeth the forme, taught before in diuision) I thinke it not nedefull, to staie any longer aboute this.

Wherfore we will goe forward to subtraction, after that I haue admonished you of fractions, in apperaunce simple, whiche in deede by addition, bee come compounde. As this $\frac{2}{3}$ ce. added to $\frac{3}{4}z$. maie firste be added by the common signe of addition, thus. $\frac{2}{3}$ ce. — + —. $\frac{3}{4}z$. whiche by reduction, vnto one denomination, will be thus written. $\frac{8ce \text{ — } + \text{ — } 9z}{12}$

But as this is easie inough to vnderstand, so maie it helpe often times, for spedie worke, as well in additiō, as in subtractiō, by the onely addyng of the signe.

As if I would subtracte this fraction $\frac{5}{7}z\,z$. out of $\frac{9}{10}z\,ce$. *Subtraction.*

$\frac{9}{10}$ Z ☾. I maie wrīte it thus. $\frac{9}{10}$ Z ☾. —— $\frac{5}{7}$ Z Z. And so is the Subtraction wrought̄e.

Yet maie you reduce theim, to one denomination, if you will, after thesame forme, as you did in addition. And then will it bee. $\frac{63\,Z\,☾ \;-\!-\!-\!-\; 10\,Z\,Z}{70}$ whiche can not bee reduced to any smaller termes, bicause the nombers are not commensurable: and one of theim (that is to saie, the denominator) is a nomber *Abstract*

Scholar. I see in this, there is no difference from Addition, but in the signes. —|— and. ——. wherfore I will proue an other example, by your leaue.

I would subtracte $\frac{3}{4}$ ſZ. out of. $\frac{4}{5}$ Z Z. and it will bee at the firste $\frac{4}{5}$ Z Z. —— $\frac{3}{4}$ ſZ. And by reduction $\frac{16\,Z\,Z \;-\!-\!-\!-\; 15\,ſZ}{20}$.

Master. Your woorke is well doen, accordyng to your firste meanyng: But as the numerator of this laste reduction doeth declare, it can not bee well, that 15 ſZ. maie bee abated out of. 16. Z Z. For the greater absolutely, can not well be abated out of the lesser: and therfore you might rather haue abated $\frac{4}{5}$ Z Z out of. $\frac{3}{4}$ ſZ.

Scholar. I see it well now: for the ſZ. is alwaies double or triple, or yet more tymes greater, then the Z Z. Bicause the. ſZ. commeth by multiplication of the Z Z by his firste roote.

Master. Yet here in is discretion to be vsed, for in fractions, sometyme the nomber of the greater signe maie be the lesser. As for example $\frac{3}{16}$ ſZ. is lesser then $\frac{3}{4}$ Z Z. as by resolution you maie proue, accompting 2. for the common roote.

Scholar. 2. beyng the roote. 32. is the. ſZ. and his $\frac{3}{16}$ maketh. 6. then. $\frac{3}{4}$ Z Z. beeyng. 12. dooeth appere double to it: and therefore greater by moche.

If I doe by the like resolutiō, proue the other fractions before, $\frac{3}{4}$ ſZ. will bee. 24: and $\frac{4}{5}$ Z Z. will bee $12\frac{1}{3}$: whiche is lesser moche.

So,

So, I perceiue the greatneſſe and ſmalneſſe of the fractions, muſt be conſidered, as well in the nombers as in the *Coßike* ſignes. And farther, if their fractions be nigh of one greateneſſe, or the fraction of the leſſer ſigne the greater, then can not the ſubtraction, appeare reaſonable.

Maſter. That is true, if thoſe. 2. fractions ſtande alone: els beyng partes of other nombers, it maie appeare reaſonable inough. As in this example of compounde fractions. $\frac{4}{10}$ ☌. —|— $\frac{3}{4}$ ♃. maie bee abated out $\frac{3}{5}$ ☌. —|— $\frac{3}{5}$ ♀. and yet in the abatemente after —|— not onely the nomber $\frac{3}{4}$ is greater, then $\frac{3}{5}$ in the other, but alſo, the *Coßike* ſigne. ♃. is greater then the other *Coßike* ſigne. ♀.

Scholar. I conſider it to be ſo: and yet $\frac{3}{5}$ ☌. doeth ſo moche excede $\frac{4}{10}$ ☌. that it ſupplieth ſufficiently the other defaulte: els could it not be well doen.

But for this woorke, I muſt craue your helpe; becauſe I haue not ſeen the like.

Maſter. You maie doe in this, as I ſaied before, generally for all ſubtractions.

Set doune bothe nombers in due order, ſo that the abatemente dooe folowe in order: and putte betwene them the ſigne of ſubtraction: as thus.

$\frac{3}{5}$ ☌ —|— $\frac{3}{5}$ ♀. ——— $\frac{4}{10}$ ☌. —|— $\frac{3}{4}$ ♃.

Howbeit, if you will firſte reduce euery cõpounde fraction, into one fraction, it will ſeme more apte. As thus. $\frac{3}{5}$ ☌. —|—. $\frac{3}{5}$ ♀. beyng reduced by additiõ will make $\frac{15☌ \,—|—\, 15♀}{25}$. and by farther reduction of nombers. $\frac{3☌ \,—|—\, 3♀}{5}$. Likewaies $\frac{4}{10}$. ☌. —|— $\frac{3}{4}$ ♃. will make by the firſte addition. $\frac{16☌ \,—|—\, 30♃}{40}$. and by farther reduction $\frac{8☌ \,—|—\, 15♃}{20}$.

Now ioyne theim together, with the ſigne of ſubtraction, and thei will ſtande thus.

$\frac{3☌ \,—|—\, 3♀}{5}$ ——— $\frac{8☌ \,—|—\, 15♃}{20}$.

Scholar.

Scholar. This doeth appeare verie straunge vnto me: but by vse I shall finde it more familiare: Seeyng I see the reason of this worke, to agree with the worke of common fractions.

*The proofe.* But for proofe of it, I will resolue eche worke, into nombers absolute, accounptyng. 2. for a roote.

Master. So shall you finde it true: But for easie wooke, take rather. 10. for the roote.

Scholar. I thanke you for your aide.

Then if. 10. be the roote, the square will be. 100. and the *Cube*. 1000. Now $\frac{3}{5}$ ℭ. that is $\frac{3}{5}$ of. 1000. is. 600. And $\frac{3}{5}$ of. 10. whiche is the roote, will bee. 6. whiche bothe put together, doe make. 606. and that is the greater nomber.

Then for the lesser $\frac{4}{10}$ ℭ. are in this example. 400 For the Cube beeyng. 1000. his $\frac{1}{10}$ is. 100. Againe the square beyng. 100. $\frac{3}{4}$ ℨ. must nedes bee 75. whiche beeyng put vnto. 400. dooeth make. 475.

Then doe I abate. 475. out of. 606. and there will reste. 131. How now?

| 606. |
|---|
| 475. |
| 131. |

Master. I perceiue you staie, as beeyng astonished, bicause in the former worke, there is not lefte a remainer: But the. 2. firste sommes onely altered by reduction, and ioyned together, with the signe of subtraction: wherein if you had continued your worke, you should haue founde thesame nombers.

For. 3. ℭ. must nedes bee. 3000. seyng. 1. ℭ. is a 1000. And also. 3ℨ. are. 30: whiche bothe added to gether, make. 3030. Diuide them by. 5. (as the denominator would) and it will be. 606. as the valewe of the firste fraction.

Then come to the later nomber: and you maie sone thinke that. 8. ℭ. are. 8000. And. 15. Squares are 1500. adde theim together, and thei will make 9500. whiche must bee diuided by. 20. (as the denominator

minator impoꝛteth) and there will amounte. 475. the valewe of the leſſer fraction: whiche nombers appeare the ſame, that were beſoꝛe: and thereby the wooꝛke is good.

| | |
|---|---|
| 11 | |
| 9500 | (475 |
| 2220 | |

But if you will bꝛyng it to a remainer, doe thus. Reduce theſe. 2. newe fractions, into one denomination.

Scholar. That can I doe, by multiplyng the numeratoꝛs together: that is. 20. by. 5. and thereof commeth. 100. whiche ſhall be the common numeratoꝛ: then muſt I multiplie in croſſe waies, the numeratoꝛ of the firſte, by the denominatoꝛ of the ſeconde, and contrarily.

So foꝛ the firſte numeratoꝛ I wooꝛke thus. And thereby dooeth amounte (as you ſee) 60.ce —|— 60.ze. And foꝛ the ſeconde numeratoꝛ, I multiplie. 8.ce —|— 15z. by. 5. and there doeth riſe. 40.ce —|—. 75. z. eche of theim hauyng one common numeratoꝛ. 100.

| | |
|---|---|
| 3. ce. —|—. 3. ze. | |
| 20. | |
| 60. ce —|— 60. ze. | |

Wherfoꝛe, ſeyng bothe nombers, haue one denominatoꝛ, I ſhall abate the leſſer numeratoꝛ out of the greater, as here in example is ſet foꝛthe: and then the

| | |
|---|---|
| 60. ce. —|—. 60. ze. | |
| 40 ce. —|—. 75. z. | |
| 20. ce. —|— 60. ze. ——. 75. z. | |

remainer will bee (as you ſee). 20. ce. —|— 60. ze. —— 75. z. vnto whiche I muſte adde the common denominatoꝛ. 100. and it will be thus.

| | |
|---|---|
| 20. ce. —|—. 60. ze. ——. 75. z. | |
| 100. | |

Now proue whether this remainer, doe not agree to thother remainer before, in your trial: which was 131

Scholar. 20 ☍ do make. 20000. ☉ 600 ☌. yelde 600: those 2 sommes I must adde together, bicause of the signe. —|—. and it will be. 20600. then. 75. ☽. are. 7500. whiche I must abate from the former somme of. 20600. and there will remaine. 13100. for the numerator, and 100. for the denominator, thus. $\frac{13100}{100}$.

20600.
7500
13100

Master. And what doe you thinke of it?

Scholar. By that I learned in the vulgare fractions, I knowe that it is iuste. 131. and so doeth it agree precisely, with the former proofe.

Master. Well yet for moare exactnesse in this worke, I will farther reduce that fractiō, by diuiding the numerator by the denominator: wherfore. 20. ☍ diuided by. 100. doeth yelde. $\frac{1}{5}$ ☍. And. 60. ☌. diuided by. 100. doeth make $\frac{3}{5}$ ☌. And lastly. 75. ☽. diuided by. 100. will yelde $\frac{3}{4}$ ☽. so is thesame fraction so reduced $\frac{1}{5}$ ☍ —|— $\frac{3}{5}$ ☌. ——— $\frac{3}{4}$ ☽. And now trie what that is, by the former proofe.

Scholar. I maie sone perceiue, that $\frac{1}{5}$ ☍. is. 200. when the Cube is. 1000: And so $\frac{3}{5}$ ☌. is. 6. whiche I must adde together, and it will be. 206. Then $\frac{3}{4}$ ☽. is 75. whiche if I dooe abate from. 206. there will remain. 131. agreably as before. And so is this woorke fully examined.

Master. Yet will I propounde one or two examples more, partly to practise your memorie, and partly to admonishe you, if you happen to see any soche misse wroughte, in some other bokes (as I haue doen) how you maie amende the errour, and not staie at it.

Firste take this example. I would subtracte.

$$\frac{-\ 48.\text{☉}.}{12.\text{☌}. \text{———} 3.\text{☽}.} \quad \text{out of} \quad \frac{48\text{☉}.}{7.\text{☽}.}$$

Scholar.

Scholar. I muſt firſt multiplie the denominatoꝛs together, and ſo it will make, as here is ſette fooꝛthe 84.ȼ.——21.ʒʒ.

Then I multiplie the numeratoꝛ of the firſte, by the denominatoꝛ of the ſeconde, and it will bꝛyng fooꝛthe. 336.ʒ: whiche is the numeratoꝛ foꝛ the abatemente.

12.ẓe——.3.ʒ.
7.ʒ.
84.ȼ——21.ʒʒ.

48.ϑ.
7.ʒ.
336.ʒ.

Afterward I multiplie the numeratoꝛ of the ſeconde, by the denominatoꝛ of the firſte, and it will make 576.ẓe.——144.ʒ.

12.ẓe——3ʒ.
48.ϑ.
96
48
576.ẓe.——144.ʒ.

Now if I ſubtracte that 336.ʒ. out of. 576.ẓe.——144.ʒ. it will bee 576.ẓe——480.ʒ. foꝛ the abatemẽte that ſhould be ſubtracted nowe, is ſette after the ſigne——with the foꝛmer ſomme of. 144.

Finally, to make the remainer complete, as that laſte nomber is the numeratoꝛ, ſo vnto it I muſt adde the common denominatoꝛ. 84.ȼ.——.21.ʒʒ. and it will bee. $\frac{576ẓe——480.ʒ.}{84ȼ——21ʒʒ.}$. that is in leſſer termes $\frac{192ϑ——160ẓe}{28ʒ——7ȼ}$.

Maſter. Now pꝛoue your cunnyng in this cõme, $\frac{439}{12ẓe——3ʒ}$. ſubtractyng it out of. $\frac{232ẓe——576ϑ}{84ʒ——21ȼ}$.

Scholar. Firſte I muſt reduce theim, to one common denominatoꝛ: by multiplyng bothe denomina-

84.ʒ.——.21.ȼ.
12.ẓe.——.3'.ʒ.
1008.ȼ.——.252.ʒʒ.
63.ſʒ.——.252.ʒʒ.
63.ſʒ.——.1008.ȼ.——.504.ʒʒ.

tozs together. And so wil it be. 63.ſz. — + — 1008.ce ——— 504.z.z. as by ſpeciall wooꝛke, I haue here pꝛoued.

Then doe I multiplie the numeratoꝛ of the totall, by the denominatoꝛ of the abatemente, as here also I haue perticularly set foꝛthe in wooꝛke, foꝛ my owne ease, and auoidyng of erroure: And so I finde it to be 1056.z. — + — .6912.ze. ——— .696.ce. whiche shall bee the numeratoꝛ of the totalle.

232.ze. — + — .576.9.
12.ze — + — .3.z.

2784z. — + — .6912.ze.
— 696.ce. ——— 1728.z.

1056.z. — + — .6912.ze. ——— .696.ce.

Then doe I multiplie the numeratoꝛ of the abatemẽte, by the denominatoꝛ of the totalle (whiche thing is easily dooen, bicause the one nomber, is a nomber *Abstracte*) and so haue I foꝛ the numeratoꝛ of the abatemente. 4032.z ——— 1008.ce.

And seyng these two nombers, haue one common denominatoꝛ, I shall abate the lesser numeratoꝛ, out

1056.z. — + — 6912.ze. ——— .696.ce.
4032.z ——————————— .1008.ce.

6912.ze — + — .312.ce. ——— .2976.z.

of the greater, & so wil there be left foꝛ the numeratoꝛ of the remainer 6912 ze — + — 312 ce — 2976 z vnto whiche, I shall adde the common denominatoꝛ, and then will it be.

6912.ze. — + — .312.ce. — + — .2976.z.
63.ſz. — + — .1008.ce. ——— .504.z.z.

That

That is in leſſer termes.

2304.ꝙ. — 104.ʒ. — 992.ꝛ.

21.ʒ ʒ — 336. ʒ. — 168. ꝛ.

Maſter. You haue wrought it well. And hereby I coniecture, that you are experte inough in ſubtraction. Wherfore now we will goe in hand, with multiplication and diuiſion.

## *Of Multiplication.*

AND firſte, concernyng multiplication, here is no more to bee ſaied, then hath been taughte before. *Multiplication.*

For the nombers ſhall bee multiplied, as common fractions are wonte to bee: that is to ſaie, numerator, by numerator, and denominator, by denominator.

And for the chaunge of their denominations *Coſsike*, the rules giuen before ſhall ſuffice: ſo that a fewe examples ſhall ſufficiently inſtruct you, in the worke of it. As this for the firſte.

20.ʒ. — 19.ꝛ.
6.ꝛ — 3.ꝙ.

120.ſʒ. — 114.ʒ ʒ.
— 60.ʒ. — 57.ꝛ.

120.ſʒ — 114.ʒ ʒ — 60.ʒ — 57.ꝛ.

Where I dooe multiplie.

20.ʒ. — 19.ꝛ. by 6.ꝛ. — 3.ꝙ.

31.ꝛ. 4.ʒ.

And firſte I ſhall multiplie, numerator by numerator:

rator: where. 20. ☉. multiplied by. 6. ☌. doe make 120. ☉☉. as the former table of multiplication, for chaunge of *Cossike* signes doeth declare. And so in all the reste, there is no difficultie, if you remember that, that you haue learned before.

Scholar. I perceiue it well. And so the whole newe numerator will bee. 120. ☉☉. —+— 114. ☉☉. —— 60. ☉. —+— 57. ☌. And the denominator will be. 124. ☉☉.

So will the whole fraction bee.

$$\frac{120.\odot\odot \text{ —+— } 114.\odot\odot \text{ —— } 60.\odot \text{ —+— } 57.\subset}{124.\odot\odot}$$

That is not to bee reduced to smaller termes of nombers, bicause thei be vncommensurable, but in *Cossike* signes, it mighte bee brought to one lesser, as.

$$\frac{120.\odot\odot \text{ —+— } 114.\subset \text{ —— } 60.\subset \text{ —+— } 57.\mathfrak{Q}}{124.\odot\odot}$$

Now will I proue an other nomber, as fortune doeth offer it to mynde. That is $\frac{32\subset \text{ —— } 29\odot}{21\odot \text{ —— } 12\subset}$. to bee multiplied by. $\frac{12\odot \text{ —+— } 9\mathfrak{Q}}{36\subset}$.

*An Absurde nomber expresseth lesse then naught*

Master. It appeareth that you take theim, at all aduentures. For your firste nomber, semeth to be an *Absurde* nomber. Seyng his numerator, is lesse then naughte, in appearaunce. And then maie it not bee diuided by any nomber: and moche lesse by so greate a denominator.

Scholar. It is easie to see, now that I am admonished thereof. For it is not possible, that any *Surfolide* nomber, can bee lesse then fower tymes so moche, as the *Cube* of thesame nature. Seeyng euery *Surfolide* is made, by multiplyng the *Cube* by the *square* of the like *Roote*, but lesse then. 4. is there no *Square*. And therefore euery *Surfolide*, doeth excede his *Cube* fower times at the leaste.

So

So that. 32. ☍. ——— 8. ſʒ. were nothyng, and ſo is an *Abſurde* nõber. And therfore. 32. ☍ ——28. ſʒ. is moche leſſe then nothyng, and is therby an *Abſurde* nomber alſo.

Maſter. Yet maie your example ſerue, to teache and practiſe multiplication by, as well as any other.

And farthermore, I will tell you by this occaſion, that I ſpake to you, more after the opinion of the cõmon nomber of artes men, then after my owne iudmente.

Scholar. I might thinke ſo, by termynge of your ſentence: but yet was your ſaiyng true.

Maſter. Yet maie that fraction ſtand well, if you take a broke nomber *Abſtracte* for the roote. Although in whole nombers, it bee an *Abſurde* nomber.

Scholar. That will I proue, by ſettyng. $\frac{3}{4}$. for a *Roote*. Then will the *Square* be $\frac{9}{16}$. and the *Cube*. $\frac{27}{64}$. Alſo the *Square of ſquares* will bee. $\frac{81}{256}$. And the *Surſolide* $\frac{243}{1024}$.

$\frac{3}{4}$ The *Roote*.
$\frac{9}{16}$ The *Square*.
$\frac{27}{64}$ The *Cube*.
$\frac{81}{256}$ The *zenzizenzike*
$\frac{243}{1024}$ The *Surſolide*.

And now to proue by reſolution, how my nomber will riſe, I take. 32. ☍. that is. $\frac{864}{64}$, or 13. $\frac{1}{2}$. whiche I note as the firſte ſomme. Then I take likewaies. 28. ſʒ. whiche yeldeth $\frac{6804}{1024}$, that is. $6\frac{165}{256}$. And now I ſee that I maie abate it very well, out of. $13\frac{1}{2}$.

Maſter. So maie you ſee, that as in whole nombers, euer moare the greater *Coſsike* ſignes, will haue the greateſte nombers: So in fractions reſolued by *Coſsike* ſignes, the greateſt fraction, aunſwereth to the leaſte ſigne: and the leaſte fractiõ, agreeth to the greateſte ſigne.

The reaſon of it is this. That the moare any fraction is multiplied by a fraction, the leſſer it wareth. For as whole nombers by multiplication, maie increaſe infinitely: ſo fractions by multiplication, maie decreaſe

decreaſe infinitely.

But before wee paſſe from multiplication, I will proue you with one example moare. I would haue $\frac{12\mathcal{C} + 1 - 3\mathcal{Z}}{73 - 6\mathcal{Q}}$ multiplied by $\frac{24\mathcal{C}}{5\mathcal{Z}} - 4\mathcal{Z}$.

Scholar. I am troubled with the multiplier. For I knowe not what to make of it.

Maſter. You doubte (I thinke) of the numeratō of it, becauſe you had not the like example before: for it is a mixte nomber of a fraction, and a whole nomber. But ſeyng the ſigne of abatemente, is ſet againſt the whole fraction, and nother againſte the numerator, nor denominator, therfore muſt that $4\mathcal{Z}$. be vnderſtande, to be abated out of the full fraction.

Scholar. Now I perceiue the mater. For there might be. 3. diuerſe formes, to place that abatemente. As here I haue ſet them. $\frac{24\mathcal{C} - 4\mathcal{Z}}{9\mathcal{Z}}$ & $\frac{24\mathcal{C}}{9\mathcal{Z} - 4\mathcal{Z}}$.

And as it was ſet by you, $\frac{24\mathcal{C}}{9\mathcal{Z}} - 4\mathcal{Z}$. whiche I will reſolue into abſolute nombers, to ſee their difference the better. And ſo, taking 3. for the roote, theſe will be their. 3. formes.

*The firſte.* For the firſte $\frac{648 - 12}{81}$. or els $\frac{636}{81}$ that is $\frac{212}{27}$.

For the ſeconde $\frac{648}{81 - 12}$ or els $\frac{648}{69}$ that is $\frac{216}{27}$.

And for the thirde nomber, whiche is our ſpecialle nomber. $\frac{648}{81} - 12$. that is. 8. $- 12$. and is an *Abſurde* nomber. For it betokeneth leſſe then naught by. 4.

Maſter. If you would haue it no *Abſurde* nomber, you muſt increaſe the proportion of the fraction, by augmentyng the numerator, or abatyng the denominator, or els thirdly, by abatyng the nomber, after the ſigne of abatemente. As $\frac{40\mathcal{C}}{9\mathcal{Z}} - 4\mathcal{Z}$: or els ſecondarily, thus. $\frac{24\mathcal{C}}{4\mathcal{Z}} - 4\mathcal{Z}$. or thirdely $\frac{24\mathcal{C}}{9\mathcal{Z}} - 2\mathcal{Z}$.

Howbeit for examples ſake, you maie worke, as well with *Abſurde* nombers, as with any other.

But

But for you eaſe, I will ſhewe you the wooꝛke of this example, in twoo ſoꝛmes.

Firſt, you ſhall multiplie the firſte whole nomber, by the fraction of the ſeconde nomber, that is. $\frac{19\text{c} + 3\text{z} - 9}{7\text{z} - 6\text{q}}$ by $\frac{24\text{c}}{9\text{z}}$. and it will bee.

$$\frac{456.\text{z}.\text{c}. + 72.\text{z}.\text{z}. - 120.\text{c}.}{63.\text{z}.\text{z}. - 54.\text{z}.}$$

As here in wooꝛke you maie ſee it plaine.

19.c. + 3.z. − 5.q.
24.c.

456.z.c + 72.z.z. − 120.c.

7. z. − 6.q.
9. z.

63.z.z − 54.z.

That is in leſſer termes, bothe of nombers, and of ſignes Coſsike.

$$\frac{152.\text{z}.\text{z}. + 24.\text{z}. - 40.\text{z}.}{21.\text{z}. - 18.\text{q}.}$$

And this is the firſte parte of your ſomme.

Then for the nerte parte, multiplie your firſte nõber, that is $\frac{19\text{c} + 3\text{z} - 9}{7\text{z} - 6\text{q}}$ by the abatement of the ſeconde nomber, that is by. − 4. z. and it will be.

$$\frac{20.\text{z}. - 76.\text{z}.\text{z}. + 12.\text{z}.}{7.\text{z}. - 6.\text{q}.}$$

As by this wooꝛke you maie ſee.

19.ce.—+—.3.ze.———.5.9.
———.4.ze.

20.ze.———.76.zz.———.12.z.

whiche being reduced to the denomination of the foꝛmer nomber, will be tripled (ſith that denominatoꝛ is triple to this) and ſo will it be $\frac{60ze——228zz——36z}{21z——18q}$

Now adde thoſe two nombers together, by puttyng their bothe numeratoꝛs in one, and it will be.

20.ze.———.76.zz.———.12.z.

21.z.———.18.q.

As here appeareth in wooꝛke.

152.zz.—+—.24.z.———.40.ze.
60.ze.———.228.zz.———.36.z.

20.ze.———.76.zz.———.12.z.

whiche will not bee reduced to any ſmaller fraction, bicauſe the nombers be incommenſurable. and one of the *Coſsike* ſignes is.9. And ſo is that the ſomme of the multiplication.

An other waie you maie wooꝛke it, and all ſoche like, by reducynge the multiplier, into one vniſoꝛme fraction. As here in. $\frac{24ce}{9z}$ ——— 4.ze. you ſhall mulpiplie ——— 4ze. by. 9.z. whiche is the foꝛmer denominatoꝛ, and it will be ———.36.ce. Then putte that to.24.ce. ouer the line, and ſet the common denominatoꝛ.9.z. vnder the line, and it will bee in one fraction reduced $\frac{24ce——36ce}{9z}$.

Scholar. Here I maie ſee at the firſte bewe, that this fraction is an *Abſurde* nomber: foꝛ the abatement after the ſigne ———. is greater then the nomber beꝛe

foꝛe it.

Maſter. That was cõfeſſed beſoꝛe. But yet maſe you woꝛke the eꝛample by it.

Scholar. That is true: and ſo will the numeratoꝛs, beeyng multiplied together, make eractely, 60.ce.——228.ze ce.——.36.ze ze. As here in eꝛample of wooꝛke, I haue ſet it, foꝛ my owne caſe and certentie.

19.ce.—|—.3.ze.——.5.ze.
24.ce.——.36.ce.

456.ze ce.—|—72.ze ze.——.120.ce.
——684.ze ce.——108ze ze.—|—.180.ce.

60.ce.——.228.ze ce.——.36ze ze.

And that is the newe numeratoꝛ.

And then foꝛ the ſecande nomber, if the firſte denominatoꝛ.7 ze——6.ze. be multiplied by the ſeconde denominatoꝛ, 9.ze. it is eaſily ſeen, that thei will make.63ze ze——54ze. whiche ſhall be the newe denominatoꝛ.

And ſo the intere fraction ſhall bee.

60.ce.——.228.ze ce.——.36.ze ze.

63.ze ze.——.54.ze.

That is in the ſmalleſte nombers and figures *Coßike*. $\frac{20ze——76ze——12ze}{12ze——18ze}$: whiche ſomme, dooeth in all thynges fully agree, with the foꝛmer nomber that you woꝛought.

Maſter. Pꝛoue theim bothe by reſolution: And then ſhall you knowe, the reaſon of their agremente.

Scholar. I ſee that the wooꝛke of the denominatoꝛs, doeth agree. Wherfoꝛe I will take.3. foꝛ a roote to pꝛoue how the woꝛke of the numeratoꝛs will agree

And ſo foꝛ.19.ce. I ſhall haue.513. And foꝛ.3.ze.

I shall haue.9.to be added to.513. And so haue I.522 out of whiche somme I must abate.5. And then remaineth.517. to bee multiplied by.24.ce.that is by.648. And the totalle will bee (as here in woꝛke appeareth).335016. whiche somme must be abated to a smaller nomber, in like rate as the other was reduced, firste by partition into.3. And then will it be.111672. And again, it must bee diuided by.9. foꝛ that is the quantitie of a square, by whiche the foꝛmer reduction, was wꝛoughte foꝛ the *Coßike* signes: and then will it bee. 12408. And that is the firste parte of the first woꝛke. Then foꝛ the seconde parte of that wooꝛke, I shall multiplie the firste nombers, that is 517 by the abatemente of the fraction, that is by —— 4ze, oꝛ —— 12. (sith.3. is the roote) and thereof will come —— 6204. whiche somme I must triple, as I did his equalle (that is.20.ze. —— 76.3z.3. —— 12.3z.) And so wil it bee —— 18612. Now shall I adde this somme, with the firste parte, whiche was.12408. and it will bee.12408. —— 18612. that is.6204. lesse then nothyng: and is the numeratoꝛ of the firste wooꝛke.

| | |
|---|---|
| | 648 |
| | 517 |
| | 4536 |
| | 648 |
| | 3240 |
| | 335016 |

Wherfoꝛe I pꝛocede to the seconde wooꝛke, where the numeratoꝛ of the fraction, beeyng reduced to the common denominatoꝛ, is.24.ce. —— 36.ce. whiche is —— 12.ce. and in nombers resolute (keping 3. still as a roote) it is —— 324. by whiche if I multiplie.517. it will yelde.167508. And that somme beyng abated, by diuision into.3. and.9. as the other was, oꝛ els diuided by.27. whiche is all one, it giueth 6204. as the foꝛmer wooꝛke did.

Maister. Thus I see, you are experte inoughe in multiplication: Wherfoꝛe I will shewe you now, the oꝛder and foꝛme of diuision.

## Of Diuiſion.

THere is noe ſpeciall rule to be giuen, for the woorke of Diuiſion, other then ſoche as are all ready taughte in other woorkes of diuiſiõ before. Wherfore I will by one or 2. examples, ſhewe you the woorke of it.

### The firſte example of Diuiſion.

$\frac{14.ce\ +\ 9.z.}{15.q.}$ to be diuided by $\frac{5.z.\ +\ 2ze.}{3.ze.}$ doeth yelde. $\frac{42.zz.\ +\ 27.ce.}{75.z.\ +\ 30.ze.}$ that is in a leſſer fraction, by bothe reductions of nombers & ſignes.

$\frac{14.ce.\ +\ 9.z.}{25.ze.\ +\ 10.q.}$

### An other example.

$\frac{12.ſz.\ \ 16.z.}{2.ce.\ \ 5.ze.}$ diuided by $\frac{19.z.\ \ 3.q.}{4.z.\ \ 5.q.}$ doeth make.

$$\frac{48.bſz\ +\ 60.ſz\ \ 64.zz\ \ 80z.}{38.ſz\ +\ 15.ze.\ \ 101.ce.}$$

whoſe nombers bee incommenſurable, and therefore maie not bee reduced, but by abatyng one denomination Coßike. And ſo will it be.

$$\frac{48.z.ce\ +\ 60.zz\ \ 64.ce\ \ 80.ze.}{38.zz.\ +\ 15.q.\ \ 101.z.}$$

Scholar. I see that you multiplie crosse wales (as in vulgare fractions) the numerator of the one nomber, by the denominator of the other. And so is diuision of noe difficultie, to hym that remembreth the former rules.

## *Of the golden rule.*

Maister.

THe golden rule, that is the rule of proportion, should folowe now, by the commō order. But seyng there is no difficultie in it, nother any other forme of woorke, then is in vulgare nombers, I will not staie any tyme aboute it. Saue that for your pleasure, I haue set here certaine examples, as wel in whole nombers *Cossike*, as in broken.

32. ☉. ⟌ 4. ☉.
6 ☾. ⟌ $\frac{3}{4}$ ☉ ☾.

250. ☾. ⟌ 20. ☿.
26. ☉. ⟌ $2\frac{3}{25}$ ☾.

5. ☉. — ⊢ — . 3. ☿. ⟌ 4. ☾. — — — . 5. ♀.
15. ☾ — — — . 9. ♀. ⟌ $\frac{60\text{☉}\,☾ \text{ — — } 111☾ \text{ ⊢ } 41♀}{5♂ \text{ ⊢ } 3☿}$

$\frac{3☾ \text{ — ⊢ — } 15♂}{12☿}$
61 ♃ — 7 ♀ ⟌ 14. ☿. — ⊢ — . 4. ♀.
$\frac{1024♄\,♃ \text{ — ⊢ — } 192♀☾ \text{ — — — } 1176♃ \text{ — — — } 336☿}{3☾ \text{ — ⊢ — } 15♂}$

Scholar. These fewe examples, doe sufficiently teache the forme of the whole rule. So that here neadeth noe farther explication.

Wherfore, if in this arte, there be any forme of extraction of rootes, I praie you to procede therto.

Of

## *Of extraction of rootes.*

Maſter.

AS in nombers *Abſtracte*, euery nomber is not a rooted nomber, but ſome certaine onely emongeſt theim, ſo in nombers *Coßike*, all nombers haue not rootes: but ſoche onely emongeſt ſimple *Coßike* nombers are rooted, whoſe nomber hath a roote, agreable to the figure of his denomination.

So that. 16. ℭ. is not a Square nomber, nother hath any roote. For although. 16. bee a ſquare nomber, and hath. 4. for his roote, yet the denomination (whiche is. ℭ.) hath noe ſquare roote: but. 16. ☉. is a ſquare nomber: and hath. 4. ☌, for his roote.

Likewaies. 8. ℭ. is a *Cubike* nomber, and his roote is. 2. ☌: but. 8. ☉. hath noe roote. For bicauſe. 8. hath no ſquare roote, agreable to the ſigne. ☉. nother is it a *Cubike* nomber, although it haue a *Cubike* roote, bicauſe the roote is diſagreable from the ſigne. ☉.

Scholar. I perceiue that in theſe nombers, as wel as in all other, the roote beeyng multiplied by it ſelf, will make the nomber, whoſe roote it is. And therefore can no nomber be called ſquare, or *Cubike*, or any waies els a rooted nomber, excepte the roote of the nomber agree with his ſigne: Whereby I perceiue well, that. 32. ♂☉. is a rooted nomber, forbicauſe that 32. hath a *Surſolide* roote, agreable to the ſigne. So likewaies. 125. ℭ. is a rooted nomber, ſeyng 5. is the *Cubike* roote of. 125. But. 27. ☉. is no rooted nōber.

Maſter. Thus you vnderſtande ſufficiently, the iudgemente of rooted nombers, and their knowlege, in ſimple *Coßike* nōbers, that be vtterly vncōpounde.

Wherfore, for extraction of their rootes, take this briefe order.

Extracte

Extracte the roote of your nomber, as if it were absolute, and put to it. ze. for the denomination.

So. 27. *Cubes* hath for his roote. 3. ze.

And. 49. z. hath. 7. ze. for his roote.

Again, the roote of. 216. ce. is. 6. ze.

Scholar. This I perceiue. And by like reason, the roote of. 243. zz. is. 3. ze. But why dooe you name nōbers *Cossike* vtterly vncompounde? For as I vnderstande, that there bee nombers compounde, in their signes, so I see that thei maie haue rootes also.

As. 16. zz. hath for his roote. 2. ze. And likewaies. 64. zce. hath. 2. ze. for his roote.

Maister. And dooe you not see, that those compounde nombers, maie haue moare rootes then one? Sith. 16. zz. hath for his square roote. 4. z. as wel as it hath. 2. ze. for his *zenzizenzike* roote.

So. 4. zz. hath for his Square roote. 2. z. And hath no *zenzizenzike* ze agreable to his whole signe.

Likewaies. 9. zce. hath no *zenzicubike* roote, according to his whole signe: but it hath a square roote agreable to parte of the signe, and that is. 3 ce.

Scholar. I see that also. And so hath. 8. zce. noe *zenzicubike* roote, but a *Cubike* roote: whiche is. 2. z.

Maister. Therfore in cōpoūde signes, if the signe maie haue sothe a roote, as the nomber will yelde, it is a rooted nomber, els not.

Whereby you maie perceiue, that if any nomber cōpounde in signe, haue a roote agreable to his whole signe, then maie it haue also, as many rootes, as ther be partes in that compounde signe.

So 4096 zz ce hath not onely a *zenzizenzicubike* roote, whiche is. 2. ze: but it hath a *Square* roote, that is. 64. z ce. And also it hath a *Cubike* roote, that is, 16. z z: Farther more, it hath a *zenzizenzike* roote, whiche is. 8. ce. And fourthly, it hath a *zenzicubike* roote, that is. 4. z.

And

And so shall you iudge, of all other like.

Scholar. This shall suffice, as I will practise the mater, at moare leiser. But and if the nombers bee compounde, with signes of addition, is there then any speciall order for their rootes? As in this example. 81.ʒ.ʒ.—+—27.ce. where I haue made eche parte to be a rooted nomber.

Maister. In deede. 81 ʒ.ʒ. hath bothe a Square roote, and also a *Zenzizenzike* roote. But. 27 ce. hath none of those twoo rootes, although it haue a *Cubike* roote, whiche the other nomber wanteth. And therfore is not that whole nomber, a rooted nomber.

But to the intente, that you maie be the more certein of rooted nombers, I will tell you certein notes, how it maie bee knowen, whether your nomber be a rooted nomber.

Firste, if the nomber annexed to the greatest signe of that compounde *Cossike* nomber, bee not a rooted nomber, the whole nomber can not be a rooted nōber

Secondarily, if the nomber that is ioyned with the leaste *Cossike* signe, be not a rooted nomber, the whole nomber can not be a rooted nomber.

And eche of these bothe rootes (if thei haueany) are partes of the whole roote, for the compounde *Cossike* nomber.

Thirdly, if the nomber be a rooted nomber, euery parte of it, that is not a rooted nomber, is a meane nomber, betwene the greatste and the leaste.

Fourthly, if. ze. bee any denomination in it, then is. 9. an other denomination in it also.

Fiftly, and generally, all rooted nōbers, other are specially framed, by orderly multiplication, or els are nombers equalle to some one rooted nomber *Abstract*.

Now specially framed are soche, as are made by multiplicatiō of one nomber by it self, and litle or nothyng altered from that very forme.

*Of square rootes.*

Exãple of. 529.z.ce +— 184.z.z. +— 16.z. whiche is a *Square* nomber, made by multiplication of. 23.ce +— 4.ze. by it self. This nomber maie haue his Roote orderly extracted thus.

529.z.ce +— 184.z.z. +— 16.z. (23.ce +4.ze
23 46.ce.

In the firste nomber, I finde the *Square* roote to bee 23. And for his denomination, I take halfe the *Cossike* signe z.ce, and that is. ce. For as. ce. multiplied by ce. doeth make. z.ce. So in diuision by. 2. and in extraction of *Square* rootes, I shall take the. ce. for the halfe of z.ce and the denomination of his roote: and so set it doune in the *quotiente*.

Then I shall double the nomber *Abstracte* of that *quotiente* (keppyng his *Cossike* signe vnaltered) and that double shall I set euermore vnder the nexte nomber, toward the righte hande. As here, you see, I haue set 46 (whiche is the double of 23) with his signe ce. vnder the seconde nomber. And there I perceiue, I maie haue it. 4. tymes, if I doe diuide (as I ought) 184. by 46. And that. 4. I sette in the *quotiente*, with the signe +—, and the denomination. ze: seyng. z.z. diuided by. ce. doeth yelde. ze.

Laste of all, I muste multiplie that parte of the *quotiente*. 4. ze. by it self, and it will yelde. 16. z. whiche beyng subtracted also (as it should) leaueth nothyng remainyng of the square nomber.

This order must you kepe in all square nombers, how greate so euer thei be. As in this seconde exãple.

—90.z.z.

25.z.ce —+— 80.z. —— 26.z.z. —— 144.ce —+— 81.z. (5.ce +8.z. —9
5.ce. 10.ce —+— 64.z.z.
—+— 10.ce —+— 16.z. —— 9.ze.

The

The roote of the firſt nomber is. 5 ☍, whiche I ſet in a *quotiente*.

Then doe I double that. 5, and it maketh. 10, to be ſette vnder. 8. with his denomination, whiche is. ☍. like to the roote.

That. 10. ☍. maie be founde in. 80. ſ ʒ. 8. times, therfore I ſet. 8. in the *quotiente*, with the ſigne —+— and the denomination. ʒ. And then doe I multiplie that. 8. ʒ. ſquaredly, whiche giueth. —+— 64. ʒ ʒ. to be ſubtracted out of ——— 26. ʒ ʒ. and ſo remaineth ——— 90. ʒ ʒ.

After this I double all the *quotiente* again, whereof commeth —+— 10. ☍. —+— 16. ʒ. And bicauſe there is a remainer, ouer the nomber that I wrought laſte, I muſt ſet. 10. ☍. vnder the remainer, and the other nomber in order, as you ſee it ſet.

Then ſeke I how often tymes maie. 10. ☍. diuide 90. ʒ ʒ, and I finde the *quotiente* to be ——— 9. ze. And likewaies —+— 16 ʒ. multiplied by ——— 9 ze doeth make ——— 144. ☍. equalle to the ſomme ouer it: And ſo ſubtracteth it cleane. Wherfore to ende that woorke, I multiplie the laſte *quotiente*, by it ſelf ſquare, and it yeldeth. —+— 81. ʒ. whiche is to bee ſubtracted out of the like ſomme, in the ſquare nomber: and ſo reſteth nothyng. Wherefore I iuſtly affirme, that the firſte nomber is a ſquare nomber, and hath for his roote. 5. ☍. —+— 8. ʒ ——— 9. ze.

Scholar. That maie I ſone proue, if I multiplie

5 ☍. —+— 8 ʒ. ——— . 9. ze.
5 ☍. —+— 8 ʒ. ——— . 9. ze.

25 ʒ ☍ —+— 40 ſ ʒ ——— 45 ʒ ʒ
—+— 40 ſ ʒ —+— 64 ʒ ʒ
81. ʒ. ——— 72. ☍ ——— 45 ʒ ʒ
——— 72. ☍.

25 ʒ ☍ —+— 80. ſ ʒ ——— 26 ʒ ʒ ——— 144 ☍ + 81 ʒ.

that roote by it self, as here I haue doen it. Wherby I haue not onely confirmed it to be a square nomber: but also I haue espied, that you vsed the nomber not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the .ʒ.ʒ. where the particulare multiplication hath —+—64.ʒ.ʒ. and ———90ʒ.ʒ. For which twoo nombers you sette one, that resulteth of thē bothe, that is ———26ʒ.ʒ.

Master. But if you would take the nōber in that sorte, the woorke would be not onely all one: but also somewhat plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forthe here, the example of that woorke. And loe, here it is.

25ʒ.ȼ +80.ſʒ +64ʒ.ʒ. —90ʒ.ʒ. —144ʒ +81ʒ (5ȼ +8ʒ -9
5. ȼ 10ȼ +64ʒ.ʒ. 10. ȼ. —+—16.

Scholar. By comparynge these bothe formes of woorke together, I dooe better vnderstande, the reason of the firste woorke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generalle patrone, for all the varieties, in this sorte of rooted nombers. And if you examine it diligently, and marke it well, you shall neade fewe other examples, for this kinde of square nombers.

The *Square* nomber, with the woorke of extraction of his roote followeth here.

The

Where for myne owne eaſe, and aied of memorie, I haue ſet vnder euery doublyng of the *quotiente*: And the ſomme that amounteth, by the multiplication of theſame, into the newe *quotiente*, with the Square of theſame newe *quotiente*.

Whereby I perceiue that the nombers, doe not go in ſoche order, that euery odde place, maketh a newe roote, as it doeth in nombers *Abſtracte*. But ſometime I muſt take. 2. places nexte together, and at an other tyme, I ſhall ſcippe. 2. or. 3. places.

Maſter. You marke it well. And yet that is a good and true rule, that ſome menne teache: that in theſe *Coſsike* nombers, as well as in other *Abſtracte* nōbers, you ſhall marke euery odde place, and vnder eche of them to finde a Square roote. But that is to be vnderſtande, when the nombers are ſette, in their brefeſte and eracteſte order.

Theſe fewe examples maie ſuffice, for a declaratiō of extractyng the roote of Square nombers, made by multiplication. And now touchyng thoſe nombers, that bee equalle to ſome rooted nomber, and namely ſoche as be equalle to a ſquare nomber, I will teache you how their roote maie be extracted. *The rootes of nōbers equal to be ſquare.*

But firſte you ſhall marke, that a Square beeyng compared, as equalle to rootes and nombers, the rootes maie bee coupled with the nombers onely, in. 3. formes. That is. ℨ. —+— .9 (whiche is all one with 9. —+—. ℨ) or els thus. 9. ——. ℨ. Or thirdly, ℨ ——9. And for eche of theſe. 3. formes, there is ſome varietie, in the extraction of the roote. And in them all moche agremente.

For the firſt forme, where ℨ —+—9 is equalle to ℨ. take theſe exāples 1 ℨ. is equall to. 4. ℨ —+—21 9. or. 1 ℨ. is equalle to 35. 9. —+—2. ℨ. Likewaies 1 ℨ. is equalle to. 10 ℨ. —+— 75. 9. or. 1. ℨ. is equalle to 10 5. 9. —+—. 8. ℨ. *The firſte forme.*

In all these exãples, and other soche like, you must first consider the nomber annered with the signe. ☍. (whiche is the middell quantitie) and the halfe of it shall you note, for with it shal you worke twise. First you shall multiplie halfe of that nomber by it self, and this is the firste worke, and to it shall you adde the other whole nomber, that is ioyned with. ☐. And thei will euer more make a square nomber: out of whiche square you shall ertracte the roote. And to that roote shall you adde halfe the nomber, that was annered with the signe of. ☍. (whiche was the nomber that I bade you to marke). And this is the seconde worke. The totall that commeth of this addition, is the roote of the compounde *Coßike* nomber.

*An example* Example of the firste. 4. ☍. —+— 21. ☐. halfe the nomber annered with. ☍. is. 2. whose Square is. 4. that shall I put to. 21. and there riseth. 25. beeyng a square nomber, and hauyng. 5. for his roote. To that 5. I ioyne halfe the nomber annered with. ☍. and it maketh. 7. whiche is the nomber that I seke for: and is the roote to. 4. ☍. —+— 21. ☐.

*The proofe.* For triall whereof take. 4. rootes, that is. 28. and putte to it. 21. and thereof commeth. 49. whiche is a square nomber, and hath. 7. for his roote.

*An other example.* Scholar. Then can I doe the like with the second exampl. 35. ☐. —+—. 2. ☍. And firste the halfe of. 2. is 1. and the Square of it is. 1. whiche I put to. 35. and it maketh. 36. a Square nomber: whose roote is. 6. To that. 6. if I adde. 1. that was the halfe before reserued, it will make. 7. whiche is the roote that I doe seke.

*The proofe.* The proofe is this: 2. rootes maketh. 14. and. 35. giueth. 49. whose roote is. 7.

*The thirde example.* Likewaies for the thirde example 10 ☍ —+— 75 ☐ I worke thus. Halfe. 10. is. 5. and his Square is. 25. that dooe I adde to. 75. and there riseth. 100. whose roote is. 10. to whiche roote I add. 5. and there commeth

meth.15.that is the roote whiche I would haue.

And that I maie proue by triall in this ſorte. 10. rootes giue.150.vnto whiche if I adde.75.there will amounte.225.whiche is a Square nomber: and hath 15.for his roote.

The fourthe example is.105.9—+—8.ze. where I take firſte the halfe of.8. that is.4. and it in Square giueth.16. whiche I adde to. 105. and there amounteth.121.beyng a Square nomber, and the roote of it 11.vnto whiche I ſhall adde. 4. for halfe the nomber of rootes: and ſo there riſeth. 15. as the roote that I ſeke for. And to approue it I take.8.times.15. whiche is.120.and adde it vnto. 105. and ſo commeth. 225. For the ſquare, and the roote of it is.15. *The fourthe example.* *The proofe.*

Maſter. The like order of worke ſhall you vſe, in all nombers *Coſsike* compounde, whē any.2.nombers with immediate denominatiōs *Coſsike*, are equalle to one of the nerte denomination, in order aboue them. *Other formes in like ſorte.*

As.1.ce. is equalle to.3.ze.—+—10.ze.

And again.1.ze. equalle to.6.ze ze—+—40.ce.

Likewaies.1.ze ce. equalle to.3.ze—+—28.ze ze.

But in al theſe the roote ſhal beare name of the greater quantie.

Scholar. By the former order of worke, I ſhall in the firſte of theſe.3.examples, take halfe.3.(bicauſe it is the nomber of the middell quantite). And that is $\frac{3}{2}$. and that ſhall I multiplie ſquarely, and ſo will there riſe $\frac{9}{4}$, vnto whiche I ſhall adde 100 or $\frac{400}{4}$. And that maketh $\frac{409}{4}$. whiche is a ſquare nomber, and his roote is $\frac{7}{2}$. vnto whiche I muſt put the firſte halfe, that is $\frac{3}{2}$, and then will it be $\frac{10}{2}$, or els 5. whiche is the *Cubike* roote of that nomber.3.ze—+—10.ze. beyng equalle to 1 ce *The firſte example.*

For proofe whereof, I multiplie.5. *Cubikely*, and it maketh.125. Then doe I multiplie it ſquarely, and it will be.25. Now.3.ze. is.75. and.10.ze. maketh.50 whiche bothe added together, giue.125. *The proofe.*

In

*The seconde example.* In the seconde example, where. 1. ſʒ. is equalle to 6. ʒ ʒ. —+—. 40. ¢. I ſhall take halfe. 6. (whiche is the nomber of the middell quantitie) and that is. 3. and the ſquare of it is. 9. whiche I muſt adde vnto 40 and thereof commeth. 49. whiche is a ſquare nomber and hath. 7. for his roote, vnto whiche I adde 3. and ſo haue I 10 for the *Surſolide* roote, of 6 ʒ ʒ —+— 4 ¢ ¢

*The proofe.* And for proofe I ſaie, if. 10. bee the roote, then is 100. the ſquare, & 1000. the *Cube*, the ʒ ʒ is 10000. And the *Surſolide*. 100000. Wherfore. 6. ʒ ʒ make 60000. and. 40. ¢. yelde. 40000. And bothe thei together doe make. 100000. whiche is the quantitie of the *Surſolide*.

*The thirde example.* In the thirde example. 1. ʒ ¢. is equalle to. 3. ſʒ. —+—. 28. ʒ ʒ. whoſe *zenzicubike* roote, I ſeke in this ſorte.

Firſte I take halfe. 3 (as the nomber of the middell quantitie) that is $\frac{3}{2}$, & that maketh in ſquare $\frac{9}{4}$. whiche I adde vnto 28 (that maketh $\frac{112}{4}$) & it yeldeth $\frac{121}{4}$ whiche is a ſquare nomber, and his roote is $\frac{11}{2}$. vnto whiche I adde $\frac{3}{2}$, and it will be $\frac{14}{2}$, or. 7. whiche is the *zenzicubike* roote vnto the forſaid nomber. 3. ſʒ —+—. 28. ʒ ʒ.

*The proofe.* For proofe whereof I multiplie. 7. *zenzicubikely*, and it maketh 117649. Then muſt the ſʒ. be 16807 and. 3. ſʒ. 50421. Again the ʒ ʒ. is. 2401. and ſo 28. ʒ ʒ. ſhall bee. 67228. And thoſe bothe together yelde. 117649.

*A thirde forme.* Maſter. Yet one other forme is there, that in all thinges, ſaue in one poincte onely: followeth theſame rule. And that is whē the 3 denominations doe not go immediatly together, but yet are equally diſtante. As ʒ ʒ. ʒ. and. 9. where the diſtaunce is one onely quantitie. Likewaies. ʒ ¢. ¢. and. 9. whiche differ by. 2. quantities. And in like ſorte. & ¢. ſʒ and ze. are diſtante by. 3. quantities. And ſo of other, how many ſo euer bee omitted, ſo that the difference bee

equalle

equalle. In all whiche you ſhall woꝛke, as you did in the foꝛmer rule, till you haue eanded all that woꝛke. But then haue you here, one thing moꝛe to bee conſi-red. Foꝛ the laſte nomber, whiche you haue founde, is not the roote, but a rooted quantitie: And his roote is the roote that you ſeke foꝛ.

Scholar. Doe you meane the ſquare roote of that quantitie, oꝛ ſome other?

Maſter. It maie be any kinde of roote, in diuerſe nombers, but not in one nomber. Wherfoꝛe foꝛ your certeintie marke this rule.

If the denominations of your nombers, differ one-ly by one, then is it a ſquare nōber, that you doe finde by the pꝛactiſe of the laſte rule. And therfoꝛe ſhall you take his ſquare roote, foꝛ the roote of your nomber.

But if the denomination differ by. 2. quantities, then ſhall you extracte a *Cubike* roote, outof your laſte nomber. And if the diſtaunce bee. 3. quantities, the roote muſt bee a *zenzizenzike* roote: and foꝛ. 4. quan-tities diſtante, a *Surſolide* roote, and ſo foꝛthe.

As foꝛ erample. 1. ȝ ȝ. is equalle to. 8 0. ȝ —+—. 2000. ℨ. Now foꝛ to finde the roote of. 8 0. ȝ —+—. 2000. ℨ. I woꝛke thus. Firſte I take the halfe of 8 0. (bicauſe it is the nomber of the middle quantitie) and that halfe is. 4 0. whiche I multiplie Square, and it maketh. 1600. to it I adde. 2000. and it will bee 3600. whiche is a ſquare nomber, ſ. 60. is his roote: to that. 60. I ſhall adde the foꝛeſaied. 40. and then will it bee. 100. whiche nomber in the firſte rule, had been the true roote. But here conſideryng the diſtāce is of one quantitie, I muſte extracte his ſquare roote, whiche is. 10. And that is the *zenzizenzike* roote, that my nomber containeth. *An example*

An other erample. 1. ȝ cc. is equalle to. 400. cc. —+— 57344. ℨ. I take 200. foꝛ the halfe of the mid-dell quantities nomber, and multiplyng it ſquare, I *The ſeconde example.*

finde. 40000. whiche I put to. 57344. and then I haue. 97344: whiche is a Square nomber, and his roote is 312 vnto whiche I shall adde the halfe of 400 and so will it bee. 512. But now must I take the *Cubike* roote of this nomber (that is. 8) for my roote, that I desire: Bicause the denominations in the nomber, differ by. 2. quantities.

Scholar. I see very well the order of this worke: And the proofe is in like sorte, whiche I maie practise by my self at any tyme. Wherfore I praie you, proceede forthe to other rules.

*The seconde sort of equal nombers.*

Master. This is sufficiente for the firste sorte. Now for the seconde sorte, in nombers *diminute* or *residualle* where. ʒ. is equalle to. ϙ——ϛ. the forme of worke is like vnto the other, in all poinctes saue in one. For in stede of the laste addition, you shall vse in these nombers, Subtraction. As here for example, when I saie. 1. ʒ. is equalle to. 60. ϙ.——.4. ϛ. to finde the roote, firste I take the halfe of. 4. (bicause it is the nomber of the middell signe) and that halfe beyng. 2. doeth make in square. 4. whiche I put to 60 and so is it. 64. a square nomber, and hath. 8. for his roote. From whiche roote (by the order of this rule) I must abate. 2. that is the halfe of the firste nomber of rootes. And then will there remaine. 6. for the verie roote of. 60. ϙ.——4. ϛ. beyng equalle to. 1. ʒ.

*Example.*

*The proofe.*

Scholar. That is sone proued. For. 6. beeyng the roote, then. 4. ϛ. maketh. 24. whiche beyng abated out of. 60. leaueth 36 and that is the iuste square vnto. 6. as the equation saieth.

*The seconde example.*

Master. An other example is this. 1. ſʒ. is equall to. 162. ϲ.——.9. ʒ ʒ.

Scholar. That can I worke, thus: Firste I take the halfe of. 9. (bicause it is the nomber of the middell signe) and it is $\frac{9}{2}$, whiche I multiplie squarely, and it will be $\frac{81}{4}$, that must bee added to. 162. or $\frac{648}{4}$, and then

will

will there amounte $\frac{729}{4}$. whiche is a Square nomber, and hath for his roote $\frac{27}{2}$ out of whiche, by this rule, I must abate $\frac{9}{2}$, and then riseth $\frac{18}{2}$, that is. 9. whiche is the very roote to 162. ce ——— 9. ʒ ʒ. beyng equall to. 1. ſʒ.

And for the proofe, I multiplie. 9. *Cubikely*, and it giueth. 729. so that. 162. ce. doe make. 118098. out of whiche I must abate. 9. ʒ ʒ. that is. 59049. (by thesame roote, sith. 1 ʒ ʒ. is. 6561). And then will there remaine. 59049. whiche is the iuste quantitie of. 1. ſʒ. *The proofe.*

Master. Yet one example more shall you haue of a thirde sorte. *The thirde example.*

When. 1 ʒ ce is equalle to. 275456. ϑ ——— 26 ce I demaunde of you, what is the valewe of. 1. ze?

Scholar. I searche it thus. The nomber of the middell signe is. 26. whose halfe I must take, and first multiplie it squarely, and there will rise 169. whiche I adde to. 275456. and it will bee. 275625. whiche is a square nomber, and hath for his roote. 525. from whiche nomber I must abate halfe the nomber, of the middell signe, that is. 13. and so there will remaine 512. whose *Cubike* roote I must extract, because the denominations differ by. 2. quantities, and that roote will be. 8. whiche is the *Cubike* roote to. 512. but to the nomber propounded, it is the *zenzicubike* roote.

Master. This is inoughe for the worke of the seconde sorte. Now for the thirde sorte of equation, where. ʒ. is equalle to ze. ——— ϑ. I will giue you a briefe admonition onely, though it differ from bothe the other. 2. rules, in forme of worke. For as the equalitie maie be in diuerse sortes, so some tymes you maie vse the worke of the firste sorte, by Addition of halfe the nomber of the middle signe: and some times you shall worke by subtraction. Wherein this is the difference, from the seconde rule. That there you doe *The thirde sorte of equal nombers.*

ſubtracte halfe the nomber of the middell ſigne, from the roote whiche you fonde. And in this thirde rule, you ſhall ſubtracte the roote from the halfe, and not the halfe from the roote. Forbicauſe that that roote, is euer leſſer then that halfe.

And in this rule, this is ſpecially to bee obſerued: that the Square of halfe the nomber, of the middell ſigne, will euer more bee greater, then the nomber of the leſſer ſigne: And therfore ſhall the nomber of the leſſer ſigne, bee abated out of that ſquare. And the remainer will bee a Square nomber, with whiche you ſhall woorke, as I haue taught you before.

And farther in this rule, it is commonly ſeen, that euery ſoche equalle nomber, hath. 2. valuations for his roote. I meane that any of thoſe. 2. nombers, will bee as the roote in this equation. For otherwaies no nomber can haue. 2. rootes of one denomination.

Scholar. I vnderſtande you thus. That no nomber can haue. 2. ſquare rootes, or. 2. *Cubike* rootes, and ſo forthe: Els one nomber maie haue. 3. or. 4. rootes. As. 64. hath. 8. for his Square roote: 4. for his *Cubike* roote: and. 2. for his *Zenzicubike* roote.

Maſter. You take it well. And farther for the eaſie knowledge of thoſe. 2. nombers, or rootes: Thei muſt bee ſoche, as beeyng added together, will make the nōber of the middell ſigne: and beeyng multiplied together, will make the nomber of the leaſt ſigne. And ſo maie you finde theim without farther multiplication, or extraction of rootes.

*The firſte example.*

For example, I ſette firſte. 1. ☉. equalle to. 16. ☉. ——. 63. ☉. where I maie eſpie quickely, that. 63. cā haue no more partes to his compoſition, but. 3. 7. 9. 21 And if I take. 3. and. 21. then their addition will bee greater then. 16. but 7. and. 9. maketh iuſte 16. by addition, and. 63. by multiplication. And therefore thei ſhall be the. 2. rootes.

Scholar.

Scholar. I will proue that by examination, thus. If. 7. be the roote, then is. 49. the ſquare. And. 16. to make. 112. out of whiche I muſt abate. 63. and there reſteth. 49. equalle with the Square: ſo is that a true roote. Then for. 9: his ſquare is. 81. And. 16. to. doe yelde 144 frō whiche I ſhal abate 63. And the remainer will be. 81. equalle to the ſquare. And ſo is that alſo a true roote.

Maſter. Now worke it by the other rules, that I taught you.

Scholar. Firſte I take. 8. as halfe the nomber of the middell ſigne, and that multiplied Square, doeth giue 64 from whiche I ſhall abate 63 and then doeth there remain but. 1. whiche is coumpted as a Square nomber, and his roote to bee. 1. alſo, whiche if I adde to. 8. it will make. 9. that is one of the rootes: And if I abate it from. 8. it will leaue. 7. whiche is the other roote. And thus I ſee one worke cōfirmeth the other.

Maſter. Take this for the ſeconde crāple. 1 ☉ & is equalle to. 8. ☉. ——— . 12. ☉ ☉. what is the roote ſaie you? *The ſeconde example.*

Scholar. To finde it, firſte I loke for the partes of 12. And thei be. 2. 3. 4. 6. of whiche. 2. and. 6. doe ſerue my purpoſe, for their addition maketh. 8. and ſo doeth not. 3. and. 4. Wherefore I ſaie, that. 2. maie bee the roote, and ſo maie. 6. But for farther trialle of it: I worke it by the other rule, ſaiyng halfe. 8. is. 4. and his ſquare is. 16. From whiche I abate. 12. and there remaineth. 4. whoſe roote is. 2. that I maie adde to. 4 and ſo haue I. 6. for one roote: or els abatyng it from 4. I ſhall haue. 2. for the other roote.

The proofe is manifeſte for. 6. beeyng a roote, the *Zenzicube* is. 46656. The *Surſolide* is. 7776. And the *Zenzizenzike* is 1296. So that 8. ☉. doe make 62208 And. 12. ☉ ☉. are. 15552. whiche being abated out of 62208 do leaue 46656. the true quantitie of 1 ☉ & *The proofe.*

And so is that worke good, 6. beyng a roote.

Now if. 2. be sette for a roote: then is the. ȝ ȝ. 16. the ſȝ. 32, and the. ȝ ȣ 64. And so are. 8. ſȝ. equall to. 256. And. 12. ȝ ȝ. yelde. 192. Wherfore abating 192. out of. 256. there resteth. 64, the iuste quantitie of. 1. ȝ ȣ. And so is that woorke also good, and. 2. a true roote.

*The thirde example.*

Maister. Now proue this thirde example, where 1. *b* ſȝ is equalle to. 2000. ȝ ȝ —— 470016 ℞.

Scholar. Halfe the nomber of the middell signe is 1000. And the square of it is. 1000000. From whiche I shall abate. 470016. and there will remaine 529984. whose square roote by trialle of extraction, I finde to be 728. whiche I maie other adde to. 1000 and so there riseth. 1728. whiche I finde to bee (as it ought) a *Cubike* nomber. And his roote to be. 12.

But and if I abate 728. from 1000, there will remain. 272. whiche is no *Cubike* nomber.

Maister. So that here semeth to be but one roote. And yet these. 2. nombers. 1728. and. 272. kepe soche a rate, that beeyng multiplied together, thei make 470016. whiche is one of the nombers, and beeyng added together, thei make 2000. whiche is the other nomber of the same *Coßike residualle*.

But now proue in other like nõbers, whiche haue some distaunce, betweene their denominations, whether it will so happen still. As namely in this, where 1. *b* ſȝ. is equalle to. 12. ȝ ȝ. —— 32. ℞.

*The fourthe example.*

Scholar. Halfe. 12. is. 6. and his Square. 36. from whiche abatyng. 32. there is lefte. 4. whose roote is. 2 And if I adde that 2. to. 6. it maketh. 8. whiche is a *Cubike* nomber, and hath. 2. for his roote. But if I abate 2. from. 6. there remaineth. 4. whiche is no *Cubike* nõber, and therfore hath no soche roote. And yet these. 2. nombers. 4. and. 8. by addition, make the middell nõber, and by multiplication, thei make the laste nõber.

Maister.

Maſter. Proue yet ones againe in a nomber, where one quantitie onely is omitted. As when 1.ſz. is equalle to. 24.ℰ. ——— 135.z. *The fifte example.*

Scholar. 12. maketh in ſquare. 144. from whiche I ſhall deducte. 135. and then reſteth. 9. whoſe ſquare roote is. 3. whiche if I adde to. 12. it will bee. 15. and hath no ſquare roote, as here is required. But if I abate. 3. from 12. then remaineth 9 whoſe ſquare roote is. 3. and ſerueth to the nomber, as I haue here proued in my tables. And. 9. and. 15. kepe the cuſtomable rate. For by addition thei make. 24. And by multiplication, thei yelde. 135.

But in all theſe eramples, where the denominations be are a diſtaunce, I can finde but one roote, and not. 2. As it was in the other eräples of theſame rule.

And in ſome of theim, the greater nomber containeth the roote: but in other, the leſſer nomber hath the roote.

Maſter. Bicauſe I can not ſtaie now, about this varietie, I will remitte it till an other tyme. But this by the waie, I muſt admoniſhe you, that I doe folowe here, the common forme of writers, in callyng theſe rootes, that riſe in equatiō, where as thei are not the rootes of thoſe nombers, but are the value of a roote. For of a *Coſsike* nomber, the roote muſt neades bee a *Coſsike* nomber alſo. And ſoche as by multiplication will make the rooted nomber: But ſo can not thoſe nombers doe.

And here will I make an eande, of the workes of *Coſsike* ombers. And now will I applie them to practiſe in the rule of *equation*, that is commonly called *Algebers* rule.

¶ The

## The rule of equation, commonly called Algebers Rule.

*The rule of equation.*

HEtherto haue I taughte you, the common formes of worke, in nombers *Denominate*, Whiche rules are vsed also in nõbers *Abstracte*, & likewaies in *Surde* nombers. Although the formes of these worke be seuerall, in eche kinde of nomber. But now will I teache you that rule, that is the principall in *Cossike* woorkes: and for whiche all the other dooe serue.

This Rule is called the Rule of *Algeber*, after the name of the inuentoure, as some men thinke: or by a name of singular excellencie, as other iudge. But of his vse it is rightly called, the rule of *equation*: bicause that by *equation* of nombers, it doeth dissolue doubtefull questions: And vnfolde intricate ridles. And this is the order of it.

### *The somme of the rule of equation:*

*WHen any question is propoũded, apperteinyng to this rule, you shall imagin a name for the nomber, that is to bee soughte, as you remember, that you learned in the rule of false position. And with that nomber shall you procede, accordyng to the question, vntil you finde a Cossike nomber, equalle to that nomber, that the question expresseth, whiche you shal reduce*

*reduce euer more to the leaſte nombers. And then diuide the nomber of the leſſer denomination, by the nomber of the greateſte denomination, and the quotient doeth aunſwere to the queſtion Except the greater denominatiõ, doe beare the ſigne of ſome rooted nõber. For then muſt you extract the roote of that quotiente, accordyng to that ſigne of denomination.*

Scholar. It ſemeth that this rule is all one, with the rule of falſe poſition: and therefore might so bee called: ſeyng it taketh a falſe nõber, to worke with al.

Maiſter. This rule doeth farre excell that other. And dooeth not take a falſe nomber, but a true nomber for his poſition, as it ſhall bee declared anon. Wherby it maie bee thoughte, to bee a rule of wonderfull inuention, that teacheth a manne at the firſte worde, to name a true nomber, before he knoweth reſolutely, what he hath named.

But bicauſe that name is common to many nombers (although not in one queſtion) and therefore the name is obſcure, till the worke doe detect it, I thinke this rule might well bee called, the rule of darke poſition, or of ſtraunge poſition: but not of falſe poſition.

And for the more eaſie and apte worke in this arte we doe commonly name that darke poſition. I. Ze. And with it doe we worke, as the queſtion intendeth, till we come to the *equation*.

This rule of *equation*, is diuided by ſome men, into diuerſe partes. As namely *Scheubelius* dooeth make. 3. rules of it. And in the ſeconde rule, he putteth. 3. ſeueralle cannõs. Some other men make a greater nõber of diſtinctiõs in this rule. But I intende (as I thinke beſte for this treatice, whiche maie ſerue as farre *The partes of the rule.*

as their woꝛkes doe ertende) to diſtincte it onely ſnto twoo partes. Whereof the firſte is, *when one nomber is equalle vnto one other.* And the ſeconde is, *when one nomber is compared as equalle vnto 2. other nombers.*

Alwaies willyng you to remeber, that you reduce your nombers, to their leaſte denominations, and ſmalleſte foꝛmes, beſoꝛe you pꝛocede any farther.

And again, if your *equation* be ſoche, that the greateſte denomination *Coſſike*, be ioined to any parte of a compounde nomber, you ſhall tourne it ſo, that the nomber of the greateſte ſigne alone, maie ſtande as equalle to the reſte.

And this is all that neadeth to be taughte, concernyng this woꝛke.

Howbeit, foꝛ eaſie alteratiō of *equations*. I will pꝛopounde a fewe eraples, bicauſe the ertraction of their rootes, maie the moꝛe aptly bee wꝛoughte. And to auoide the tediouſe repetition of theſe woꝛdes: is equalle to: I will ſette as I doe often in woꝛke vſe, a paire of paralleles, oꝛ Gemowe lines of one lengthe, thus: ═══, bicauſe noe. 2. thynges, can be moare equalle. And nowe marke theſe nombers.

1. 14.ze. —+— .15.ꝙ ═══ 71.ꝙ.

2. 20.ze. ——— .18.ꝙ ═══ .102.ꝙ.

3. 26.ʒ —+— 10ze ═══ 9.ʒ —— 10ze —+— 213.ꝙ.

4. 19.ze —+— 192.ꝙ ═══ 10ʒ —+— 108ꝙ —— 19ze

5. 18.ze —+— 24.ꝙ. ═══ 8.ʒ. —+— 2.ze.

6. 34ʒ —— 12ze ═══ 40ze —+— 480ꝙ —— 9.ʒ

I. In the firſte there appeareth. 2. nombers, that is 14.ze.

14.ze.—+—15.ꝙ. equalle to one nomber, whiche is 71.ꝙ. But if you marke them well, you maie ſee one denominatiō, on bothe ſides of the *equation*, which neuer ought to ſtand. Wherfore abating the leſſer, that is. 15.ꝙ. out of bothe the nombers, there will remain. 14.ze ===56.ꝙ. that is, by reduction, 1ze ===4.ꝙ.

Scholar. I ſee, you abate. 15.ꝙ. from them bothe. And then are thei equalle ſtill, ſeyng thei wer equalle before. Accordyng to the thirde common ſentence, in the patthewaie:

*If you abate euen portions, from thynges that bee equalle, the partes that remain ſhall be equall alſo.*

Maſter. You doe well remēber, the firſte groundes of this arte. For all ſpringeth of thoſe principles *Geometricalle*. Wherfore call to your minde likewaies the ſeconde common ſentence, in theſame booke, and then haue you another reaſon, whiche will helpe you not onely, in the other formes of woorke here, but alſo very often in the practiſe of this arte.

Scholar. That is this.

*If you adde equalle portions, to thynges that bee equalle, what ſo amounteth of them ſhall be equalle.*

Maſter. Theſe twoo ſentences doe inſtructe you that when you ſee on bothe the ſides of the *equation*, anny one denominatiō *Coſsike*, you ſhall marke the ſigne that is annered to the leſſer of them bothe: and if it be the ſigne of addition.—+—. then ſhall you abate that leſſer nomber, from bothe the partes of the *equation*. As I did in this firſte example. But if the ſigne be of abatemente ———, then ſhall you adde that leſſer nōber, to bothe partes. And ſo ſhall you doe, till there be noe one denomination on bothe partes, but diuerſe and diſtincte.

So the ſeconde nomber will be. 20.ze ===120ꝙ 2.
and in the leaſte termes. 1.ze ===.6.ꝙ.

Scholar. I ſee that you adde. 18.ꝙ. to bothe par-

3. tes of the equation. But by that reason, I doubte in the thirde somme, bicause. 10. ze. is in bothe partes of the equation: in the firste parte with —+—, and in the seconde parte with ——, whether I shall adde 10. ze, or abate them.

Maister. In soche a case, you maie doe either of bothe, at your libertie: and all will be to one eande.

Scholar. If I adde. 10. ze. then will it be. 26. z. —+— 20. ze. ——. 9z. —+— 213. 9.

Maister. And doe you not see. z. on bothe sides of the equation?

Scholar. I did loke but for one alteration onely.

Maister. If there were twentie like denominations, you should alter them all. For that is the principalle and peculiare reduction, that belongeth to equations.

Scholar. Then must I abate. 9. z. on bothe partes, and so will there remaine. 17. z. —+—. 20. ze. ——. 213. 9.

Maister. Now reduce it by abatyng. 10. ze.

Scholar. So it will bee. 17. z. ——. 213. 9. ——. 20. ze.

And now I remēber, that this is the better forme of reduction. Bicause the greater denomination, that is. z, is alone with his nomber on the one side of the equation, and the. 2. lesser denominations, on the other side.

Maister. How doe you reduce the other equatiōs, to their smallestе formes?

4. Scholar. In the fourthe example, there is noe denominatiō, before the signe of equation, or in the first parte, but the like is in the seconde parte also, after the signe of equation. Wherfore firste, bicause I see 19. ze. on bothe sides, I will abate it on bothe sides. And then will it be thus.

192. 9. —— 10. z. —+— 108. 9. ——. 38. ze.

But

But bicauſe I ſee ☉. yet remainyng on bothe partes, I abate the leſſer, that is .10 ☉. from bothe partes, and it will be. 84. ☉. ——. 10. ☽ ——. 38. ☌.

Maſter. This equation would bee better, if the greater denomination, did ſtande as one parte of the equation alone. Whiche thyng you maie eaſily doe, by addyng. 38. ☌. to bothe partes: bicauſe ſo moche folowth ——, on the one parte.

And euermoze when occaſion ſerueth, to tranſlate nombers compounde, —— on the one ſide is equalle to —+— on the other ſide. *Tranſlation of nombers.*

Scholar. Then it will be thus.

84. ☉. —+— 38. ☌. ——. 10. ☽.

Maſter. It were better thus.

10. ☽. ——. 38. ☌. —+—. 84. ☉.

And in ſmaller termes.

5. ☽. ——. 19. ☌. —+—. 42. ☉.

But nowe pzocede with the examples.

Scholar. The fifthe is eaſily reduced, by abatyng 2. ☌. on bothe ſides: Foz ſo will it bee. 5.

8. ☽. ——. 16. ☌. —+—. 24. ☉.

The ſixthe equation will be, by addyng. 12. ☌. on bothe ſides. 34. ☽ —— 52. ☌ —+— 480. ☉ —— 9☽. 6.

But yet I muſt reduce it farther, by addyng. 9. ☽. on bothe ſides. And then it will ſtande thus.

43. ☽. ——. 52. ☌. —+—. 480. ☉.

Maſter. Nowe will I ſhewe you the varieties of equatiõs, taught by *Scheubelius*, bicauſe you maie perceiue, howe thei bee conteined in thoſe. 2. fozmes, named by me. As foz the manyfolde varieties, that ſome other doe teache, I accoumpte it but an idle bablyng, oz (to ſpeake moare fauourably of them) an vnneſſary *Varieties of equations.*

distinction.

*The firste equation.* The firste equatiõ after *Scheubelius*, & after my meanyng also, is, when one nomber is equall to an other: meanyng that thei bothe must be simple nombers *Cossike*, and vncompounde. As. 6. ☉. equalle to. 18. ☽:

4. ☽. ——. 12. ☉. 14. ☌. ——. 70. ☽:

15. ☽☽. ——. 90. ☽☽: 20. ☽☌ —— 180. ☽☽:

26. ☽☽☽. —— 117. ☌☌.

In all these examples, as you see but one nomber, compared to an other: so to finde the quantitie of one roote, you shall diuide the nomber of the lesser Character, by the nomber of the greater Character, and so shall the *quotiente* bryng forthe the quantitie of. 1. ☉.

Scholar. It semeth at the firste vewe, that it is against reason, to diuide the nomber of the lesser signe, by the nomber of the greater. But when I consider, that if I compare a nomber of crounes, or any like denomination, to a nomber of shillynges in equalitie, the nomber of crounes, or other sorthe like, must neades be lesser, then the nõber of shillinges. And so diuiding the nõber of the shillinges (or other lesser name) by the nomber of crounes (or other greater name) the *quotiente* will shewe, how many shillynges make a croune: and generally, how many of the lesser, dooe make one of the greater.

As if. 20. crounes bee equalle to. 100. shillynges, then. 5. shillynges dooeth make a croune. So when 6. ☉. bee equall to. 18. ☽. then. 3. ☽. doeth make. 1. ☉. And. 4. ☽. —— 12. ☉. dooeth cause that. 3. ☽. must be a roote.

Maister. As your examplarie proofe is good, so reduction will be a sufficiente proofe in this.

Scholar. I see it manifestly. For if. 14. ☌. bee equalle to. 70. ☽. then. 1. ☌. is equalle to. 5. ☽. by that

reduction

reduction in nombers. And again by reduction in ſignes. 1. ☉. is equalle to. 5. ☾.

Like waies. 15. ☽. beyng equalle to. 90. ☽☽. reduction by ſignes and nombers alſo, will make 1. ☉ ——6 ☾. So ſhall. 20. ☽☉. —— 180. ☽. be reduced to. 1. ☉. —— 9. ☾. And. 26. ☽☽. ——. 104. ☉☉. will make. 1. ☉. ——. 4. ☾.

Maſter. And ſo generally, when there is noe denomination omitted, betwene thoſe. 2. that bee compared in equalitie, ſtill the diuiſion of the nomber, of the leſſer denomination, by the nomber of the greater denomination, will bryng forthe in the *quotiente*, the quantitie of. 1. ☉.

But if there bee any denominations omitted, betwene thoſe. 2. whiche be compared together in equalitie: loke how many denominations are omitted, and ſo many in order is the rooted quantitie, whoſe roote you muſt extract, for the aunſwere to the queſtiō. For in ſoche a caſe, euer more you ſhall extracte the roote of your laſte nomber. *The ſeconde forme of the firſte equatiō*

As for example, when. 6. ☉. be equalle to. 24. ☉. by the former rule, you ſhall finde. 4. in the *quotiente*. But here that. 4. is not the quantitie of a roote, but is a rooted nomber, whoſe roote I ſhall extracte. And ſeyng betwene. ☉. and. ☉. there is no quantitie omitted, but one, that is. ☽. Therefore I ſhall accoumpte. 4. the firſte quantitie, that is to ſaie, a *Square* nomber, and ſo take his *Square* roote, beyng. 2. for the quantitie of a roote.

Again if. 7. ☽☽. be equalle to. 567. ☉. the *quotinete* will be. 81. and declareth a *zenzizenzike* nomber, becauſe there are omitted betwene. ☽☽. and. ☉. three nombers: and *zenzizenzike* is the thirde quantitie: as you did learne in the beginnyng of this treatice, of nombers denominate.

Scholar. I perceiue it. And therfore I muſt take the

the *zenzizenzike* roote of. 81. whiche is. 3. and that is the true roote, where. 7. ſz. be equalle to. ſ 67. ze.

Maſter. And if thoſe. 7 ſz. were accõpted equalle to. ſ 6. z. the *quotiente* will be. 8. And bicauſe there are omitted. 2. quantities, that is. c. and. z z. therfore you ſhall accompte that. 8. to be 1 c. or a ſeconde quantitie. And his roote *Cubike* is. 2. whiche ſtandeth as the valewe of a roote, in the former equation.

And it is not poſſible that any other nomber, maie be placed as a roote, in that equation: or in any other forme of this firſte kinde. Howbeit in one ſorte of equation, of the ſeconde kinde, there maie be. 2. diuerſe rootes, when one nomber hath. 2. rootes in valewe. As I taught you before in the extraction of rootes.

*The ſeconde kinde of equation.*

The ſecõd kinde of equatiõ, after *Scheubelius* minde and myne alſo, is, when one ſimple nomber *Coßike*, is compared as equalle to. 2. other ſimple nõbers *Coßike*, of ſeueralle denominations, and like diſtaunce.

And in ſoche equation, beyng reduced as is taught before, the roote of thoſe. 2. nombers compounded, as in one (or rather the valewe thereof) ſhal be extracted: As I haue before taughte alſo. And that roote doeth aunſwere to the queſtion.

*The ſeconde forme of the ſecond kinde*

Howbeeit, here is the like obſeruation, as was in the ſeconde forme of the firſte kinde. For if thoſe. 3. denominations be not immediate, but doe omit ſome other betwene them, then ſhall you extracte the roote of that laſte nomber, in all pointes, as you did in the firſte equation.

Examples of the firſte ſorte.

4. z. ——. 6. ze —+— 4. ſ.

Whiche beyng reduced, will bee:

1. z. ——. ½. ze. —+—. 1. ſ. And the roote wil be. 2

And. 6. ſz. ——. 12. z z. —+—. 18. c.

That

That is by reduction.

1.ſz. ——— 2.z z. —+— 3.ce. or

1.z. ——— 2.ze. —+—. 3.q. And the roote. 3.

5.ſz ——— 25.z z ——— 30.ce. Or by reduction.

1.ſz. ——— 5.z z. ——— 6.ce. Or.

1.z. ——— .5.ze ——— .6.q. whoſe roote is. 3. or. 2.

Likewaies. 2.z ——— 120.q. ——— 8.ze.

Or by reductiõ. 1.z ——— 60.q ——— 4.ze. whoſe roote is. 6.

## Examples of the ſeconde ſorte.

5.z z ——— 60.z —+— 320.q.

That maketh by reduction.

1.z z. ——— 12.z. —+—. 64.q.

And the ſquare roote. 4.

Likewaies. 8z ce. ——. 40.ce. —+—. 30208.q.

Or by the orderly reduction.

1.z ce ——— 5.ce. —+— 3776.q. whoſe *Cubike* roote is. 4.

Again in reſidualles.

8.z ce ——— 864.z. ——— 24.z z.

That maketh by reduction.

1.z ce ——— 108.z. ——— 3.z z. Or els.

1.z z ——— 108.q ——— 3z. whoſe roote is. 3.

So. 9. *b*ſz ——— 90.z z. ——— 144.ze.

Or by reduction. 1.z ce ——— 10.ce. ——— 16.q. whoſe roote is. 8. or. 2.

But now bicauſe *Scheubelius* dooeth make. 2. ſeueralle equations of theſe. 2. formes: And giueth. 3. diuerſe rules, or canons for eche of them, I will declare his. 6. canons to be all contained in this ſeconde kind of equation.

he maketh his diuision thus. When .1. nomber is compared as equalle to .2. other, other that one nō-ber is of the smalleste denomination. And then is it of the firste Canon. As. 1.ʒ. —+— 8.ze ——— 65.9. or els that one nomber, is of the greateste denominatiō: As. 3.ze. —+— 4.9 ——— 1.ʒ. And then is it of the seconde Canon: Or els thirdely, the alone nomber is of the middle denominatiō: and then is it of the thirde Canon. As. 1.ʒ. —+—. 12.9. ——— 8.ze.

The like forme he vseth, for the nombers of denominations distaunte.

Wherby you maie perceiue, that in my rule there is noe forme of nombers, like thē of the firste Canon, nother yet of the thirde: but onely of the seconde. But then again in my rule, there are .2. sortes of examples whiche he hath not. And if you compare them well to gether, you shall perceiue, that thei bee agreable to-gether.

As for exāple. In his firste canon, this is the forme 1.ʒ. —+— 6.ze. ——— 27.9. whiche equation in my rule, by tradation, is expressed thus.

1.ʒ. ——— 27.9 ——— 6.ze. bicause I doe still set the greateste denommination alone.

Again in his thirde Canon, this is an example.

1.ʒ. —+— 15.9 ——— 8 ze and that nomber doe I translate into this forme 1ʒ. ——— 8.ze ——— 15 9.

Now where as he giueth seueralle rules, for euery Cannon, I saie for them all: extracte the roote of that compounde nomber. For all his rules doe teache nothyng els.

Scholar. I doe vnderstande the diuersitie, and agremente of your rules and his. But for my exercise, I dooe couette some apte questions, appertainyng to these equations.

*A question of ages.* Master. Take this for the firste question.

Alexander beyng asked how olde he was, I am .2. yeres

yeres elder (quod he) then Epheſtio. Yea, ſaied Epheſtio. And my father was as olde as we bothe, and. 4. yeres moare. And my father hauyng all thoſe yeres, ſaied Alexander, was. 96. yeres of age. I demaunde now of you, how olde was eche of them.

Scholar. I praie you aunſwere the queſtion your ſelf, to teache me the forme.

Maſter. I will begin with the yongeſte mannes age, and that will I call. 1. ☜. whiche is the common ſuppoſition in all ſoche queſtions. Then is Alexanders age. 2. yeres moare, that is. 1. ☜ —+— 2. ☉. And thoſe bothe together dooe make. 2. ☜ . —+— . 2. ☉. whereunto if you put. 4. more, then haue you the age of Epheſtio his father, that will be. 2. ☜ . —+— . 6. ☉. And all theſe put together, that is. 4. ☜ —+— . 8. ☉. will make 96 whiche is the equation that ſhall open the queſtion. *1. ☜. is the common ſuppoſition.*

Wherfore I ſet doune the equation thus.
4. ☜ . —+— 8. ☉ ═══ 96. ☉. And bicauſe I ſee on bothe ſides, one denomination of. ☉. I doe abate. 8. ☉. frō bothe ſides: & then there remaineth. 4 ☜ ═══ 88 ☉
And by reduction or diuiſion, 1. ☜. ═══ 22. ☉.

Scholar. Then maie I eaſily ſaie, that Epheſtio was. 22. yeres olde, ſeyng you did putte. 1. ☜. for his age: and now. 1 ☜. is founde to be. 22. And therby all the other yeres be manifeſte. For Alexander beyng. 2 yeres elder, muſt be. 24. And Epheſtio his father had in age. 22. and. 24: and. 4. more, that is. 50. yeres. All whiche make. 96. So is that queſtion fully aunſwered. *The proofe.*

| |
|---|
| 22 |
| 24 |
| 50 |
| 96 |

Maſter. An other queſtion is this. I had a ſomme of money owing vnto me: whereof I did receiue at one tyme $\frac{1}{4}$ and afterward I receiued $\frac{2}{7}$ of that reſidue, whiche remained vnpaied. And ſo remained the reſte of the debte 27. l. I would knowe what was the firſt debte, & what wer the. 2. ſeuerall paiementes *A queſtion of debte.*

Scholar. This muste I obserue still, to name the firste doubtfull thyng. 1. ☜. wherefore I saie that the firste debte was 1. ☜. whereof I receiued $\frac{1}{4}$. And so did there remain. $\frac{3}{4}$. ☜. of whiche reste, againe I receiued $\frac{2}{5}$, that is $\frac{6}{10}$. of the whole somme, or $\frac{6}{10}$ ☜. And that beyng abated also, then did there remaine $\frac{9}{10}$ ☜. whiche you named to be. 27. ℞. Then if $\frac{9}{10}$. ☜. bee equalle to 27. ℞, diuide. 27. ℞. by $\frac{9}{10}$, and the *quotiente* will bee $\frac{140}{9}$, that is. 60. whiche was the whole debte: And then is it plaine, that $\frac{1}{4}$. of it is. 15. and $\frac{2}{5}$. of the residue is. 18. whiche maketh. 33. and then remaineth. 27.

Maister. There is nothyng better then exercise, in attainyng any kynde of knowlege: And therfore I will proue you with diuerse questions, to make you the moare experte in this rule. And this is one.

*A question of pauyng.*

There is a floore Paued with Square Bricke, the lengthe of that floore beyng longer then the breadth, by $\frac{1}{7}$, and the whole Pauemente containeth. 3584. brickes: I require to knowe the bredthe and lengthe.

Scholar. The lesser quãtitie, whiche is the bredth I doe name. 1. ☜. And then the lengthe will bee, by your proportion. $1\frac{1}{7}$. ☜. Now must I multiplie the bredthe by the lengthe (for that is the orderly worke in all flatte formes, to finde out the whole platte) that is here. 1. ☜. by. $\frac{8}{7}$. ☜. and there will amounte the whole platte. $\frac{8}{7}$. ☡. whiche by your supposition is equalle to. 3584.

Wherfore accordyng to your rule, I diuide. 3584. by $\frac{8}{7}$. and the *quotiente* will be. 3136. whiche is a *Square* nomber, bicause there is one denomination omitted in this equatiõ. For betwene ☡ and ☉. there is omitted. ☜. And therfore muste I extracte the square roote of. 3136. and it will bee the quantitie of. 1. ☜. that I woorke in my tables, and finde it. 56. whiche muste be the bredthe: for that I named. 1. ☜. Then the length muste be moare by $\frac{1}{7}$. of it: and so shall it be. 64.

Now

Now for to confirme my woorke, I multiplie. 56. by. 64 and it will make. 3584. whiche is the nomber that you did name.

Maſter. That queſtion is well aunſwered: And if you had put. 1.ze. for the lengthe, as you might do, then the bredthe will be $\frac{7}{8}$.ze. and the ſquare $\frac{7}{8}$z. and ſo. 1.ze. would bee. 64. as you maie proue at leiſer: but in the meane time, what ſaie you to this queſtiō: *An other woorke of that queſtion*

There is a capitain, whiche hath a greate armie, & would gladly Marſhall thē, into a ſquare battaile, as large as mighte bee. Wherefore in his firſte proofe of ſquare forme, he had remainyng. 284. to many. And prouyng again by puttyng. 1. moare in the fronte, he founde wante of. 25. men. How many ſouldiars had he, as you geſſe? *A queſtian of an armie.*

Scholar. I call the firſte fronte. 1.ze. and then multiplyng it Squarely: I ſhall haue for the whole battaile. 1.z. and ſo by your ſaiyng, there was lefte 284. men, wherefore the whole nomber of men, was 1.z.—+—.284.9.

Now for the ſeconde proofe, when the fronte was increaſed by. 1. man: I ſhall ſet the former fronte, and 1. manne moare, that is 1.ze—+—1.9. And multiplyng that nomber, ſquarely: there will ariſe for the whole armie.

1.ze.—+—.1.9.
1.ze.—+—.1.9.
1.z.—+—.1.ze.
1.ze—+—1.9
1.z—+—2.ze.—+—1.9.

1.z—+—2ze—+—19. out of whiche I muſte abate 25 that, you ſaie, did wante, to make vp that ſquare battaile. And then it will be. 1.z.—+—2.ze.———24.9.

Now haue I one nomber of menne, expreſſed by. 2 *Coſsike* nombers: Of neceſſitie therefore muſt theſe. 2. nombers be equalle: ſeyng thei repreſente one armie.

Wherfore I ſet them thus.

1.ȝ. + 284.ϙ. ═══ 1.ȝ. + 2.ʒ. ——— 24.ϙ.

And findyng 1.ȝ. on bothe partes of the equation, I doe abate it, & then standeth. 284ϙ ═══ 2ʒ ——— 24ϙ. Yet again I see. ϙ. on bothe sides of the equation, and therfore, seing the lesser of them hath the signe of subtraction, I doe adde. 24. to bothe nombers, and then will there be. 308. ═══ 2.ʒ. that is. 154 ═══ 1.ʒ.

So that seing 1ʒ was set for the first fronte: the same front must be. 154. whose Square is. 23716. vnto whiche I muste adde the. 284. that did abounde, and then will the whole nomber be. 24000.

For farther trialle whereof, I take the seconde fronte to be. 155. that is. 1. moare then the firste: and his Square will bee 24025. And so is there. 25. moare then the iuste nomber of the armie, as the question supposed.

| | |
|---|---|
| | 154. |
| | 154. |
| | 616. |
| | 770 |
| 154 | |
| 23716. | |
| | 284. |
| 24000. | |

*An other woorke of that questiō.*

Maister. That question maie be wrought also, by namyng the seconde fronte. 1.ʒ. and then will his square bee. 1.ȝ. but seyng there wanteth. 25. menne, to make that Square battaile, the nomber shall bee 1.ȝ. ——— 25.ϙ.

Then for the firste front, you must set. 1. man lesse, as the question importeth, & that will be. 1.ʒ ——— 1.ϙ whose square will be 1.ȝ + 1.ϙ. ——— 2.ʒ.

1.ʒ. ——— .1.ϙ.
1.ʒ. ——— .1.ϙ.

1.ȝ. ——— .1.ʒ.
——— .1.ʒ. + .1.ϙ.

1.ȝ. + .1.ϙ. ——— .2.ʒ.

vnto whiche I must adde the. 284. menne that did abounde, whē that battaile was framed, and then will the

the nomber be. 1.ʒ. —|— .285.ϥ. —— .2.ƺ. And it muſt bee equalle to. 1.ʒ. —— .25.ϥ. Wherefore to reduce that equation, firſte I adde on bothe ſides 25.ϥ & then reſteth. 1.ʒ. equalle to. 1.ʒ —|— 310 —— 2.ƺ Then I adde. 2.ƺ. bicauſe I will haue noe —— in the equation: and it will be,

1.ʒ —|— 2.ƺ. ═. 1.ʒ. —|— 310.ϥ. Thirdely I abate. 1.ʒ. on bothe ſides of the equation: and then remaineth. 2.ƺ ═. 310.ϥ. that is. 1.ƺ. ═ 155.ϥ. wherby it appeareth that the ſeconde fronte was. 155 and the firſte fronte. 154. ſo forthe, as you wrought it before.

An other queſtion is this. *A queſtion of an armie.*

There is a kyng with a greate armie: And his aduerſarie corrupteth one of his heraultes with giftes, and maketh hym ſwere, that he will tell hym, how many Dukes, Erles and other ſouldiars there are in that armie. The Heraulte lothe to leaſe thoſe giftes, and as lothe to bee vntrue to his Prince, diuiſeth his aunſwere, whiche was true, but yet not ſo plain, that the aduerſarie could therby vnderſtande that, whiche he deſired. And that aunſwere was this.

Looke how many Dukes there are, and for eche of them, there are twiſe ſo many Erles. And vnder euery Erle, there are fower tymes ſo many ſoldiars, as there be Dukes in the fielde. And when the muſter of the ſoldiars was taken, the. 200. parte of them, was 9. tymes ſo many as the nomber of the Dukes.

This is a true declaratiō of eche nomber, quod the Heraute: and I haue diſcharged my othe. Now geſſe you how many of eche ſorte there was.

Scholar. Although the queſtion ſeme harde, I ſee many tymes, that diligence maketh harde thynges eaſie, and therfore I will attempte the worke of it.

And firſte for the nomber of Dukes, I ſette. 1.ƺ. then will the nomber of Erles bee. 2.ʒ. that is. 1.ƺ

by

by. 1. ze multiplied twise: And the nomber of soldiars are. 8. ce. that is. 2. zz. multiplied by. 1. ze. sower tymes, but bicause the. 200. parte of the soldiars is. 9. tymes so moche as the nomber of the Dukes, therfore must the. 200. parte of. 8. ce be equalle to. 9. ze. And so consequently. 8. ce ======= 1800. ze and 1. ce ==== 225. ze. or. 1. z. ======= 225. 9.

For if I set $\frac{8}{200}$ and. 9. as equalle together, & would by the arte of fractions, bryngе thesame proportion in whole nombers, I shall haue for. 9. this fraction $\frac{1800}{200}$. And seyng the denominators, be all one in $\frac{8}{200}$ and in $\frac{1800}{200}$ the proportion consisteth betwene the numerators.

Then to procede, if. 225. be equalle to. 1. z. I shall take the square roote of. 225. for. 1. ze. and that is. 15 whiche must be the nomber of Dukes.

And so haue I the firste nomber, wherefore the seconde nomber, that is the nomber of Erles, must bee 15. tymes. 15. twise: that is. 450. And the nomber of soldiars shall be. 4. tymes. 15. multiplied by. 450. that is. 27000. And for iuste trialle of this worke, I take the. 200. parte of the soldiars that is. 1350. and I finde it to bee. 9. tymes. 15. that is. 9. tymes so moche as the nomber of the Dukes. And so is that question solued, and tried.

| |
|---|
| 450. |
| 60 |
| 27000 |

*An other question of walles.*

Master. This is an other question. There is a grounde inclosed with. 4. walles, beyng like iambes and of one heigthe. The longest. 2. walles are in proportion to the shortestе, as. 5. to. 3. And vnto the height thei bee double *Sesquialter*. Now multiplyng the longestе by the shortestе, and that totalle by the heighte, there will rise. 39930. foote. I demaunde then, what is the lengthe and the heighte of eche walle?

Scholar. The least quãtitie is the heighte, whiche I call. 1. ze. and vnto it the longestе walle is double

*Sesquialter:*

*Seſquialter*: that is. $2\frac{1}{2}$. ☉. Now that ſame longeſte is in proporcion *Superbipartiente quintas*, to the ſhorteſte walle. So muſt the ſhorter wall be $1\frac{1}{2}$ ☉. Then muſt I multiplie all thoſe. 3. nōbers together, that is 1. ☉. by. $1\frac{1}{2}$ ☉. whereof doeth come. $\frac{1}{2}$ ☽. then ſhall I multiplie that totalle, by $\frac{5}{2}$ ☉. and it will be $\frac{15}{4}$ ☌. or $3\frac{3}{4}$ ☌ whiche muſt be equalle, by the woordes of the queſtion, to. 39930.

So by reducyng them to one denomination. $\frac{15}{4}$ ☌. ſhall be equalle to $\frac{159720}{4}$ that is. 15. ☌ ═ 159720. ♀. and. 1. ☌. ═. 10648. wherfore I ſhall extracte the *Cubike* roote out of. 10648. and that is the quantitie of. 1. ☉. or the heighte of the walle.

In my Tables I woorke that extraction of *Cubike* roote, and finde it to be. 22. And therfore muſt the lōgeſte walle bee double *Seſquialter* to it, that is. 55. And the ſhorteſte walle will be. 33.

For proofe whereof I dooe multiplie. 22. with. 55. and it maketh. 1210. whiche nomber I ſhall multiplie by. 33. and it will be. 39930. according to the ſuppoſition of the queſtion. *The proofe.*

Maſter. You doe chooſe ſtill the leaſte nomber, to be equalle to. 1 ☉. as the eaſieſte forme. Howbeit you maie put. 1. ☉. for the lengthe of any of the walles.

And if you ſette it for the longeſte walle, then the ſhorteſte walle will be $\frac{3}{5}$ ☉. and the heighte $\frac{3}{5}$. ☉ and all thoſe. 3. nombers will make, by multiplication together $\frac{9}{25}$ ☌. equalle to. 39930. And ſo will. 6. ☌. be equalle to. 99825. ♀. and. 1. ☌. ═. 166375. ♀. whereof the *Cubike* roote is 55. and aunſwereth to the quantitie of. 1 ☉. *An other forme of woorke.*

But if. 1. ☉. be ſet for the meaſure of the ſhorteſte walle, then the longeſte walle will bee $\frac{5}{3}$. ☉. And the heighte $\frac{3}{5}$. ☉. And ſo all. 3. nombers multiplied together will make $\frac{10}{3}$ ☌. ═. 39930. So ſhall. 10. ☌. be equall to. 35937 0. And. 1. ☌. ═. 35937. where- *The thirde forme of woorke.*

of the *Cubike* roote is. 33. and is the value of. 1. ☉. in this position.

Scholar. This varietie of woorke, is not onely pleasaunte, but it maketh the reason of the woorke to appeare moare plainly. So that I could neuer be weric to heare soche questions.

Master. Then will I propounde one or 2. moare before we passe from this kinde of equation. Whereof one shall be somewhat like that last. And this it is.

*A question of Bricke.*

A Brickeleiar had a pile of Bricke, whiche he sold by the yarde. The lengthe of it was $\frac{7}{1}$ to the bredthe, that is *Tripla sesquialtera*. And the heighte was fiue tymes so moche as the lēgthe. This pile the owner sold for. 980. crounes. By soche rate that he had for euery yarde so many Crounes, as the Pile had yardes in bredthe. Now is the question, what was the lengthe, bredthe, and heighte of this pile?

Scholar. I suppose the bredthe to bee. 1. ☉. then was the lengthe $3\frac{1}{2}$ ☉. and the heighte. $17\frac{1}{2}$ ☉. These 3. sommes dooe I multiplie together, and thei make $\frac{245}{4}$ ☌. whiche standeth as equalle to all the yardes in the whole pile. But yet what that is, I knowe not.

Wherfore to procede farther, I consider that euery yarde coste as many crounes, as the bredthe contained yardes. Now the bredthe being 1. ☉ I must saie, that euery yarde did coste. 1. ☉. of crounes. And then by the Golden rule: if. 1. yarde coste. 1. ☉. of Crounes, what shall $\frac{245}{4}$ ☌. coste?

| | |
|---|---|
| 1. | 1. ☉. |
| $\frac{245}{4}$ ☌ | $\frac{245}{4}$ ʒ. ʒ. |

Woorkyng by the rule, I finde that it shall coste $\frac{245}{4}$ ʒ. ʒ. And the question doeth suppose that it coste. 980. crounes. Wherfore I must saie, that. 980. crounes, are equalle to $\frac{245}{4}$ ʒ. ʒ. And consequently. 245. ʒ. ʒ. ═. 3920. ϑ. wherfore diuidynge the nomber of the lesser name, by the other, the *quotiente* will be 16. whose *zenzizenzike* roote is 2

And

And that therfore must be the value of a roote, and the bredthe of the pile. So shall the lengthe be. 7. yardes, and the heighte. 35. yardes.

For trialle of it, I mutiplie the lengthe, by the bredthe, and that totalle by the heighte, and so haue I 490. for all the yardes of Bricke. Then considering that euery yarde coste. 2. crounes, bicause. 2. yardes is the bredthe of the pile: the nomber of crounes must be twise. 490. that is. 980. And so is the woorke good. *The proofe.*

Master. Now woorke that question, by settynge 1. ☉. for the lengthe. *An other forme of woorke.*

Scholar. If the lengthe be. 1. ☉. the bredthe must bee $\frac{1}{7}$. ☉. that is *Subtripla sesquialtera* to. 1. ☉. And the heighte must bee. 5. ☉. All whiche sommes make by multiplication $\frac{10}{7}$ ☐.

Then farther, if 1. yarde coste $\frac{2}{7}$ ☉. $\frac{10}{7}$ ☐. shall coste $\frac{20}{49}$. ☉☉, whiche is equalle to 980. And so is. 20. ☉☉. equal to. 48020. and by diuision 1. ☉☉. ═. 2401. whose *zenzizenzike* roote is. 7. And that is the lengthe of the walle, and is the value of. 1. ☉.

1. $\frac{2}{7}$ ☉.
$\frac{10}{7}$ ☐. $\frac{20}{49}$ ☉☉.

The reste of this woorke, is like as before.

Master. Yet proue the thirde wate.

Scholar. The heighte beeyng. 1. ☉ the lengthe must be the first part of it, that is $\frac{1}{5}$ ☉. And the bredth $\frac{2}{35}$ ☉. All these make by multiplication $\frac{2}{175}$ ☐. Then for the price, if. 1. yard coste $\frac{2}{35}$ ☉ what shall $\frac{2}{175}$ ☐. coste? By the Golden Rule there is founde, $\frac{4}{6125}$. ☉☉, whiche is equalle to 980. And so shall. 4. ☉☉. be equalle to. 6002500. And. 1. ☉☉. ═ 1500625. whose *zenzizenzike* roote is. 35. And that is the value of. 1. ☉. and the heighte of the Pile. *A thirde forme of woorke.*

1. $\frac{2}{35}$ ☉.
$\frac{2}{175}$ ☐. $\frac{4}{6125}$. ☉☉

Master. One question moare will I propounde,

and so eande with this equation.

*A question of a Testament.* A poore man died, whiche had fower children, and all his goodes came to. 72. crounes: whiche he would haue parted so, that the seconde & thirde childe should haue. 7. times so moche as the firste. And that the portions of the thirde and fourthe childe should bee. 5. tymes so moche as the secondes parte: And that the first and the fourthe, should haue twise as moche as the thirde. If you worke the solution wel, you maie seme worthy to be master of those wardes.

Scholar. I trust to obtaine moare benefite by the question, then by that office. Wherefore I will giue good hede vnto it. And for the firste nōber, I set. 1. ze then muste the seconde and thirde portions make together. 7. ze. And the fourthe must bee all the reste of the. 72. that is. 72 —— 8 ze. Now the thirde must be halfe the firste & the fourthe, that is. 36 —— $3\frac{1}{2}$ ze. And the third & fourthe, is. 5. tymes the second. wherfore the seconde shall be the. 5. part of. 108 —— $11\frac{1}{2}$ ze that is. $21\frac{3}{5}$ —— $2\frac{3}{10}$ ze, whiche nombet I shall set in order with Letters, as here I haue dooen for my owne ease, and aide of memory. And then shal I adde them all together. Whereof there commeth.

| | |
|---|---|
| A | 1. ze. |
| B | $21\frac{3}{5}$ —— $2\frac{3}{10}$ ze. |
| C | 36. —— $3\frac{1}{2}$ ze. |
| D | 27. —— 8. ze. |
| | $129.\frac{3}{5}$ —— $12\frac{4}{5}$ ze. |

$129\frac{3}{5}$ —— $12\frac{4}{5}$ ze, whiche is equalle to 72. Firste therfore I do adde all that foloweth —— to bothe partes of the equatiō. And so haue I $129\frac{3}{5} = 12\frac{4}{5}$ ze + 72. But bicause there are nombers Absolute on bothe sides, I shall abate the lesser somme, that is. 72. from bothe partes, and then will there bee lefte, $57.\frac{3}{5} = 12\frac{4}{5}$. ze. that is. 288. = 64. ze. And by diuision $4\frac{1}{2}$. = 1. ze.

*The proofe.* So shall the firste mannes portion bee $4\frac{1}{2}$. And the seconde and thirde mannes portion. 7. times so moche that

that is.31 $\frac{1}{2}$. Whereby it followeth, that the fourthe manne, ſhall haue the reſte of 72. that is.36.

Then ſeeyng the thirde manne, hath halfe ſo moche as the firſt and the fourthe, his portiõ ſhall be 20 $\frac{1}{4}$. And then by diuerſe reaſons, the ſeconde mãnes part ſhall bee. 11 $\frac{1}{4}$. And all theſe partes added together, doe make iuſte 72. Wherfore the woorke is good.

| | | |
|---|---|---|
| A | 4 $\frac{1}{2}$. | |
| B | 11 $\frac{1}{4}$ | 31 $\frac{1}{2}$. |
| C | 20 $\frac{1}{4}$ | |
| D | 36. | |
| | 72. | |

Maſter. You haue wroughte it well. And yet maie you woorke it thus. Firſte ſette doune. 1.ze. for the firſte mannes parte. And then for the ſeconde and thirde ioyntly. 7.ze. ſo ſhall the fourthe manne haue 72.9.———.8.ze. And bicauſe the ſeconde mannes parte is $\frac{1}{7}$. of the thirde and fourthe mannes portion, if you ioyne all their. 3. partes together, the ſeconde mannes portion will be $\frac{1}{8}$ of that totalle. Put therfore 7.ze, whiche is the partes of the ſecond and the third vnto. 72———8ze, whiche is the fourthe mannes parte, and the totalle will be. 72.9———1.ze. whoſe firte parte is 12.9.———$\frac{1}{8}$ze, for the ſeconde mannes ſhare. Whiche ſomme if you abate out of. 7. ze. there wil remain for the thirde mannes parte 7 $\frac{1}{8}$ze——12.9.

*An other forme of woorke.*

| | | |
|---|---|---|
| A | 1.ze. | |
| B | 12.9.——— | $\frac{1}{8}$ze |
| C | 7 $\frac{1}{8}$ze——— | 12.9 |
| D | 72.———.8.ze. | |
| | 72. | |

And ſo haue you euery mannes portiõ allotted to hym duely. As I haue here ſet it forthe for you. And all thei added together, doe make. 72.

Scholar. But here is noe equatiõ yet, though the partes be diuided iuſtly.

Maſter. Now ſhall you ſee it.

The queſtion ſaieth, that the thirde mannes portion is halfe the portions of the firſte and fourthe man. wherefore ſeyng the firſte and fourthe mannes portions doe make. 72———7.ze. the thirde mannes por-

tion beeyng doubled, shall make as moche. But the double of the thirde mānes parte, is $14\frac{1}{3}$ ze —— 24 ꝙ. and therfore I saie, that those. 2. nombers be equalle, that is. 72. ꝙ. —— 7. ze. ═ $14\frac{1}{3}$ ze. —— 24. ꝙ. Firste adde. 7. ze. to eche parte, and it will bee 72. ꝙ. ═ $21\frac{1}{3}$ ze. —— 24. ꝙ. Then adde. 24. ꝙ. on bothe sides, and there will be. 96. ꝙ. ═ $21\frac{1}{3}$ ze. that is by reduction. 288. ═. 64. ze. as you made it. And then all agreeth.

Likewaies for the equation, you maie set the third mannes portion, with the halfe of the firste & fourthe mennes partes. And so will. $7\frac{1}{6}$. ze. ——. 12. ꝙ. be equalle to. 36. ꝙ. —— $3\frac{1}{2}$ ze. And by reduction, $10\frac{2}{3}$ ze. ═. 48. ꝙ. That is in other termes of whole nomber. 32 ze. ═. 144. And by diuision it will bee 1. ze. ═. $4\frac{1}{2}$. And thus will we eande the examples of the firste equation, for this tyme. And will shewe you some questions of the seconde equation.

## *Examples of the seconde equation, by questions propounded.*

*A question of silkes.*

THere are two men that haue silke to sell. The one hath. 40. elnes, and the other. 90. And the firste man his silke is not so fine as the seconde man his silke. So that he selleth in euery angell, price more by $\frac{1}{3}$ of an elne, then the seconde mā doeth. And at the eande, bothe their moneis made but 42. angelles. Now I demaunde of you, how moche eche man solde for an angell?

Scholar. I will folowe my olde forme, in putting 1. ze. for the leaste quantitie, whiche is the seconde mannes somme, and then shall the firste mannes somme be. $1\frac{1}{3}$ ze.

Maister. You are deceiued all readie. For you set

1. ze.

1.ϱ. for an elne. Seyng you name $\frac{1}{3}$ of an elne, to be $\frac{1}{3}$.ϱ. And ſo were the poſition neadeleſſe, and likewaies all the wooꝛke.

Scholar. I ſee my faulte: but I knowe not how to amende it. For that. 1.ϱ. maie bee a parte oꝛ partes of an elne: and ſo maie it be moare then. 1. oꝛ. 2. elnes ſo that I ought not to haue ſet $\frac{1}{3}$ (whiche is certainly referred, in this queſtion, to an elne) as the parte of a doubtfull quantitie, but rather as the parte of a quantitie certaine. Whereas. 1.ϱ. is euer put foꝛ a nomber vnknowen.

Maſter. To helpe you herein, I will ſet the firſte nombers, as you began theim. The ſeconde man his nombers of elnes, ſhall bee. 1.ϱ. as you did name it, and then ſhall the firſte mannes poꝛtion be as moche, and $\frac{1}{3}$ of an elne moare: whiche $\frac{1}{3}$ I maie beſte call $\frac{1}{3}$9. And ſo ſhall it bee diſtaunte from 1.ϱ. clerely in all wooꝛke *Arithmeticall*.

A 1.ϱ —|— $\frac{1}{3}$9.
B 1.ϱ.

But now to pꝛoceade, I ſhall diuide eche mannes nomber of elnes, whiche he had, by the nomber of elnes, whiche he ſolde foꝛ an angelle, and the *quotiente* will declare how many angelles eche man had receiued. So that the firſte mannes nomber of elnes, beeyng. 40. ſhall bee the numeratoꝛ, and the ſomme of meaſure, whiche he ſolde foꝛ an Angelle, that is 1ϱ —|— $\frac{1}{3}$9. ſhall bee the denominatoꝛ. And ſo is the diuiſion eanded. And that fraction is the *quotiente*.

$$\frac{40.}{1.ϱ \text{ —|— } \frac{1}{3}9.}$$

Scholar. Now I perceiue the wooꝛke. And by like reaſon: the ſeconde mannes ſomme of elnes beeyng. 90. ſhall bee the numeratoꝛ, and. 1.ϱ. beyng the ſomme of meaſure, ſolde foꝛ one Angelle, ſhall be the denominatoꝛ, that is in one fraction $\frac{90}{1ϱ}$: accoꝛdingly as I haue ſette bothe nombers

$$\frac{90.}{1.ϱ.}$$

here

here distinctly.

Master. It were moare ease for you in worckyng, if you did tourne that fractiō of $\frac{1}{3}$ into an intere vnitie.

Scholar. That wil easily be doen, by multiplyng euery nomber, of that whole fraction by. 3. And then will it be $\frac{120}{3ze \text{ —+— } 1q}$, whiche is all one in value with $\frac{40.}{1.ze \text{ —+— } \frac{1}{3}q.}$ And this I consider farther, that as these. 2. fractions, seuerally doe expresse the sommes of angelles, that eche of them receiued, so ioyntly bothe together, doe declare the full somme, of all their angelles. Wherefore I shall adde theim bothe together. And thei will make. $\frac{390ze \text{ —+— } 90q}{33 \text{ —+— } 1ze}$ As here in woorke I haue expressed.

$$\frac{390.ze.\text{—+—}.90.q.}{\frac{120.}{3.ze.\text{—+—}.1.q.} \quad \frac{90.}{1.ze.}}$$

$$3.z.\text{—+—}.1.ze.$$

And by your supposition, their bothe sommes of Angelles made. 42. So that those. 2. sommes are equalle: and therefore am I come to an equation. In whiche I see a nomber absolute, equalle to a fraction *Cossike* compounde.

Master. When so euer that, or the like dooeth chaunce, you shall reduce the whole nōber, to the like denomination: and then their numerators will bee equalle.

Scholar. Then shall I multiplie. 42. by the denominator $3z \text{—+—} 1ze$ & it wil be $126z \text{—+—} 42ze$ whiche muste bee equalle to. $390.ze. \text{—+—}.90.q.$ That is in lesser termes.

$$21z \text{—+—} 7ze. \text{———}.65.ze. \text{—+—}.15.q.$$

Where firste I dooe abate. $7.ze.$ on bothe sides: and there remaineth then. $21z. \text{———}.58.ze. \text{—+—}.15.q.$

But

But now I remēber your admonitiō, that bicause the nomber annered to the greatest signe, is moare then. 1. I shall diuide all the nombers by it, and sette their *quotientes* in their stede, with their signes. And so will the nomber of the greateste signe, euermoze be 1. And this equation will be 1.ʒ. —— $\frac{18}{21}$ ☍. —+— $\frac{15}{21}$ ☉. Where I must extracte the square roote of the later part, accoꝛding to your doctrine, and it will be. 3. As it appereth in this woꝛke folowing, whiche I did frame in my tables.

$\frac{29}{21}$. in square doeth make $\frac{841}{441}$, vnto whiche I muste adde $\frac{315}{441}$, whiche is all one with $\frac{15}{21}$, by reduction to one denominatiō. So is the full additiō $\frac{1156}{441}$. whose square roote is $\frac{34}{21}$. vnto whiche I shall adde $\frac{29}{21}$, and it will bee $\frac{63}{21}$, that is. 3.

Master. This is well doen. Now woꝛke the same questiō, as it was pꝛoponed, and you shall easily finde all the other nombers to bee true, and agreable to the question.

Scholar. Seyng the seconde manne solde. 3. elnes *The proofe.* foꝛ an angell, the firste manne did sell. 3. elnes and $\frac{1}{3}$. So. 40 (whiche is the somme of elnes of the first mans silke) diuided by. $3\frac{1}{3}$. doeth yelde. 12. and sheweth how many angelles that man receiued.

Again foꝛ the seconde man, whiche had. 90. elnes, diuide that. 90. by. 3. and so shall you finde. 30. foꝛ the nomber of his Angelles. And that. 30. and. 12. dooe make. 42, it neadeth not to be pꝛoued.

Master. Now againe foꝛ your exercise, suppose *An other forme of woorke.* the firste mannes somme to be. 1.☍.

Scholar. Then muste the seconde manns sell foꝛ an angelle. 1.☍ ——— $\frac{1}{3}$.☉. And their nombers of elnes, diuided by those nombers will make. $\frac{40}{1☍}$. and $\frac{90}{1☍ \text{ ——— } 3☉}$. whiche bothe added together, will bee $\frac{390☍ \text{ ——— } 40☉}{3☉ \text{ ——— } 1☍}$ equalle to. 42.☉. That is by reduction.

390.☍. ———. 40☉. ═══. 126.ʒ. ———. 42.☍.

And by addition of. 42. ze. on bothe partes. 432. ze ——— 40. ☉. ═══. 126. ʒ. And by diuision it will be. $\frac{24}{7}$ ze ——— $\frac{20}{63}$ ☉. ═══ 1. ʒ.

So that now I muste extracte the roote of that compounde *Cossike* fraction, thus. $\frac{12}{7}$ squarely, dooe make $\frac{144}{49}$ out of whiche I shall abate $\frac{20}{63}$. And therfore, firste of all I doe reduce thē to one denomination, & thei make $\frac{9072}{3087}$. and $\frac{980}{3087}$. wherefore if I abate the lesser out of the greater: there will remaine $\frac{8092}{3087}$. that is in lesser termes $\frac{1156}{441}$ and is a square nomber, whose roote is. $\frac{34}{21}$ vnto whiche if I adde $\frac{12}{7}$ that is $\frac{36}{21}$. it will make $\frac{70}{21}$, or $\frac{10}{3}$. that is the valewe of. 1. ze. And is the firste mannes nomber of elnes, agreably as I tried it before. And so doe bothe workes agree.

But now commeth to my remembraūce, that this nomber, whose roote I did extract, in this laste worke is of that sorte, where. ze. ——— ☉. is equalle to. ʒ. And therfore hath in it. 2. rootes: thone by addition, as this, whiche I now founde: And the other by subtraction, whiche in this example, by abatyng $\frac{34}{21}$ out of $\frac{36}{21}$, will bee $\frac{2}{21}$. But how I maie frame that roote, to agree to this question, I doe not see.

Maister. That varietie of rootes dooeth declare, that one equation in nomber, maie serue for. 2. seuerall questions. But the forme of the question, maie easily instruct you, whiche of those. 2. rootes, you shall take for your purpose. Howbeit sometymes you shall take bothe. As for example again, marke this question.

*A question of money.*

A gentilman, willyng to proue the cunnyng, of a braggyng *Arithmetician*, saied thus: I haue in bothe my handes. 8. crounes: But and if I accounpte the somme of eche hande by it self seuerally, and put therto the squares and the *Cubes* of thē bothe, it will make in nomber. 194. Now tell me (quod he) what is in eche hande: and I will giue you all for your laboure.

Scholar.

Scholar. Soche incoragementes, would make me ſtudie harde, and trauell very willyngly in learned exercises: though learnyng bee moste to be loued, for knowledges sake. But for to finde the true aunſwere thus I doe proceade.

Firſte I ſuppoſe the one nomber in one hand, to be 1.ze. And then muſt the other nedes be 8.9——.1.ze Then doe I make theim bothe Squares. And for the firſte I haue. 1 z. and for the seconde. 1.z——+——64 9 ——16.ze. Thirdely I multiplie theim bothe *Cubikely*: and so haue I for the firſte. 1.c and for the other 24. z. —+—.512. 9. —— 1. c.——192. ze. Then muſt I adde bothe the nõbers, with their ſquares, and their *Cubes*, into one ſomme. As here in worke

| | | | |
|---|---|---|---|
| 1. ze. | —+— . 1. z. | ——+—— . 1. c. | |
| | 8. 9. | ——— . 1. ze. | |
| 1. z. | —+— . 64. 9. | ——— . 16. ze. | |
| 24. z | —+— . 512 9 | —— 1 c —— | 192. ze. |
| 26. z | —+— 584. 9. | ——— . 208. ze. | |

it is ſet forthe. Where for eaſe I haue ſet. 1. ze, 1. z. and. 1. c (whiche is the Roote, the Square and the Cube of one nomber) all in one line: and the other Roote, Square, and Cube, I haue ſet ſeuerally. And ſo all thei doe make. 26 z —+— 584 9 —— 208 ze whiche is equalle to. 194. by the intente of the queſtion. Wherefore I adde firſte. 208. ze. to bothe partes, and there remaineth.

26. z. —+— 584. 9 ==== 208. ze —+— 194. 9.

Then I abate. 194. from bothe ſides, and ſo reſtethe the equatiõ thus. 26. z —+— 390. 9 ==== 208 ze That is by diuiſion. 1. z. —+— . 15. 9. ==== 8. ze. And by tranſlation of. 15. 9. toſette. 1. z. alone, it wil be. 1. z ==== 8 ze —— 15. 9. And now haue I the exacte and complete equation, where I muſt ſeke for

the value of. 1. ☉. by extractyng the roote. Therefore firste I take halfe of. 8. and that is. 4. whose square is. 16. out of whiche I abate. 15. and the remainer is 1. whiche I maie either adde to. 4. and so haue I. 5. other, I maie abate it from 4 and so haue I 3. Whiche nombers also according to thesame rule, beyng added together dooe make. 8. that is the nomber of the middell denomination. And beyng multiplied together, thei dooe make. 15. that is the other parte of thesame compounde *Cossike* nomber.

Master. And if you had marked that firste, you might easily haue found bothe those nombers, by the partes of. 15. whiche can be none other, but. 5. and 3.

And farther, seyng thei. 2. doe make. 8. and. 8. is the nomber (named in the questiō) that thei should make, therfore you shall take them bothe. And name whiche of them you liste to be. 1. ☉. And the other shall be of necessitie, the reste of. 8.

*The proofe.* Scholar. To examine theim, by the order of the question, I doe proceade thus. 3. with his Square. 9. and his *Cube*, 27. dooeth make. 39. And. 5. with his square 25 and his *Cube*. 125. doe yelde 155. And bothe thei together doe byng forthe. 194. according to the saiyng of the question: and therfore it is certain, that the woorke is good.

*An other woorke for equations.* Master. Before you passe any farther, I will admonishe you of one waie, whiche I oftē vse in reduction of soche equations, as this is, when there is noe denomination on the one side, but the like is on the other side, with a greater nomber annered to it. Then maie you abate all the lesser nōbers, out of their greaters, and the reste shall bee accounpted equalle to nothyng. Whiche chaunce can neuer happen: excepte there bee some nombers on the greater side, with the signe of abatemente. ——.

As here you had.

26 ☉. —+— 584 ☐. ——— 208 ☉. ═══ 194 ☐.
Bicauſe on the one ſide, there is noe nōber but 194 ☐ and on the other ſide, there is. 584. ☐. beeyng a greater nomber, and of theſame denomination: therefore maie you abate. 194. from bothe ſides, and then remaineth. 26 ☉ —+— 390 ☐ ——— 208 ☉ ═══ 0
Wherfore you maie well conſider, that the nombers whiche be ioined with —+—. are equalle to the nombers that bee ſet with ———. And therfore the one abatyng the other iuſtly, dooe remaine together as equalle to nothyng.

Wherefore it is reaſonable, that ſeeyng the nombers with ——— bee equalle to the nombers with —+— that I maie tranſlate the nombers with ——— from that ſide of the equation, and ſet them on the cōtrary ſide, with the ſigne of —+—. And ſo in this exāple it will bee. 26 ☉. —+—. 390 ☐. ═══ 208. ☉. And this forme ſhall eaſe you moche, in reducyng of equations.

Scholar. I thanke you moche. And I will not forget to vſe it, as occaſiō ſhall happen. But I praie you propounde yet ſome moare queſtions, that I maie ſee their diuerſe varieties.

Maſter. There were twoo ſeueralle men, which had certaine ſommes of angelles, in ſoche rate, that the ſeconde manne his ſomme, was *triple ſeſquiquarta* to the firſte: and if their. 2. ſommes were multiplied together, and to that totall the 2 firſte ſommes added, there would amounte. 142 ½. I demaunde of you, what was eche of their ſommes in angelles? *A queſtion of money.*

Scholar. The firſte mannes ſomme I call. 1. ☉. And the ſeconde mannes ſōme ſhall be. 3 ¼ ☉. which 2. ſommes beeyng multiplied together, dooe make 3 ¼ ☐. vnto whiche I muſt adde bothe the firſte nombers, that is 4 ¼ ☉. And it will be 3 ¼ ☐ —+— 4 ¼ ☉ equalle to. 142 ½. All whiche nombers, I ſhal bring

into whole nombers, if I multiplie theim by. 4. And so will it be. 1 3. ☉. —|— 17. ☽ ═══ 570. And by reducyng the greatest denomination *Cossike*, to an vnitie. 1. ☉. —|— . $\frac{17}{13}$ ☽ . ═══ . 43 $\frac{11}{13}$. And laste of all, by translatyng the nomber of. ☽ . to set. 1. ☉. alone, on one side of the equation, it will be. 1. ☉ ═══ 43 $\frac{11}{13}$ ♀ —— $\frac{17}{13}$ . ☽ . where I must extracte the value of the roote thus. $\frac{17}{26}$ . squarely dooe make $\frac{289}{675}$ . vnto whiche I shall adde the. 43 $\frac{11}{13}$ (it beeyng firste multiplied by. 52. to bryng it to the denomination of. 676. And so makyng $\frac{29640}{676}$) And it will be $\frac{29929}{676}$ whiche is a square nomber (as I haue proued in my Tables) and his roote is $\frac{173}{25}$ . from whiche roote I must abate $\frac{17}{26}$, and then wil there remain $\frac{156}{26}$, that is. 6.

And that. 6. is the value of. 1. ☽, and standeth for the firste mannes nomber. So that the seconde mannes nōber must be as. $\frac{13}{4}$ to it: that is *tripla sesquiquarta*. And so shall it be. 19 $\frac{1}{2}$.

*The proofe.* Master. Now proue those nombers, accordyng to the question.

Scholar. 19 $\frac{1}{2}$ multiplied by. 6. doeth make. 117. vnto whiche I shall adde. 25 $\frac{1}{2}$. amountyng of their. 2 additiōs, and all will be. 142 $\frac{1}{2}$, accordyng to the purporte of the question.

*An other worke of the same questiō.* Master. So is your woorke good. Yet woorke it again, by chaungyng the position.

Scholar. I maie put. 1. ☽ . to betoken the seconde manne his somme. And then shall the firste mannes somme bee $\frac{4}{13}$ . ☽ . whiche bothe multiplied together doe make $\frac{4}{13}$ ☉. And then addyng the. 2. firste sommes to it, it wil bee $\frac{4}{13}$ ☉ —|— 1 $\frac{4}{13}$ ☽ . And that is equalle to . 142 $\frac{1}{2}$ . All whiche nombers will bee reduced to whole nombers, by multiplication conueniente. And so will it be. 8. ☉ —|— 34. ☽ . equalle to. 3705: that is by reduction, 1. ☉. —|— . 4 $\frac{1}{4}$ ☽ ═══ 463 $\frac{1}{8}$ ♀. and by translation of the termes.

1. ☉.

1ℨ. ═══ 463 1/8. 9. ——— 4 1/4. ℨ. out of whiche nomber I ſhall extract the value of the roote, in this ſorte.

Firſte I ſaie 17/8 multiplied Square, doeth make 289/64, vnto whiche nomber I muſt adde. 463 1/8, reduced as it ought, and it will bee in all 29929/64. whiche is a ſquare nomber, and hath for his roote 173/8. from whiche I muſt abate 17/8. And then will there remain 156/8, that is 19 1/2, for the value of. 1. ℨ. And ſo conſequently for the ſecond mannes nōber; whiche was named in this poſition, 1. ℨ. And this maie bee proued as the other was.

Maſter. What ſaie you then to this queſtion? There is a ſtraunge iorneye appointed to a manne. The firſte daie he muſt goe 1/2 mile, and euery daie after the firſte, he muſt ſtill augemente his iorney, by 1/6 of a mile. So that his iorney ſhall procede by an *Arithmeticalle* progreſſion. And he hath to trauell for his whole iorney. 2955. miles. I demaunde in what nōber of daies, ſhall he eande his iorney? *A queſtion of iorneyng.*

Scholar. I knowe not how to procede in this queſtion.

Maſter. Doe you not heare me name it, an *Arithmeticalle* progreſſion? Wherby you might be aſſured, that it doeth appertaine to that rule. And accordyng to the canons of that rule, muſt you woorke this queſtion. But for your better inſtruction, I will helpe you in this woorke.

Firſte aunſwere to the queſtion, by the common poſition: and ſaie that the tyme of his iorney is. 1. ℨ. of daies. And then ſhall all the *exceſſes* (whiche maie alſo be called the *nomber of the ſpaces*) be. 1. ℨ ——— 19 The *common exceſſe* was ſuppoſed to bee. 1/6. of a mile. And therefore the *ſomme of all the exceſſes* muſte bee 1ℨ ——— 9 / 6 that is to ſaie, the nomber of all the *exceſſes* multiplied by 1/6, that is here, the ſixte parte of the nomber

nomber of the *exceſſes*.

And bicauſe that *the firſte nomber* is. $1\frac{1}{2}$. I muſt adde it vnto the *ſomme of the exceſſes*, and ſo haue I the *laſte nomber* of that progreſſion. Wherefore addyng. $1\frac{1}{2}$. (whiche is $\frac{3}{2}$, or in like denomination with the other, $\frac{9}{6}$) with $\frac{17\text{ℓ}-19}{6}$ it will make $\frac{17\text{ℓ}-10}{6}$. And that is *the laſte nomber* of the progreſſion.

Now you remember, that in progreſſion *Arithmeticall*, if you adde *the firſte* nomber to *the laſte*: and multiplie that totalle, by the *nomber of halfe the places*, there doeth amounte the ſomme totalle of that progreſſion.

And therfore in this exãple, if you adde. $1\frac{1}{2}$ (whiche is the firſte nõber in the progreſſion) vnto $\frac{17\text{ℓ}-19}{6}$ (that is the laſte nomber of the progreſſion) there wil amounte $\frac{17\text{ℓ}-7}{6}$, whiche beeyng multiplied by the nomber of halfe the places, that is $\frac{1}{2}$ ℓ. (For all the nomber of places is. 1. ℓ) there will riſe, $\frac{17\text{ℓ}^2-7\text{ℓ}}{12}$, whiche is the totalle ſomme of all the miles: and therfore is equalle to. 2955.

Scholar. All the reſte, and this againe can I dooe now. Seyng $\frac{17\text{ℓ}^2-7\text{ℓ}}{12}$. is equalle to. 2955. I will firſte bryng the whole nomber to the like denomination, with the fraction, and it will bee. $\frac{17\text{ℓ}^2-7\text{ℓ}}{12} = \frac{35460}{12}$. And then omittyng the like denominations. 1. ℓ — 17. ℓ. = 35460. ♀. That is by tranſlation 1. ℓ = 35460. ♀. — 17 ℓ. whoſe roote in value I ſhall finde out thus. I multiplie $\frac{17}{2}$ ſquarely, and it will be $\frac{289}{4}$. vnto whiche I ſhal adde 35460. ♀ it will make $\frac{142129}{4}$, whiche is a ſquare nomber, and hath for his roote $\frac{377}{2}$, frõ whiche I ſhall abate $\frac{17}{2}$, and then remaineth $\frac{360}{2}$, that is. 180. whiche is the valur of. 1 ℓ. And expreſſeth the nomber of dayes, whiche the queſtion requireth.

*The proofe.* Maſter. The proofe in this, and the like queſtions, is, to ſet foorthe the progreſſion with all his termes.

mes. Excepte you will for ſhortneſſe, ſette doune the firſte terme, whiche in this example is. $1\frac{1}{4}$: and then by the nomber of the *exceſſes*, or diſtaunces (whiche is euer one leſſe then the nober of places) multiplie the quantitie of one *exceſſe*: and put to it the firſte terme: and ſo haue you the laſte terme. Then hauyng the firſte terme and the laſte, with the nomber of *exceſſes* you knowe how to finde the totalle.

As in this example, the nomber of *exceſſes* beeyng 179. And the quantitie of one *exceſſe* beyng $\frac{1}{4}$. their multiplication giueth $\frac{179}{4}$. vnto whiche if you adde the firſte nomber, that is $1\frac{1}{4}$, it will be $\frac{183}{4}$. And that is the laſte nomber of that progreſſion. Then to trie the totalle ſomme of the miles, adde the firſte nomber. $1\frac{1}{4}$ to the laſte, and thei will make $\frac{197}{4}$, that you ſhall multiplie by halfe the nomber of the places, whiche in our example are. 90 (ſith the whole nomber is. 180) and there will amounte. 2955. accordyng as the queſtion ſaieth.

Scholar. This is ſufficient for this queſtion. And at ſome idle time, I will not ſticke to trie it out, by ſettyng the progreſſion foorthe at large. In the meane tyme I praie you for better exercise,, giue me ſome moare queſtions.

Maſter. There is a nomber, whiche I haue forgotten: and it is diuided into. 2. partes, whereof the one I haue forgotten alſo, but the other was. 4. And yet this I remember, that if the parte, whiche I haue forgotten, be multiplied by it ſelf, and then alſo with 4. thoſe. 2. ſommes will make. 117. Now would I knowe what was the whole nomber, and alſo what is the parte, whiche I haue forgotten. *An other queſtion.*

Scholar. I ſuppoſe the whole nomber to be. 12. And bicauſe. 4. is his one parte, the other parte muſt neades bee. 12. —— 4. Then doe I accordyng to the queſtion, multiplie. 12. —— 4. firſte by it ſelf,

and it will make. 1 ȝ. + 16. ϙ. —— 8. ze. Secondarily, I doe multiplie it that is. 1. ze. —— 4) by. 4 And it giueth. 4. ze. —— 16.

Then adde I bothe these nombers together, and it will be. 1 ȝ. —— 4. ze. whiche by the question shall be equalle to. 117.

1. ȝ. —— + ——. 16. ϙ. —— ——. 8. ze.
4. ze —— ——. 16. ϙ.

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1. ȝ. —— ——. 4. ze.

But then must I vse the accustomed translation, to bryng the greatest quantitie in denomination, to stande alone. And so will it bee.

1. ȝ. ——— ——. 4. ze. —— + ——. 117. ϙ.

Where I must seeche for the value of a roote. And therfore I multiplie. 2. by it self squarely, and so haue ϙ. 4. vnto whiche I adde. 117. and it maketh. 121. whose roote is. 11. vnto whiche I muste adde. 2. and there commeth. 13. as the value of. 1 ze and the quantitie of the whole nomber.

*The proofe.* For proofe of this worke, I abate. 4. out of. 13. and there resteth. 9. as the other parte. Then doe I multiplie. 9. by it self, and therof riseth. 81. Also I doe multiplie. 9. by. 4. and it maketh. 36, whiche bothe together, doe make. 117. as the question would.

*An other woorke.* Maister. Set. 1. ze. for the vnknowen parte, and then woorke it, to see the diuersitie of the woorkes.

Scholar. If. 4. bee one parte, and. 1. ze. the other parte, then will the whole nomber be. 1. ze + 4ϙ Therefore firste I multiplie. 1. ze. by it self, and it yeldeth. 1. ȝ. Then doe I multiplie. 1. ze. by. 4. and it giueth. 4. ze. whiche bothe sommes together, doe make. 1. ȝ. + ——. 4. ze. whiche is equalle to. 117. And by translatiō. 1. ȝ. ——— ——. 117. ϙ —— 4. ze.

Wherefore I doe multiplie. 2. squarely, and it giueth

ueth. 4, whiche added to. 117. maketh. 121. and the rotte of that is. 11. from whiche I shall abate. 2. and there will reste. 9. as the other parte of the nomber. This is verie plain, & the prose of it as it was before.

Maister. Then aunswere to this question.

*A question of proportiõ.*

There are 3 nõbers in proportion *Geometricall*. And one of the extremes is. $20\frac{1}{4}$. the other extreme, with the double of the middell terme, doeth make 22. Now would I knowe of you, what those. 3. nombers bee?

Scholar. For trialle, I name the other extreme, 1. ☉. And bicause it, with the double of the middle terme dooeth make. 22. the middell terme shall bee 11. ——— . $\frac{1}{2}$ . ☉ . for his double is. 22. ——— 1☉. whiche with. 1. ☉. doeth make. 22.

Then to proceade, I knowe the propertie of those nombers in proportion *Geometricall* to bee soche, that *the multiplication of bothe the extremes, is equalle to the square of the middell terme*, wherefore I multiplie the. 2. extremes together, and there will rise. $\frac{81}{4}$ ☉. Then dooe I multiplie. 11 ——— $\frac{1}{2}$ ☉. by it self in Square, and it will bee. 121. ♀. —+— . $\frac{1}{4}$ ♂ . ——— . 11☉, whiche must bee equalle to $\frac{81}{4}$ ☉. or. $20\frac{1}{4}$ ☉. Then to reduce it, I adde. 11. ☉. on bothe sides, and it will be. $31\frac{1}{4}$ ☉. ——— $\frac{1}{4}$ ♂. —+—. 121 ♀. and by translation. $\frac{1}{4}$ ♂ ——— $31\frac{1}{4}$ ☉. ——— . 121. ♀. That is 1. ♂. ——— 125. ☉. ——— 484. ♀.

Now resteth nothyng, but to searche the value of 1. ☉. Wherfore I take $\frac{125}{2}$ and multiplie it Square, and so haue I $\frac{15625}{4}$. from whiche I must abate. 484. that is $\frac{1936}{4}$. And there will remain $\frac{13689}{4}$ whose roote is $\frac{117}{2}$, whiche I shall abate from $\frac{125}{2}$, and there will remain $\frac{8}{2}$, that is. 4. for the other extreme.

*The proose.*

Then for the middell terme, thus shall I doe. Multiplie. 4. and. $20\frac{1}{4}$ together, and there will rise. 81. whose roote is. 9. and is the middell nomber. That 9 doubled will make. 18. and 4. ioined therto, giueth 22

So are those. 3. termes in progression *Geometricall*, according vnto the conditions limited in the question.

Master. Proue the worke now, how it wil frame if. 1. ☉. be set for the middell nomber. For it wer follie, to trie whether this question, would admitte addition of the. 2. false nombers. Although the rule doe declare that in soche sorte of equations, there is double valuation to eche roote.

Scholar. Yet I beseke you, let me examine it a litle, to see the cause, why I maie not adde them, and so take the roote.

Master. I must bere with you so moche. By addition you see, there will rise $\frac{242}{2}$, that is 121. And then the middell nomber will be. $49\frac{1}{2}$. And so the proportion is $\frac{22}{9}$. that is *Dupla superquadripartiens nonas*. Where as in the other. 3. nombers. 4. 9. $20\frac{1}{4}$. the proportion is *Dupla sesquiquarta*.

But in the question is one cõdition, that secludeth the roote, that riseth by additiõ. For the double of the middell terme, with the other vnknowen extreme, should make. 22. As in. 4. and. 9. it doeth. But in $49\frac{1}{2}$ and 121. it would be 220. that is 10. tymes so moche.

Scholar. And if you had saied in the question, that the double of the middell nomber, with the other extreme, would haue made. 220. then I should haue taken this later roote by additiõ, and not the firste roote by subtraction.

And so I perceiue the varietie of conditions in the question dooeth limite, whiche of the. 2. rootes I shall of necessitie take, and leaue the other.

*An other worke.*

But now to varie that worke, I will set. 1. ☉. for the middell terme. And then the double of it, with the other terme, will make. 22. The double of. 1. ☉. is. 2. ☉. So must the other terme be $22 ♎ —— 2. ☉$.

Then to seke out an equation, I multiplie the. 2. extremes together, that is. $22. ♎ —— 22 ☉$. by $20\frac{1}{4}$.

And

And there riſeth. $445\frac{1}{2}$. —— $40\frac{1}{2}$. ☉. And the ſquare of. 1. ☉. beyng the middell terme, is ſone perceiued to be. 1. ☽. And ſo the firſte equation is,

1 ☽ —— $445\frac{1}{2}$ ☽. —— $40\frac{1}{2}$ ☉.

Wherefore I take halfe. $40\frac{1}{2}$, that is. $\frac{81}{4}$, whoſe ſquare is $\frac{6561}{16}$. And vnto it I putte. $445\frac{1}{2}$. whereby there commeth $\frac{13689}{16}$. whoſe roote is $\frac{117}{4}$. from whiche roote I muſt abate $\frac{81}{4}$, and ſo remaineth $\frac{36}{4}$. that is. 9. As the value of. 1. ☉. And for the middle nomber.

Then for the proofe. 9. bee the middell nomber, the ſquare of it, whiche is. 81, ſhall bee equalle to the multiplications of the extremes. Wherefore if I diuide. 81. by $20\frac{1}{4}$, the *quotiente* beyng. 4. declareth the other extreme. *The proofe.*

Maſter. You ſeme experte inough in this forme of wooꝛke. Therfoꝛe I will pꝛocede to other queſtiõs, that differ ſomewhat from theſe.

There are. 2. menne talkyng together of their monies, and nother of theim willyng to expꝛeſſe plainly his ſomme, but in this ſoꝛte. The nomber of angelles in my purſe, ſaieth the firſte manne, maie bee parted into ſoche 2. nombers, whiche beyng multiplied together, will make. 24. And their *Cubes* beeyng added together, will make. 280. Then, quod the other man. And the like maie I ſaie of my money, ſaue that the *Cubes* of the. 2. partes, will make. 539. Now I deſire to knowe, what monie eche of them had. *A double queſtion.*

Scholar. The firſte mannes ſõme, I ſet to be 1 ☉ whiche I muſt parte into twoo ſoche partes, that thei bothe multiplied together, maie make. 24.

Maſter. You erre verie moche. Foꝛ it is not poſſible, that the partes of any *Coſsike* nomber multiplied together, can make an abſolute nomber. Wherefoꝛe in ſoche caſes, where you perceiue that there is required, after the firſte poſition, any multiplication to make an abſolute nomber, you ſhall call the firſte nõ-

bers, by some other name of pleasure. As here you maie call the firste mannes somme. A. And the second mannes somme. B. and then in their partition, vse the name of. 1. ☉.

And as thei are twoo questions in one, so shall you make seueralle woorkes for them.

Scholar. Then shal I saie, that the firste mannes somme is. A. and it is diuided as he declared. Wherefore for one nomber of that diuision, I set. 1. ☉. And then the other shall be $\frac{24\square}{12\odot}$. for as the one nomber multiplied by the other, doeth make. 24. So. 24. □. diuided by the one of them, must neades bring forthe the other.

Master. That is well remembred of you. For as 4. and. 5. by multiplication, doe make. 20. So. 20. diuided by. 5. bringeth forthe 4. and diuided by. 4. it yeldeth. 5.

Scholar. So $\frac{20}{5}$ is but. 4. and $\frac{20}{4}$ is. 5.

Master. Go forth then with the rest of the woorke.

Scholar. The *Cube* of. 1. ☉. is. 1. ☇. and the *Cube* of $\frac{24\square}{12\odot}$ is $\frac{1382\frac{4}{9}\square}{1\odot}$ whiche. 2. nombers I maie not adde together, vntill I haue reduced theim vnto one denomination: whiche thyng I shall doe, by setting. 1. ☇. as a fraction thus $\frac{1\odot}{1}$. And then woorkyng after the rate of fractiõs, in the firste reduction thei will stande thus. $\frac{13\frac{1}{2}\odot}{1\odot} + \frac{1382\frac{4}{9}\square}{1\odot}$. And by farther addition thus. $\frac{13\frac{1}{2}\odot + 1382\frac{4}{9}\square}{1\odot}$

And hetherto the woorke of bothe the se. 2. mennes sommes, are indifferente and agreynge. So that this one woorke serueth for theim bothe. But now thei will differ. For in the firste mannes woordes, and so in the woorke for him $\frac{13\frac{1}{2}\odot + 1382\frac{4}{9}\square}{1\odot}$ is equalle to 280: but in the seconde mannes woorke, it must be accompted equalle to. 539.

But firste to goe forward with the firste man. Seyng $\frac{13\frac{1}{2}\odot + 1382\frac{4}{9}\square}{1\odot}$ is equalle to. 280. Therefore by reduction

reduction to one denomination, $\frac{13\,\text{ce} \;+\; 13824\,\text{ꝯ}}{1\,\text{ce}}$ is equalle to $\frac{28}{1}\,\text{ce}$. And remouyng the common denominator, the numerators shal kepe thesame proportion: and therfore, 13 ce + 13824 ꝯ. shall be equalle to. 280 ce. And by translation, to leaue the greateste denomination alone, 13 ce = 280 ce — 13824 ꝯ Where I shall seke the value of. 1. ce. whiche shall not be here accoumpted the square roote, but the *Zenzicubike* roote, or the *Cubike* roote of the square roote, accordyng to the greateste denomination.

Wherfore. 140. in square, maketh 19600. from whiche I must abate 13824. And there doeth remain 5776 whose square roote is. 76. whiche beyng added vnto. 140. dooeth giue. 216. and beyng abated from it, it leaueth. 64. of whiche bothe I must extracte the *Cubike* roote, bicause in the equation there are. 2. quãtities omitted. So that of. 216. the *Cubike* roote is 6. And of. 64. the *Cubike* roote is. 4. Here I see bothe rootes serue to my purpose, that I shall take thē both.

Maister, And good reason. For as in settyng 1 ce for your position, you could not tell whether it were the greater parte, or the lesser, so maie you not now applie it to either of theim bothe, but take bothe rootes for the. 2. partes of your nomber.

Scholar. So doeth the firste mannes nomber appeare to be. 10. seyng the partes bee. 4. and. 6. whiche I maie examine thus. That thei make. 24. by multiplication, it is easily seen. And that their *Cubes* added together, doe make. 280. is sone perceiued: seyng the *Cube* of. 4. is. 64: and the *Cube* of. 6. is. 216. whiche. 2. nombers by addition, doe make. 280. *The proofe.*

Maister. Now proue the seconde mannes woorke. *The worke of the second parte.*

Scholar. In his woorke $\frac{13\,\text{ce} \;+\; 13824\,\text{ꝯ}}{1\,\text{ce}}$ is equalle to 539. And by reduction to one denomination, it is equalle to $\frac{539\,\text{ce}}{1\,\text{ce}}$. So that. 1. ce + 13824 ꝯ. is equalle to. 539. ce. and by translation.

1. ce.

1.ʒ. ☍. ——.539. ☍. ——.13824.ꝙ. whose *zenzicubike* roote I seke, thus: $\frac{539}{2}$ doth make in square $\frac{290521}{4}$, from whiche I must abate $\frac{55296}{4}$, and then remaineth $\frac{235225}{4}$, whose roote is $\frac{485}{2}$ vnto whiche I maie adde $\frac{539}{2}$. and then will it bee $\frac{1024}{2}$, that is. 512. whose *Cubike* roote is .8. And is one parte of the seconde mannes nomber. And for the other parte, I shall abate $\frac{485}{2}$ out of $\frac{539}{2}$, and there remaineth $\frac{54}{2}$. that is, 27. whose *Cubike* roote is. 3. And is the other parte of the seconde mannes nomber. As it maie sone be tried thus. For 2. 3. tymes. 8. maketh. 24. and. 27. whiche is the *Cube* to. 3. added with. 512. whiche is the *Cube* to. 8, dooeth make 539, as the question intendeth.

*The proofe.*

*A question of an armie.*

Maister. One other question I will propounde, of. 2. armies beyng bothe square, and of like nomber. And if you abate. 4. from the one armie, and adde. 10. to the other armie, and then multiplie them bothe together, there will amounte. 9853272. I demaunde of you, what is the fronte of those square battailes.

Scholar. I call the fronte 1ʒ. And then must the battaile bee. 1.ʒ. Now abatyng. 4. from the one, it will bee. 1.ʒ. ——.4.ꝙ. Then addyng. 10. to the other, it wil make. 1.ʒ. —|—.10.ꝙ. And if you multiplie those. 2. nombers together, there will amounte by it. 1.ʒ ʒ —|— 6.ʒ. —— 40.ꝙ. whiche somme must be equalle to. 9853272.

1.ʒ. —|—.10.ꝙ.
1.ʒ. ——.4.ꝙ.

1.ʒ ʒ. —|—.10.ʒ. ——.4.ʒ.
——4.ʒ. ——.40.ꝙ.

1.ʒ ʒ. —|—.6.ʒ. ——.40.ꝙ.

And if you adde. 40.ꝙ. to bothe partes of the equation, it will be. 1ʒ ʒ —|— 6ʒ. equalle to. 9853312

And

And by tranſlation. 1 ☉ ☉ —— 9853312. —— 6. ☉.
out of whiche laſte equation, I ſhall ſearche for the value of. 1 ☌. by multiplyng firſt. ☉. ſquarely, whereof commeth. 9. and then addyng it to. 9853312. And ſo commeth. 9853321. whoſe roote is. 3139. from whiche I muſt abate. 3. And then remaineth. 3136. whiche is the full nomber and Square of the one armie. And hath for his roote. 56.

For as here is one onely quantitie omitted, ſo the firſte nomber, whiche in other queſtiõs of immediate equations, was the verie roote, in theſe interrupte equations is a rooted nomber, and is here a ſquare nõber: whoſe roote therfore, I haue drawen accordyngly. And for triall of this woorke. 56. in ſquare maketh 3136. from whiche if you abate. 4. there will reſte 3132. Again if you adde. 10. there will riſe. 3146. And thoſe. 2. nombers multiplied together, doe make 9853272, as the queſtion intendeth. *The proofe.*

Maſter. This you ſee, what vſe is in theſe equations, yet are there many other equatiõs, whiche here be not ſpoken of: but here after you ſhall haue moare largely declared, if you ſhewe your ſelf diligente in this parte.

And one queſtion I will propounde, & aſſoyle without woorke for breſeneſſe, that you maie ſee there is moare behinde. There is a nomber whoſe Square abated by. 16. and the firſte nomber augemented by 8. and then bothe thei multiplied together, will bryng forthe. 2560. *A queſtion of ſtraunge equation.*

Scholar. I will proue the woorke of it. And therefore ſuppoſe the firſte nomber to be. 1 ☌. Then is his ſquare 1 ☉ whiche abated by 16. leueth. 1 ☉ —— 16 ☩. and the nõber augemẽted by. 8. yeldeth 1 ☌ —+— 8 ☩. Theſe. 2. nombers multiplied together, will make 1 ☍. —+—. 8. ☉. —— 16. ☌. —— 128. ☩. beyng equalle to. 2560.

1. ʒ. —— .16.ꝙ.
1.ꝛe. —+— .8.ꝙ.

1.ꝯ. —— .16.ꝛe.
8. ʒ. —— .128.ꝙ.

1.ꝯ. —+— .8.ʒ. —— .16.ꝛe. —— .128.ꝙ.

And addyng 128.ꝙ. on bothe ſides of the equation, it will be, 1ꝯ —+— 8ʒ —— 16ꝛe ═══ 2688ꝙ
Againe addynge. 16. ꝛe . on bothe ſides, it will bee 1.ꝯ —+— 8ʒ ═══ 16ꝛe —+— 2688ꝙ.

Maſter. Where at ſtaie you now?

Scholar. I ſee no ſhifte, but other to leaue it, as it is, 2. nombers equalle to. 2: other els to make. 1. nomber equalle to . 3. And all that is aboue my cunnyng. For hetherto I haue learned noe rule for any of them bothe. So that I can not geſſe, what the firſte nomber might bee.

Maſter. The nomber is. 12. And his Square is 144. from whiche if you abate. 16. it will bee. 128. And if you adde. 8. to. 12. it will yelde. 20. Then multipliyng. 128. by. 20. the ſomme will be. 2560. as the queſtion declared.

*Of other equations.*

But to put you out of doubte, this equation is but a trifle, to other that bee vntouched. And yet I will tourne this equation a litle, to giue you ſome light in it, and other ſoche. As here.

1.ꝯ. ═══ .16.ꝛe. —+— .2688.ꝙ. —— 8ʒ. where you ſee. 1.ꝯ. equalle to. 3. other nombers. And is it not certaine to you, that this equation is true?

Scholar. Yes, I am aſſured thereof.

Maſter. And yet to auoide doubtfulnes the more trie it by reſolution, accoumptyng. 12. for. 1.ꝛe.

Scholar. Where. 12. is. 1.ꝛe, there. 1.ʒ. is. 144. and. 1.ꝯ. is. 1728. whiche. 1728. muſt bee equalle to 16.ꝛe.

16.ze (that is.192) and to.2688. saue that you must abate.8.z, that is 1152. Now if I adde 192.to 2688 it will make. 2880. out of whiche abatynge. 1152. there will remaine.1728. wherby I see the equation is iuste.

Master. Then you see that the equation is true. And can you doubte, that any nomber, whiche is equalle to a *Cubike* nomber, hath in it a *Cubike* roote?

Scholar. It must neades be a *Cubike* nomber, that is equalle to a *Cubike* nomber: and therefore muste neades haue a *Cubike* roote: although I knowe not how to extracte that roote.

Master. Likewaies, when I saie: *Here the roote is.2.*
8.z ce.=====.12.sz.—+—.128.ꝯ. it is certaine, not onely that. 12.*sz*.—+—.128.ꝯ.containeth in it as moche as. 8. z ce. but that the. 8. parte of it is a z ce. nomber, and hath a *zenzicubike* roote.

And farther it is manifeste, that as euery. z ce. nomber, dooeth containe in it certaine. *sz*. nombers exactly, so if any nomber be annexed with those *Surfolides* (as here in this example are set 128) it is of necessitie, that that. 128. must containe in it certaine *Surfolides* exactly.

So if.8.z ce. bee equalle to *The roote is.5*
10.*sz*—+—20.z z—+—400.ce—+—3125.ꝯ. it must neades be that the.8. parte of this compounde nomber shall bee a. z ce. nomber. And also that the z z. with the other nombers folowyng dooeth containe a certain nomber of. *sz*. nombers. And the. ce. in like sorte includeth a nomber of. z z. nombers. And laste of all. 3125.ꝯ. dooeth comprehende certain *Cubike* nombers exactly.

In like sorte, when we saie, that. 1. *sz*. is equalle to *The roote here is.2.*
6.ce.—+—.8.z.—+—.9.ꝯ. All this compounde nomber is a *Surfolide*, and hath a. *sz*. roote. And 8. z.—+—.9.ꝯ. includeth certaine *Cubes*. And so

doeth. 9. 9. containe eracly. 1. 3. 02 moare.

But of these and many other verie excellente and wonderfulle woorkes of equation, at an other tyme I will instructe you farther, if I see your diligence applied well in this, that I haue taughte you.

And therefore here will I make an eande of *Coßike* nombers, for this tyme.

## *Of Surde nombers, in diuerſe ſortes*

### *And firſte of Surde nombers vncompounde.*

NOw that you haue ſomewhat learned the arte of *Coſſike* nombers, with the rule of equation, it ſemeth good time and apte place, to teache you the arte of *Surde* nõbers, whiche are diuerſe in name, accordyng as there are diuerſe natures of rootes, whiche maie giue theim name.

*A Surde nomber.*

For generally, a *Surde* nomber is nothyng els, but ſoche a nomber ſet for a roote, as can not be expreſſed by any other nomber abſolute.

As the *Square roote* of. 10, or of. 8. or of any nomber, that is not ſquare. Likewaies the *Cubike roote* of. 4, or of. 5. or of any nomber that is not *Cubike*. So the *Zenzizenzike roote* of. 8. 12. or. 20, or of any nomber that is no *Zenzizenzike*, is called a *Surde* nomber. And in like maner, any other roote of any nomber, that hath noe ſoche roote, doeth cauſe that nomber to be a *Surde* nomber.

For if you ſee thoſe ſignes annexed with nombers, that hath ſoche rootes, thoſe nombers are not *Surde* nombers properly, but ſette like *Surdes*. As the *Square roote* of. 4. or of. 9. or. 25. &c. The *Cubike roote* of. 8. 27. or. 125. &c. whiche ſometymes is vſed for apte worke, as you ſhall ſee here after.

## *Of Numeration.*

THe numeration of thē doeth conſiſte, in knowlege of their figures, whiche partly be declared before. But their common and peculiare ſignes are theſe. √. ww. w. Although there maie be moare

varieties: Yet these for this tyme maie suffice.

The firste, that is. √. is customably set, to signifie a *Square roote*. As this. √. 5. betokeneth the *Square roote* of. 5. And. √. 12. is the *Square roote* of. 12. Howbeit many tymes it hath with it, for the moare certeintie the *Cossike* signe. ʒ. And is written thus. √ ʒ. 20. the *Square roote* of. 20. And. √ ʒ. 56. the *Square roote* of. 56.

The seconde signe is annexed with *Surde Cubes*, to expresse their rootes. As this. ww. 16. whiche signifieth the *Cubike roote* of. 16. And. ww. 20. betokeneth the *Cubike roote* of. 20. And so forthe. But many tymes it hath the *Cossike* signe with it also: as ww. c̃ 25 the *Cubike roote* of. 25. And. ww. c̃. 32. the *Cubike roote* of. 32.

The thirde figure doeth represente a *zenzizenzike roote*. As. ww. 12. is the *zenzizenzike roote* of. 12. And ww. 35. is the *zenzizenzike roote* of. 35. And likewaies if it haue with it the *Cossike* signe. ʒʒ. As ww ʒʒ 24 the *zenzizenzike roote* of. 24. and so of other.

Scholar. It were againste reason, to seke reason for those signes, whiche be set voluntarily to signifie any thyng: although some tymes there bee a certaine apte conformitie in soche thynges. And in these figures, the nomber of their minomes, seameth disagreable to their order.

Master. In that there is some reason to bee shewed: for as. √. declareth the multiplication of a nomber, ones by it self: So. ww. representeth that multiplication *Cubike*, in whiche the roote is represented thrise. And. ww. standeth for. √. √. that is. 2. figures of *Square* multiplication: and is not expressed with. 4. minomes. For so should it seme to expresse moare then. 2. *Square* multiplications. But of voluntarie signes, it is inoughe to knowe that this thei doe signifie. And if any manne can diuise other, moare easie or apter in vse, thei maie well be receiued.

But

But concerning the numeration of *Surde* nombers this shal you marke: that when any compounde signe is putte before a nomber, whiche hath any roote, that maie bee expressed by parte of that signe, that nomber is not absolutely so to bee expressed, onlesse it bee for ease or aptnesse in woorke. As. ✓ʒ.ʒ.36. whiche betokeneth the *zenzizenzike roote* of. 36. Seyng it is well knowen, that. 36. hath. 6. for his *Square roote*, it were moare apte expressynge that nomber thus. ✓ʒ 6. that is the square roote of. 6.

Otherwaies, if the nōber that foloweth the signe, haue a roote agreable to that signe: it is noe *Surde* nomber. As. ✓. ʒ. 16. is. 4. and is noe *Surde nomber*. So. ✓✓. 27. is. 3. and neadeth not to bee written in *Surde* forme, excepte it bee for aptnesse of woorke. And by this maie you iudge of all other, as thei come in vse

Scholar. If this bee all that is requisite to numeration, I praie you procede to addition. For that is nexte in order.

Master. That is the common order. Howbeit in vulgare fractions, you remember that multiplication and diuision, are set before addition and subtraction: bicause of the easier formes of woorke, in multiplication and diuisiō. And so in these *Surde nombers*, bicause the woorkes of multiplication, and of diuision, be not onely moare easie, then the woorkes of addition, and of subtraction, but also be requisite to them, therefore will I begin with them, and so come to the other.

## *Of Multiplication.*

MUltiplicatiō in *Surde nombers* vncōpounde hath noe difficultie, if thei be of one denomination: els must thei be reduced to one denomination: and that by multiplication, accordyng to their signes.

But where noe reduction neadeth, you shall multiplie

tiplie the nombers together, and sette their common signe before the nomber, that resulteth of that multiplication.

## Examples of square Surdes.

IF you will multiplie. ✓. 3. 15. by. ✓. 3. 26. it will make. ✓. 3. 390.

So. ✓. 3. 32. multiplied by. ✓. 3. 48. dooeth make. ✓. 3. 1536.

And. ✓. 3. 56. multiplied by. ✓ 3 21. doeth yelde. ✓. 3. 1176.

Howbeit some tymes it happeneth, that the nomber, whiche is made by that multiplication, is a nomber absolute, and not a *Surde nomber*.

## Examples of soche as make nombers Absolute.

| | |
|---|---|
| ✓. 12. | ✓. 48. |
| ✓. 3. | ✓. 3. |
| ✓. 36. that is. 6. | ✓. 144. that is. 12. |
| ✓. $12\frac{1}{2}$. | ✓. $28\frac{4}{5}$. |
| ✓. $4\frac{1}{2}$. | ✓. $7\frac{1}{5}$. |
| ✓. $56\frac{1}{4}$ that is. $7\frac{1}{2}$. | ✓. $207\frac{9}{25}$ that is $14\frac{3}{5}$. |
| ✓. 240. | ✓. 325. |
| ✓. 15. | ✓. 13. |
| ✓. 3600. that is. 60. | ✓. 4225. that is. 65. |

And generally when any nomber is multiplied by an other, if the proportion betwene those. 2. nombers bee representeed by a Square nomber, as by. 4. 9. 16. 25. &c. then dooe thei make a square nomber by their multiplication.

Examples

## Examples of Cubike rootes.

| | | |
|---|---|---|
| ww. ☍. 91. | ww. ☍. $\frac{2}{3}$. | ww. 256. |
| ww. ☍. 12. | ww. $\frac{1}{4}$. | ww. $\frac{10}{5}$. |
| ww. ☍. 1092. | ww. 5. $\frac{1}{4}$. | ww. 196 $\frac{11}{13}$. |

## Examples of ſoche as make Abſolute nombers.

| | |
|---|---|
| ww. 54. | ww. 686. |
| ww. 32. | ww. 4. |
| ww. 1728. that is. 12. | ww. 2744. that is. 14 |

ww. 486.
ww. 96.
ww. 46656. that is. 36.

## Examples of zenzizenzike rootes.

| | | |
|---|---|---|
| w. 15. | w. 204. | w. 162. |
| w. 7. | w. 26. | w. 32. |
| w. 105. | w. 5304. | w. 5184. that is. √. 72. |

| | |
|---|---|
| w. $7\frac{2}{5}$. | w. 27. |
| w. $\frac{14}{5}$. | w. 12. |
| w. $5\frac{19}{25}$. that is. √. $2\frac{2}{5}$. | w. 324. that is. √. 18. |

## Examples of zenzizenzike rootes that make abſolute nombers.

| | |
|---|---|
| w. 32. | w. 128. |
| w. 8. | w. 32. |
| w. 256. that is. 16. | w. 4096. that is. 64. |

$\sqrt{.}$ 288.
$\sqrt{.}$ 72.

$\sqrt{.}$ 20736. that is. 12.

But here is to bee noted, that if you would multiplie any *Surde* nomber, by an absolute nomber, or any *Surde* nomber of one denominatiō, by a *Surde* nomber of an other denomination: you must firste reduce that Absolute nomber to the like denomination. And so must you reduce the. 2. *Surde* nombers to one denomination.

And bicause that this woorke doeth serue often in this arte, and that in diuerse woorkes, I will set here the arte of reduction.

## *Of reduction in Surdes.*

REduction in *Surdes*, is the bringyng of sundrie denominatiōs vnto one. Which in absolute nōbers is this doen. You shall multiplie the absolute nomber, according to the signe of the *Surde*, and then sette before it the like signe. So that if you would double. $\sqrt{.}$ 3. 8. that is to sate, if you would multiplie it by. 2. you must firste multiplie that. 2. squarely, and then multiplie those nombers together. That is to sate, you shall multiplie. $\sqrt{.}$ 3. 8. by. $\sqrt{.}$ 3. 4. and so is it doubled.

Likewaies, to *triple* any Square *Surde*, is to multiplie it by. 9. And so to *quadriple* any square *Surde*, is to multiplie it by. 16. And so forthe.

But if you double any *Cubike* nomber, you shall multiplie it by. 8. that is the *Cube* of. 2. And so if you would *triple* a *Cubike* roote, you muste multiplie it by 27. And if you would *quadriple* it, you shall multiplie

it

it by.64. And so of other like woorkes.

Again, if you will double any *zenzizenzike* roote, you must multiplie it by.16. And if you will *triple* it, you shall multiplie it by.81. And so if you will *quadriple* it, you must multiplie it by 256. And in like maner euer moare, for the nomber absolute, you shall set his *zenzizenzike* nomber. Like as in Squares, for any nomber absolute, you shall set his square. And in *Cubes* you shall take his *Cube*.

Scholar. This is plaine inoughe: yet I praie you put an example or twoo, of eche kinde.

Maister. Take these examples for square rootes.

| | | | | | |
|---|---|---|---|---|---|
| √. | 38. | √.z. | 128. | √. | 3264. |
| | 2. | | 6. | | 12. |
| √. | 152. | √.z. | 4608. | √. | 469976. |

## *Examples in Cubike rootes.*

| | | | | | |
|---|---|---|---|---|---|
| ww. | 52. | ww. | 163. | ww. | 4806. |
| | 2. | | 5. | | 8. |
| ww. | 416. | ww. | 20375. | ww. | 2460672. |

## *Examples in zenzizenzike nombers.*

| | | | | | |
|---|---|---|---|---|---|
| w. | 69. | w. | 251. | w. | 1385. |
| | .2. | | 4. | | 5. |
| w. | 1104. | w. | 64256. | w. | 2250625. |

Scholar. This I perceiue well. But now in *Surde* nombers of diuerse denominations, what the order of reductiõ is, I praie you to set forth with some exãples

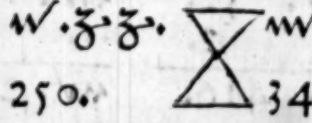
Maister. These examples with their declaration, maie sufficiently serue for a shewe, if I would multiplie. ww.12. by. √.5. I must firste multiplie the nomber of one signe, accordynge to the signe of the other

nomber, and so alter them bothe. Whiche woorke is like the reduction of fractions, to one common denomination. As here I muste multiplie. 5. *Cubikely*, and 12. must be multiplied squarely, and then shall I adde bothe signes in one, for their common signe. So shall I haue for them the. z. c. roote of. 144. to be multiplied by the *zenzicubike* roote of 125. And so will there come of that multiplication, the *zenzicubike* roote of. 18000. As here by example doeth appeare.

√.z.c. 144.
√.z.c. 125.
√.z.c.18000.

Likewaies if I would multiplie. w.z.z.250. by w.34. I shall firste multiplie. 250. *Cubikely*, and it will bee. 15625000. And 34. must I multiplie *zenzizenzikely*, and it will yelde. 1336336. Wherefore multiplying theim together, and addyng thereto the common denomination, it will bee the. z.z.c. roote of. 20880250000000.

This woorke is aptly representeed in figure, after this sorte. And then shall you multiplie crosse waies the nomber of the one, by the signe of the other. And so maie you doue in all other like nombers, of diuerse denominations.




This reduction doeth serue for any other woorke, as well as for multiplication.

## *Of Diuision.*

Diuision is as easie as multiplication. For in it there is noe regard had to the signes. But the one nomber diuided by the other as if thei were nōbers absolute. And then the firste signe added to the *quotiente*. For the more lighte and certaintie, I haue set here, examples of eche sorte.

And

## of Surde nombers.

### And first examples of square rootes.

√. 72. (√. 9. that is. 3.
√. 8.

√. 128. (√. 32.
√. 4.

√. 457 1/3. (√. 21 7/9.
√. 21.

### Examples of Cubike rootes.

ww. 96. (ww. 24.
ww. 4.

ww. 1664. (ww. 52.
ww. 32.

ww. 5624. (ww. 74 5/19.
ww. 76.

### Examples of zenzizenzike rootes.

w. 54. (w. 9. that is. √. 3.
w. 6.

w. 286. (w. 6 17/31.
w. 42.

w. 5892. (w. 109 1/9.
w. 54.

And this maie suffice for Diuision. The proofe of it is by the contrary kinde. For Multiplication proueth Diuision: and Diuision trieth Multiplication.

Scholar. All this is easie inoughe to remember.

## Of Addition.

Master.

ADdition is not so easie, but hath diuerse varieties of worke, as anon shall appere. Whereof the firste is as easie as can bee. For it requireth onely the signe of addition. —|—. As if I would adde. √. 12. to

*The firste forme of Addition.*

√.26. I shall set it thus. √.26. —+— √.12. And so √.20. put vnto. √.54. maketh. √.54. —+— √.20. This forme serueth chiefly for rootes of diuerse names. As. √. ʒ ʒ .20. —+— . √. ☾. 30. where √. ☾. 30. is added to √. ʒ ʒ .20. And so of al other.

*The seconde forme.*

The seconde forme is not so easie: and yet many times it is more certaine. And this is the order of it.

You shall sette doune your .2. nombers, that you would adde together, forseyng that thei be of one denomination. Then shall you adde in plaine forme, their nombers together, puttyng thereto the signe of the roote. And kepe that as a parte of the addition. Again you shall multiplie the .2. firste nombers together. And their totalle you shall multiplie by .4. And before that shall you sette the signe of the roote. And it shall stande as the seconde parte of that addition. So that those .2. partes, shall be added with the signe —+—. And then is the woorke eanded. Example hereof. I would adde the .2. firste sommes, that is, √. 12. to. √.26. Wherfore I set them thus. And then doe I adde thē bothe plainly together, and thei make. √.38 whiche I set by, as one part of the addition. Then doe I multiplie √.26. by. √ 12. and there riseth. √. 312. whiche I must double, or multiplie by .2. And therfore seyng the woorke is in square rootes, I set the square of 2. with the signe of. √. for .2. and then multiplyng theim together, I haue. √.1248. whiche is the seconde parte of the roote. Wherfore addyng those .2. partes together, with the signe. —+—. there commeth. √. 38. —+—. √.1248. as the totalle of that addition.

√.26—+—√12

| | |
|---|---|
| √.38. | √.26. |
| | √.12. |
| | √. 312. |
| | √. 4. |
| | √.1248. |

√.38.—+—√.1248.

Scholar. As me thinketh, the firste forme of addition,

tion serueth better for these nombers, then this seconde forme. For it is moare easie to vse, in any kinde of woorke, and moare spedily doen: and it seemeth that this laste nomber, is moare obscure then the firste.

Maister. Yet is this woorke good, and very necessarie. For in these nombers, and soche other like, it serueth onely (as appereth) to alter the state of the nōbers, whereby thei maie bee commensurable, with other, then thei were before that alteration. But in some nombers, and that very many, it reduceth them to one simple forme of roote. As by the examples folowyng you shall perceiue.

An example.

√.28.—+—√.7.

√. 28.
√. 7.

√. 196.
√. 4.

√.784.

28.
7.
35.

√.35.—+—√.784.
Or. √.35.—+—28.
That is. √. 63.

*A thirde forme of addition.*

Thesame example other waies wrought.

√.28.—+—.√.7.

√. 28.
√. 7.

28.
7.
35.

√.196. whose roote is 14

14.
2.

√.35.—+—.28.

Where firste I haue set forthe. 2. examples of one addition, that you maie see the agremente of thē both

And firste I would adde. √. 28. with. √.7. wherefore I dooe ioyne. 28. and. 7. in one somme, whiche I set a parte, as the firste portion of theaddition. Then I doe multiplie. 28. by. 7. And thereof commeth. 196. whiche is a square nōber, and hath. 14. for his roote. So that I maie vse now. 2. woorkes. For other I maie continue my woorke, as I haue doen (agreable to the firste example) in multiplyng that. √. 196. by. √. 4. (whiche is but doubling) and so there cōmeth. √. 784

whiche

whiche is a nomber absolute: bicause it hath a roote, accordyng to his signe, whiche roote is. 28. and maie be set for. √. 784.

Now in the seconde woorke, bicause the firste multiplication of. 28. by. 7. doeth make a square nomber, I doe take the roote of that nomber for it: seyng it is all one thyng to saie. √. 196, and. 14. for. 14. is the roote of. 196. And then hauyng the roote, I muste double it, accordyng to the rule, or multiplie it by. 2. and there of commeth. 28. whiche I shall adde with 35. And so haue I. 63. whose roote containeth the addition of. √. 28. and. √. 7.

Scholar. This woorke semeth straunge: and farthestе from common reason, of all other woorkes in this arte.

Master. I mighte easily by demonstration make you, to perceiue as moche reason in this woorke, as cā be in any: for it dependeth of the. 38. Theoreme of the patthewaie. But haste of other businesse, maketh me to omit the demonstration at this tyme, whiche shortly you shall haue, for all the equations, and other woorkes likewaies.

But for this presente tyme, it shall be sufficiente to woorke an example in *rationall* nombers, as if thei iwer *Surde* nombers: that therby you maie perceiue the order, and the truthe of the woorke.

Wherfore I take these twoo nombers. √. 36. and √. 49. to bee added together. Where I doe firste adde the twoo nombers plainely together: And thei make 85. for the firste parte of the addition. Then dooe I multiplie. 49 by. 36. and there riseth. 1764. whiche is a square nomber. And therefore maie I vse. 2. woorkes, as you see. In the firste I multiplie that Square nomber by. 2. or by. √. 4. whiche is all one: and there doeth amounte. 7056. a Square nomber also, whose roote is. 84.

The

**The firste forme.**

✓.36.—+—.✓.49.

| | | |
|---|---|---|
| ✓. | 49 | 49. |
| ✓. | 36 | 36. |
| | 294 | 85. |
| | 147 | |
| ✓. | 1764 | |
| ✓. | 4 | |
| ✓. | 7056 | |

✓.85.—+—.✓.7056.

Or.✓.85.—+—.84

**The seconde forme.**

✓.36—+—✓.49

| | |
|---|---|
| ✓. | 49. |
| ✓. | 36. |
| | 294. |
| | 147. |
| ✓. | 1764. |

That is. 42.
2.
84.

✓.85—+—84.

That is. ✓. 169.
Or. 13.

In the seconde woorke I take the roote of. 1764. whiche is 42 and doublyng it, I haue 84. agreable to the other woorke. Then doe I sette those. 2. nombers doune with —+—, and putte to them the signe. ✓. in token that I muste take the roote of that compounde nomber: and not of any one parte of it.

Scholar. That haue I marked well: For 85. hath no roote, nother 84. hath any roote. But 85—+—84 that is. 169. hath. 13. for his roote.

And so I see, that the roote of. 36. whiche is. 6. And the roote of. 47. that is. 7, beeyng bothe added together will make. 13. that is the roote of. 169.

*Of nombers commensurable, after the forme.*

Maister. Yet one other forme of casie woorke, I will shewe you, whiche is bothe pleasaunte and profitable: But is not generalle, for it serueth onely for *nombers commensurable*, I meane soche nombers, as by one common diuisor, maie bee brought into Square nombers. With whiche nombers, you shall woorke thus.

Firſte diuide theim by the common diuiſor: and ſet for them their rootes. Then adde thoſe. 2. rootes together, and multiplie it ſquarely. And that ſquare beyng multiplied by the common diuiſor, will bryngе forthe the Square of bothe the rootes. As here followeth in example.

Where I would adde ✓.384 vnto ✓.150 which nombers I doe eramin, til I maie finde their commō, and leaſte diuiſor, whiche here is. 6. Then diuidyng them by that. 6. I haue for 384. a ſquare nomber. 64. And for. 150. I haue an other ſquare, that is. 25. Of whiche bothe ſquares I ſet doune the rootes: and the common diuiſor alſo. Then doe I adde bothe rootes together, and thereof commeth. 13. whoſe Square is 169. that I doe multiplie by. 6. whiche is the commō diuiſor, and it will bee. 1014. whoſe roote doeth contain bothe the rootes before named. As you ſhall ſee it proued anon by Subtraction.

| ✓.384 | ✓.150. |
|---|---|
| 6.) 64 | 25. |
| 8 | 5. |
| 13. | |
| 13. | |
| 169. | |
| 6. | |
| 1014. | |

Scholar. In the meane ſeaſon I conſider, that one of theſe formes, maie confirme the other. And therefore if I worke this laſte erample, by one of the other formes, and finde theſame totall, it muſt neades be that the worke is good. Whiche I proue thus.

Firſte ſettyng doune the nombers, in forme of the eaſieſte Addition. And then addyng theim together, I finde. 534. whiche I ſette a ſide, as one parte of the nomber, that I doe ſeke for.

Then dooe I multiplie the. 2. nombers together, and thei make. 57600. whiche I dooe multiplie again by 4. And there riſeth. 230400. being a ſquare nomber, and hath. 480. for his roote. Wherefore I

ſet

√.384 —+— √.150.

384
150
19200
384
57600
4
230400

384
150
534

√.534 —+— √.230400.

Or. √.534 —+— .480.

That is. √.1014.

set. 534. and. 480. together, with the signe of Addition, thus. 534 —+— 480. And the roote of that nomber, is equalle to bothe the firste rootes. But considering that bothe those nombers, which bee ioyned laste of all with —+—, are nombers rationall and absolute, I maie adde thē in one, & so thei make 1014. agreably to the other woorke. Wherefore I iudge them bothe to be good

Maister. You might haue wrought this woorke otherwaies, bicause the firste nomber, that riseth of the multiplication is a square nomber.

Scholar. Then I perceiue, I mighte haue taken the roote of it, whiche is. 240. and doublyng it, I should haue. 480. As I had in the other woorke. And so all doe agree in one.

But my chief doubte now is, how to knowe those nombers that bee *commensurable*: For if I shall stande long in searchyng for that, I might soner woorke the other some of woorke, then to make that trialle of *commensurableneſſe*.

Maister. The easieste waie is, to diuide the greater nomber, by the lesser, as if thei were bothe nombers absolute: & the *quotiente* will declare their *Squares*. *The findyng of nombers commensurable.*

As if you doubte, whether. 384. and. 150. bee *commensurable*, diuide. 384. by. 150. and the *quotiente* will be $2\frac{14}{25}$, that is $\frac{64}{25}$. Then diuide whiche of the. 2. firste nombers you liste, by his like nomber in the *quotiente*: And the common diuisor will amounte. So if you di-

uide the greater nomber. 384. by the greater nomber in the *quotiente*, whiche is. 64, you shall finde the new *quotiente* 6. whiche. 6. is the common nomber. Or if you diuide. 150. by 25. the common nomber. 6. will be the *quotiente*.

But and if the *quotiente* be a whole nomber, and no fraction, and be a Square nomber, then is it the lesser square. Wherefore if you diuide the lesser nomber of the .2. by the *quotiente*, the common nomber will appeare in the seconde *quotiente*. And then if you diuide the greater of the. 2. nombers, by that common nomber, his *quotiente* will shewe you the other *Square*.

And if so happen, that the *quotiente* of the firste diuision be not a square nomber, then are those nombers *incommensurable*.

So. ✓. 32. and. ✓. 128. bee *commensurable*: and the *quotiente* of their diuision is. 4. whiche is the lesser square. And. 8. appeareth to be the common nomber. And the greater square is. 16.

Howbeit by this nomber it maie easily bee espied, that some nombers maie be resolued, into more squares then one. As these 2. nombers, beyng diuided by. 2 doe giue. 16. and. 64. And beeyng diuided by. 8, they beyng for the. 4. and. 16.

But for their addition, what Squares so euer you take, that redounde by one common diuisor, the triall will be like, and the roote one.

Scholar. I praie you let me proue that varietie.

Master. Then proue it in soche nombers, where you maie finde moare varietie. As these bee. ✓. 288. and. ✓. 1152.

Scholar. If I diuide. 1152. by. 288. the *quotiente* will bee. 4. whiche I must take for the leaste Square. Then by it I diuide. 288. and the *quotiente* will be. 72. as the common diuisor. By whiche if I diuide. 1152. there will rise. 16. as the seconde square. Then set I

the

the nōbers in order thus. And vnder. 1152. I set the one Square. 16. And vnder. 288. I putte the other square. 4. And vnder eche of theim his roote. Then adde I the Rootes together, whiche maketh. 6. whose square is. 36. And that beyng multiplied by 72. the common nomber, doeth yelde. 2592. whose roote doeth containe bothe the other. 2. rootes by addition.

√. 1152 — + — √. 288.

16 4.

72) 4 2.

6.

6.

36.

72.

72.

252.

√. 2592.

But now how I shall finde any other Squares in those nōbers, to make any farther trial, I knowe not.

Maister. Diuide alwaies one of the nombers, by some square nōber, that will parte it exactly, without any remainer. And marke the *quotiente*. For by it shal you diuide the other nōber, and if the *quotiente* in that last diuision, be a square nomber, then haue you your purpose. Els musts you proue with an other Square nomber.

Scholar. I vnderstande you. And therfore in these nombers, I will make trialle with. 9. by whiche I diuide. 288. And finde the *quotient*. 32. Then by thesame 32. I diuide 1152. and the *quotiente* is. 36. So haue I 9 and. 36. for the. 2. squares, and. 32. for the cōmon diuisor. Wherfore I set the nōbers in order as thei ought. And vnder them I place the. 2. square nombers with their rootes. Then addynge the rootes together, I finde. 9. whiche I multiplie square, and it yeldeth. 81. that. 81. I doe multiplie by the common nomber. 32. and there amounteth. 2592. As it did before in the other worke. Whereby I perceiue that these woorkes doe confirme one an other.

| √.1152 | + | √.288. |
|---|---|---|
| 36 | | 9 |
| 32) 6 | | 3 |
| 9 | | |
| 9 | | |
| 81 | | |
| 32 | | |
| 162 | | |
| 243 | | |
| √ 2592 | | |

And therefore I will proue, how many varieties of this worke, I may finde in these nombers. And for my purpose, I will diuide the lesser of the .2. nombers, by as many Squares as I can, for that seameth to be the readiest waie. And firste I proue with .16. And so the *quotient* is .18. by whiche .18. I diuide .1152. and the *quotiente* is .64. whiche is a square nōber. So that I haue that varietie more.

Then again I proue with .25. But I see, that will not frame. Wherfore I assaie with .36. And finde the *quotiente* 8. by whiche I diuide the greater square, and the *quotiente* is .144. a square nomber also. And therfore I note that for an other varietie.

Thirdly, I proue with .49. but that wil not agree. Then attempte I with .64. And that serueth as euil. Verte that I assaie .81. 100. and. 121. but none of them will diuide .288. wherefore I passe vnto .144. whiche is twise contained in 288. by that .2. I diuide 1152. and finde the *quotiente*, 576. whiche is a Square nomber also. And so haue I .3. other varieties beside the .2. former woorkes: whiche .3. varieties, for my remembraunce I set doune, thus.

√.1152.

| √.1152 — + — √.288 | | √.1152 — + — √.288 | |
|---|---|---|---|
| 64 | 16 | 144 | 36 |
| 18) 8 | 4 | 8) 12 | 6 |
| 12 | | 18 | |
| 12 | | 18 | |
| 144 | | 324 | |
| 18 | | 8 | |
| 1152 | | √. 2592 | |
| 144 | | | |
| √. 2592 | | | |

| √.1152 — + — √.288 | |
|---|---|
| 576 | 144 |
| 2) 240 | 12 |
| 36 | |
| 36 | |
| 1296 | |
| 2 | |
| √. 2592 | |

Master. Then for to gratifie you, I will sette doune .2. other nombers with 6 varieties. Whiche maie seame to suffice for this worke, without more exãples. And bicause you know the order to trie thẽ I will sette theim doune without any explication, other declaration. As here you see.

| √.28800 — + — √.7200 | | √.28800 — + — √.7200 | |
|---|---|---|---|
| 14400 | 3600 | 3600 | 900 |
| 2) 120 | 60 | 3) 60 | 30 |
| 180 | | 90 | |
| 180 | | 90 | |
| 32400 | | 8100 | |
| 2 | | 8 | |
| √. 64800 | | √. 64800 | |

√.28800

| √.28800 — + — √.7200 | | √.28800 — + — √.7200 | |
|---|---|---|---|
| 1600 | 400 | 900 | 225 |
| 18) 40 | 20 | 32) 30 | 15 |
| 60 | | 45 | |
| 60 | | 45 | |
| 3600 | | 2025 | |
| 18 | | 32 | |
| 28800 | | 4050 | |
| 36 | | 6075 | |
| √. 64800 | | √. 64800. | |

| √.28800 — + — √.7200 | | √.28800 — + — √.7200 | |
|---|---|---|---|
| 576 | 144 | 400 | 100 |
| 50) 24 | 12 | 72) 20 | 10 |
| 36 | | 30 | |
| 36 | | 30 | |
| 1296 | | 900 | |
| 50 | | 72 | |
| √. 64800 | | √. 64800 | |

Scholar. This varietie is pleasaunte.

Maister. I will satisfie your delite better at more leisare. But yet one thyng moare will I saie, before we eande this sorte of Additiõ: that if you would adde any roote to it self. As. √.6. to. √.6. or. √.10. to. √.10. &c. you shall onely *quadriple* the nomber: and so haue you doen.

Scholar. I see good reason in that: For addition of any nomber to it self, is but doublyng that nomber or multiplication by. 2. And that must be doen by that *quadriplation*, as you taught before.

*Addition of cubike rootes*

Maister. Now will I set forthe some examples of addition in *Cubike* rootes. For the worke is like vnto this laste forme in Square rootes, saue that the multiplications,

tiplications, whiche were Square in that woꝛke, muſt be *Cubike* in this woꝛke. And that onely in nombers *commenſurable*. Foꝛ nombers *incommenſurable* bee added with the ſigne. —+—. without moare woꝛke.

I call ſoche *Cubike* rootes *commenſurable*, whiche beyng diuided by any common nomber, will make *Cubike* nõbers in their *quotiente*. As. ww. 24. and. ww. 81 whiche diuided by. 3. doe make. 8. and. 27. bothe beyng *Cubike* nombers. So. ww. 320. and. ww. 135. beyng diuided by. 5. doe make. 27. and. 64. bothe *Cubike* nombers. Likewaies. ww. 2744. and. ww. 1000. be *commenſurable*, bicauſe thei make. 343. and. 125. bothe *Cubike* nombers: If thei be diuided by. 8. *Cubike rootes commenſurable.*

Scho. I praie you make your exãples with theſe.

Maſter. There nedeth noe woꝛdes in this woꝛke it is ſo like the Addition of ſquare rootes. And theꝛeꝛoꝛe marke theſe examples well.

| ww. 81. —+— ww. 24. | ww. 320. —+— ww. 135 |
|---|---|
| 27 8 | 64 27 |
| 3.) 3 2 | 5.) 4 3 |
| 5 | 7 |
| 5 | 7 |
| 125 | 343 |
| 3 | 5 |
| ww. 375 | ww. 1715 |

ww. 2744 —+— ww. 1000

343 125.

8.) 7 5.

12

12

1728

8

ww. 13824

Do. j. Scholar.

Scholar. Here is noe diuersitie, from the former worke, but in settyng the *Cubike* roote, for the square roote. And in multiplyng the addition of the. 2. rootes *Cubikely*.

Master. That is all. And therefore will I stande noe longer aboute it: But will proceade to an other forme of addition, whiche serueth also for *Cubike rootes commensurable*. The rule is this. Set doune the *Cubike* rootes, with their common diuisor, and the *Cubes* that rise therby, and their rootes also. All this you did in this former worke. But now peculiarly in this rule, you shall set doune. 3. other nombers orderly, vnder those. 3. former nombers. The firste is the square of that laste *Cubike* roote: the secõde is the *triple* of that Square: and the thirde is a nomber resultyng of the multiplication of that triple by the other roote.

*Another forme of addition.*

Then take the. 4 extreme nombers, that is those 2 laste nombers, and the. 2. *Cubes*, and adde theim together into one somme. And that somme beyng multiplied by the common diuisor, will make a *Cubike* nomber, whose *Cubike* roote shall containe bothe the firste rootes, whiche you intended to adde. Now marke these examples: and cõferre theim well with the wordes of the rule.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ww. 384 | + | ww. 48 | | ww. 15972 | + | ww. 2592. | |
| 64 | | 8 | 12) | 1331 | | 216 | |
| 6) 4 | | 2 | | 11 | | 6 | |
| 16 | | 4 | | 121 | | 36 | |
| 48 | | 12 | | 363 | | 108 | |
| 48 | | 96 | | 1188 | | 2178 | |
| | 216 | | | | 4913 | | |
| | 6 | | | | 12 | | |
| | | | | | 9826 | | |
| | | | | | 4913 | | |
| ww. | 1296 | | | ww. | 58956 | | |
| | | | | | | ww. 52488. | |

. ww.52488.—+—.ww.24696.

| | | | |
|---|---|---|---|
| | 5832. | | 2744. |
| 9) | 18 | | 14. |
| | 324 | | 196. |
| | 972 | | 588. |
| | 10584 | | 13608 |
| | | 32768. | |
| | | 9. | |
| ww. | | 294912. | |

Scholar. In these examples I see, the woordes of your rule obserued. For vnder eche *Surde Cubike* roote, there is set a true *Cubike* nomber, whiche is founde by the common diuisor: then foloweth the roote of that true *Cube*: and beside it standeth the common diuisor. Then in the fourthe roome is the Square of the true *Cubike* roote. And vnder it his nomber tripled (as. 48 vnder. 16, and. 12. vnder. 4) whiche triple beeyng multiplied by the roote of the other side, dooeth make the loweste nomber in that rowe. So. 48. multiplied by. 2. maketh. 96. whiche is set vnder the roote. 2. And. 12. multiplied by. 4. yeldeth. 48. whiche is placed vnder that. 4.

Then those. 4. extreme nombers. 64. and. 48, 8. & 96. doe make by addition 216. whiche somme is multipled by. 6, that is the common diuisor, and so riseth 1296. whose *Cubike* roote comprehendeth bothe the firste rootes.

Master. The like maie you iudge of the other. 2. examples. But bicause you maie vnderstande the certaintie of this woorke the better, I haue here sette forthe. 2. examples of true *Cubike* rootes, formed like *Surde* nombers.

ww.4096. —+— .ww.1728.

| | | |
|---|---|---|
| | 512. | 216. |
| 8) | 8. | 6. |
| | 64. | 36. |
| | 192. | 108. |
| | 864. | 1152. |

2744
8

ww. 21952

ww.19683. —+— .ww.3375.

| | | |
|---|---|---|
| | 729 | 125 |
| 27) | 9 | 5 |
| | 81 | 25 |
| | 243 | 75 |
| | 675 | 1215 |

2744
27

19208
5488

ww. 74088

Scholar. I perceiue by examination of wooꝛke in my Tables here, that 4096. is a *Cubike* nomber, and hath 16 foꝛ his roote. So 1728 is a *Cubike* nōber also, & his roote is. 12. those bothe rootes added together, doe make. 28. And that. 28. is the *Cubike* roote to. 21952. as the firſte example would.

And foꝛ the ſeconde example, I ſee likewaies that 19683. hath. 27. foꝛ his *Cubike* roote. And. 3375. hath 15. foꝛ his roote. And thei bothe make. 41, whiche is the *Cubike* roote to. 74088. accoꝛdyng to the wooꝛke of the ſeconde example.

*Addition of zenzizenzike rootes.*

Maſter. Seyng you are conuenientlye inſtructed, in theſe nombers, wee will goe in hande with *zenzizenzike* rootes, and their additiō: wherein is no differēce of woꝛke, but onely foꝛ the multiplicatiō, whiche muſt be agreable to the nature of the nombers, *zenzizenzikely*. And the reduction by the common diui-

ſoꝛ,

for, in like forme, into *zenzizenzike* nombers, whē the firſte nombers bee *commenſurable*. But if thei be *incommenſurable*, then muſt the addition be wrought by the ſigne. —+—, without any other buſineſſe.

## *Examples of zenzizenzikes beeyng commenſurable.*

| ✓.648—+—✓.5000 | ✓.1280—+—✓.6480 |
|---|---|
| 81 625 | 256 1296 |
| 8) 3 5 | 5) 4 6 |
| 8 | 10 |
| 8 | 10 |
| 4096 | 10000 |
| 8 | 5 |
| ✓. 32768 | ✓. 50000 |

| ✓.38416—+—✓.65536 |
|---|
| 2401 4096 |
| 16) 7 8 |
| 15 |
| 15 |
| 50625 |
| 16 |
| 3037.50 |
| 50625 |
| ✓. 810000 |

In the firſte and ſeconde examples the nōbers are *Surdes*, but in the thirde example thei are rationall nombers, framed like vnto *Surdes* to the intente that you mighte the better perceiue the forme of the worke. For 38416. is a *zenzizēzike* nomber, & hath. 14. for his roote So. 65536. is a *zenzizenzike* nomber, and hath. 16. for his roote. And theſe. 2. rootes do make. 30. whiche is the *zenzizenzike* roote vnto. 810000. And therefore maie it bee truely ſaied, that. ✓. 810000 doeth containe the twoo firſte rootes.

Scholar. I praie you procede to Subtraction. For all this I doe well perceiue.

# The Arte
## Of Subtraction.
### Master.

Subtraction doeth differ from addition, in litle moare then the signe ———. Whiche signe serueth generally, for all nombers *incommensurable*. And considering there is litle difficultie in Subtraction: If you remember well the arte of Addition, I wil lightly passe it ouer in thesame examples, that I haue wrought in Addition, because it maie bee a proofe of that woorke: and that woorke also a confirmation of this.

Onely this shall you obserue in this rule peculiarly: that as in the seconde forme of Addition, you must adde the rootes together, before you multiplie theim. So here you shall Subtracte the lesser roote, from the greater, before you doe multiplie theim.

Example of Subtraction, with ———.

√.12. abated out of √.26. maketh. √.26———√.12. and so of other.

## *Examples of the seconde forme of Subtraction*

√.63.———.√.28.

63
28
504
126
√.1764
√. 4
√.7056

√91———√.7056

Dz. √.91———84.

63
28
91

The seconde forme of that woorke.

√.63.———.√.28.

63
28
1764

whose roote is. 42.

42.
2
84

√.91——84.

That is. √.7.

√.169

√.169——√.36

169
36
1014
507
√. 6084
√. 4
24336
√.205——√.24336.
or.√.205——√.156.

169
36
205

An other foꝛme of that wooꝛke.

√.169.——.√.36.
169
36
√. 6084
whose roote is. 78.
78
2
156
√.205——156.

That is. √.49.

Scholar. I see in all these examples, you take the same nombers, that you had befoꝛe in Addition. And firste you set the totalle, out of whiche you abate one of the nõbers, that befoꝛe were added, & the remainer bringeth foꝛthe the other. Foꝛ in the firste of these. 2. examples. √.28. is abated out of. √.63. and there remaineth. √.91. ——84. that is. √.7. foꝛ. 84. taken out of. 91. leaueth. 7. And in the seconde exãple. √.39 abated out of. √.169. doeth leaue remainyng. √.49.

Maister. The thirde foꝛme of Subtraction, is like the thirde foꝛme of Addition: saue that we set ——. foꝛ —+—. And here wee muste abate the lesser roote frõ the greater (as I said) befoꝛe we doe multiplie that nomber by it self. As by this exãple, you maie perceiue Where I dooe Subtracte. √. 105. out √. 1014, and the remainer is. √. 384. Now marke the wooꝛke

√.1014——√.105.

169 25.
6) 13 5.
8
8
64
6
√. 384

Here you see all thinges agree, with the foꝛme of Addition, saue ——. foꝛ —+—. and when I begin to gather the nomber, that stãdeth in the middle, whiche I multiplie by it selfe, and I dooe not make that nomber,

nomber, by addyng bothe rootes together: For so. 13. and. 5. would make. 18, but I abate. 5. out of 13. and so there doeth remain. 8. with whiche I procede as I did in Addition. And then commeth forthe the remainer. √.384.

Scholar. I vnderstande it very well. And I praie you that for a proofe, I maie varie the other examples of addition. Partly for my exercise, and partly for examinatiō of the former additions, by the cōtrary kind.

Master. With good will.

Scholar. Then will I set them, and worke them, as here foloweth.

But firste I will begin, with the worke of this laste example, after the seconde forme of Subtraction: for a double confirmation of it.

√.1014——√.150

| | |
|---|---|
| 1104 | 1014 |
| 150 | 150 |
| 50700 | 1164 |
| 1014 | |
| √. 152100 | |
| √. 4 | |
| √.608400 | |

√.1164——√.608400

Or. √.1164——780

An other forme of thesame worke.

√.1014——√.150.

1014
150
50700
1014
152100

whose roote is. 390.
390:
2
√.1164——780.

That is. √.384.

And now here are the variations of the other examples.

√.2592.

√.2592 —— √.288.

36 4.
72) 6 2.
4
4
16
72
32
112
√. 1152

√.2592 —— √.288.

81 9.
32) 9 3.
6
6
36
32
72
108
√. 1152

√.2592 —— √.288.

144 16.
18) 12 4.
8
8
64
18
512
64
√. 1152

√.2592 —— √.288.

8) 324 36.
18 6.
12
12
144
8
√. 1152

√.2592 —— √.288.

1296 144.
2) 36 12.
24
24
576
2
√. 1152

√.2592 —— √.1152.

1296 576.
2) 36 24.
12
12
144
2
√. 288

Other examples varied, for proofe of the like. 6. examples in Addition.

√.64800——√.7200

32400 3600
2) 180 60
120
120
14400
2
√. 28800

√.64800——√.7200.

8100 900.
8) 90 30
60
60
3600
8
√. 28800

√.64800——√.7200.

3600 400.
18) 60 20
40
40
1600
18
12800
16
√. 28800

√.64800——√.7200.

2025 225.
32) 45 15.
30
30
900
32
√. 28800

√.64800——√.7200.

1296 144.
50) 36 12
24
24
576
50
√. 28800

√.64800——√,7200.

900 100.
72) 30 10.
20
20
400
72
√. 28800

*Subtraction of Cubike rootes.*

Maker. Like difference is there in Subtraction of *Cubike* rootes *commensurable*. And therfore I set the examples onely, without any larger declaration.

ww.375——ww.81.

| | |
|---|---|
| 125 | 27. |
| 3) 5 | 3 |
| 2 | |
| 2 | |
| 8 | |
| 3 | |
| ww. 24 | |

ww.1715——ww.135.

| | |
|---|---|
| 343 | 27. |
| 5( 7 | 3. |
| 4 | |
| 4 | |
| 64 | |
| 5 | |
| ww. 320 | |

ww.13824——ww.1000

| | |
|---|---|
| 1728 | 125 |
| 8) 12 | 5 |
| 7 | |
| 7 | |
| 343 | |
| 8 | |
| w. 2744 | |

*An other woorke of Subtraction for Surde Cubes.*

In the seconde forme of addition of *Surde Cubes*, you remember that you added 4 nombers together. But in Subtraction, you shall adde to ethe roote seueralie that, that commeth of his owne multiplication, with the other *triple*. And then shall you Subtracte the lesser nomber, out of the greater. And the remainer you shall multiplie by the common diuisor. And so shall you haue the roote that remaineth of the Subtraction. As in example.

ww.1296——ww.48.

| | |
|---|---|
| 216 | 8 |
| 6) 6 | 2 |
| 36 | 4 |
| 108 | 12 |
| 72 | 216 |
| 64 | |
| 6 | |
| ww. 384 | |

ww.8956——ww.15972

| | |
|---|---|
| 4913 | 1331 |
| 12) 17 | 11 |
| 289 | 121 |
| 867 | 363 |
| 6171 | 5537 |
| 216 | |
| 12 | |
| ww. 2592 | |

pp.y. ww.294912

| ww.294912 | ww.24696 |
|---|---|
| 32768 | 2744 |
| 9) 32 | 14 |
| 1024 | 196 |
| 3072 | 588 |
| 18816 | 43008 |
| 5832 | |
| 9 | |
| ww. 52488 | |

Scholar. In all these examples I se the confirmation of the former additiō. And in these laste woorkes, this I see peculiare from additiō, that the *Cube* is added with the lowesse nomber in that rowe (as in the firste example. 216. is added with 72. and maketh. 288: And. 8. is added with. 216. that yeldeth. 224.) And then is the lesser abated from the greater (as. 224. from 288.) And the remainer (whiche there is. 64) set in the middle vnder bothe the rewes of nombers. And then is multiplied by the common nomber, to make the remainer.

So in the firste example, the remainer is. ww. 384. where. ww. 48. is abated out of. ww. 1296. And in the seconde example where. ww. 15972. is subtracted out of. ww. 58956. the remainer is ww. 2592. Likewaies in the thirde example. ww. 24696. is abated out of. ww 294912 & leaueth remainyng. ww 52488

Maister. But now in addition there folowеth. 2. other examples, whiche by subtraction maie bee proued thus: as here you see.

| ww21952 | ww4096 |
|---|---|
| 2744 | 512 |
| 8) 14 | 8 |
| 196 | 64 |
| 588 | 192 |
| 2688 | 4704 |
| 216 | |
| 8 | |
| ww. 1728 | |

| ww74088 | ww 19683 |
|---|---|
| 2744 | 729 |
| 27) 14 | 9 |
| 196 | 81 |
| 588 | 243 |
| 3402 | 5292 |
| 125 | |
| 27 | |
| ww. 3375 | |

Scholar.

Scholar. I see, in these examples of Subtraction: that the firste nomber is the totalle, or laste nomber in addition. And the seconde nomber, whiche folowethe ——. is the nomber to be abated: and then laste and loweste of all, is the remainer, whiche was one of the firste sommes in addition.

And though there remaine. 3. other exãples of *Zenzizenzike* nombers, I see no difficultie in theim, but that I can woorke them: As here I haue set thẽ forth.

| | |
|---|---|
| ℳ 32768 —— ℳ .648 | ℳ .50000 —— ℳ 1280 |
| 4096 81 | 10000 256 |
| 8) 8 3 | 5) 10 4 |
| 5 | 6 |
| 5 | 6 |
| 625 | 1296 |
| 8 | 5 |
| ℳ . 5000 | ℳ . 6480 |

| |
|---|
| ℳ 810000 —— ℳ 65536 |
| 50625 4096 |
| 16) 15 8 |
| 7 |
| 7 |
| 2401 |
| 16 |
| ℳ . 38416 |

Master. Seeyng you are experte inough in the 5. woorkes of these *Surdes* vncõpounde, I wil teache you the like woorkes in cõpounde *Surdes*.

Scholar. Is there thẽ noe reduction, nother extraction of rootes, to bee taughte in these vncompounde *Surdes*? *Of reduction and extraction of rootes.*

Master. As for reduction, I haue taughte you all readie in multiplication, as moche as is required in these nombers.

And for extraction of rootes, you maie sone vnderstande, that here can be none. For then were thei not *Surde* nombers. And therfore I saied vnto you before,

that. ✓. 3. 100. is not a *Surde* nomber, although it be written like a *Surde* nomber, bicause it hath a Square roote, accordyng to his signe: and that is. 10. Likewayes. ✓. 256. is no *Surde* nomber: for his Square roote is knowen to be. 16.

Scholar. I might haue cõsidered as moche, by the definition of *Surde* nombers, that their rootes can not be assigned in nombers absolute. And therfore I see that. ww. 125. is noe *Surde* nõber, sith his *Cubike* roote is. 5. And. w. 256. is a nomber *rationalle*, and no *Surde* nomber: for his *zenzizenzike* roote is. 4.

Maister. But. w. 64. is a *Surde* nomber, and yet hath. 64. a *Square* roote, and a *Cubike* roote also, but not a *zenzizenzike* roote, accordyng to his signe. And therfore ought better to be written thus. ✓. 8.

Scholar. I praie you to procede to *Surde* nombers compounde.

## *Of Surde nombers compounde.*

Maister.

*Surde* nombers compoũde, are made not onely of. 2. or. 3. or moare *Surde* nombers vncompounde, but also of *rationalle* or *Abstracte* nombers ioyned with *Surde* nombers. As. ✓. 10 —+— ✓. 12. and. 8. —+— ✓. 6. likewaies. ✓. 20. ——— 3. and. w. 40. —+— ✓. 14. ——— .3.

*Compounde Surdes.*

But here shall you marke, that I call compounde nombers, not onely soche as haue the signe of. —+—, but also soche as haue the signe of ——— for although in nature of the nomber ✓. 10 ——— ✓. 5. be not compounde, but abated, yet in name he is compoũde, and augemented. For. ———. doeth as well augemente

the

the name, as —+— doeth.

Scholar. It semeth reasonable. For when I saie, √.12.———√.7. the name is compounde, an well as if I had saied. √.12.—+—√.7. although the quantitie bee not so greate. For ——— doeth euer abate the quãtitie of the nõber, though it do increase the name.

Master. Yet for a difference, the nombers that be compounde with —+— be called *Bimedialles*: and those that be compounde with ———, be named *Residualles*. And if the *Bimedialles* haue all their nombers and partes of one denominations, then bee thei called onely by their generalle name *Bimedialles*. But if their partes be of. 2. denominations, then are thei named *Binomialles* properly. Howbeit, many vse to call *Binomialles* all compounde nombers that haue —+—. And so wil I let the names passe. *Bimedialles.* *Residualles.* *Binomialles.*

*Euclides* definitions doe not very aptly agree to this place, as at an other tyme I will shewe you, and therfore I doe omitte them for this tyme.

But touchyng our principalle intente, whiche is to declare the practike woorke of *Binomialles*, and *Residualles*, there is litle difficultie, if you marke well that whiche is taught before. For as *Binomialles* and *Residualles*, bee made of *Surdes*, or els of *rationalle* nombers with *Surdes*, so the woorke of the compounde nombers dependeth of the woorke of the simple nombers, and is all one with them. And concernyng the signes —+— and .———. here is no moare to bee saied, then was taughte in *Cossike* nombers compounde.

Scholar. Yet of euery kinde, it maie please you to set forthe some examples.

Master. I thinke that mete, without many woordes els. Not forgettyng by the waie, that *vniuersalle rootes*, are not accompted emongeste these compounde *Surdes*: but are reserued to their peculiare treatice, as rootes of compounde *Surdes*.

that. √. 3. 100. is not a *Surde* nomber, although it be written like a *Surde* nomber, bicause it hath a Square roote, accordyng to his signe: and that is. 10. Likewayes. √. 256. is no *Surde* nomber: for his Square roote is knowen to be. 16.

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Master. But. w. 64. is a *Surde* nomber, and yet hath. 64. a *Square* roote, and a *Cubike* roote also, but not a *zenzizenzike* roote, accordyng to his signe. And therfore ought better to be written thus. √. 8.

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But here shall you marke, that I call compounde nombers, not onely soche as haue the signe of. —+—, but also soche as haue the signe of ——— for although in nature of the nomber √. 10 ——— √. 5. be not compounde, but abated, yet in name he is compoũde, and augemented. For. ———. doeth as well augemente the

the name, as —+— doeth.

Scholar. It semeth reasonable. For when I saie, √.12.———√.7. the name is compounde, an well as if I had saied. √.12.—+—√.7. although the quantitie bee not so greate. For ——— doeth euer abate the quãtitie of the nõber, though it do increase the name.

Master. Yet for a difference, the nombers that be compounde with —+— be called *Bimedialles*: and those that be compounde with ———, be named *Residualles*. And if the *Bimedialles* haue all their nombers and partes of one denominations, then bee thei called onely by their generalle name *Bimedialles*. But if their partes be of. 2. denominations, then are thei named *Binomialles* properly. Howbeit, many vse to call *Binomialles* all compounde nombers that haue —+—. And so wil I let the names passe. *Bimedialles.* *Residualles.* *Binomialles.*

*Euclides* definitions doe not very aptly agree to this place, as at an other tyme I will shewe you, and therfore I doe omitte them for this tyme.

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Scholar. Yet of euery kinde, it maie please you to set forthe some examples.

Master. I thinke that mete, without many woordes els. Not forgettyng by the waie, that *vniuersalle rootes*, are not accompted emongeste these compounde *Surdes*: but are reserued to their peculiare treatice, as rootes of compounde *Surdes*.

Of

# The Arte

## Of Numeration.

Numeration is moare plain, then that I neade to ſtande in declaryng it, otherwaies then by examples: As here you ſee.

### Examples af Binomialles.

6.—|—√.8. That is 6 moze the *Square* roote of 8.
√.20—|—.3. Is the *Square* roote of. 20. moare. 3.
ww.30—|—w.9. Signifieth the *Cubike* roote of. 30. moze the *zenzizenzike* roote of. 9. And ſo of other.

### Examples of Reſidualles.

24.———√ 96. That is 24. abating the roote of 96
√.150.———.9. Is the *Square* roote of 150. abating 9
ww5208——√35. The *zenzizenzike* roote of. 5208. ſaue the *Square* roote of. 35. And ſo forthe.

Scholar. So I ſee any *Surdes* maie bee compounde with other: And any nõbers *rationalle* ioined with thẽ.

## Of Addition.

Maſter. Addition is as plaine. For as the partes bee, ſo ſhall the Addition bee, accordyng as you haue learned before.

### Examples of Binomialles.

| | | |
|---|---|---|
| √.50—|—10 | 15.—|—√.15. | √.1264—|—8. |
| √.2.—|—.8. | 18—|—√.60. | 28—|—√316 |
| √.72—|—18 | 33.—|—√.135. | 36—|—√2844 |

| | |
|---|---|
| w.48.—|—√.5. | ww.32—|—√10. |
| w.243.—|—√.45. | ww.4.—|—√19. |
| w.1875—|—√.80. | ww108—|—√29—|—√760 |

Examples

## *Of Surde nombers.*

### *Examples of Residualles.*

| | | |
|---|---|---|
| √.75. —— .4 | 14 —— √.3. | 250 —— √.108. |
| √. 3 —— 1. | 16 —— √.27 | √.44 —— 76. |
| √.108 —— 5. | 30 —— √.12. | 174 —— √.275. |

| | |
|---|---|
| ww.72. —— w.96 | w. 32. —— √.5. |
| ww. 9. —— w.6. | w. 32. —— √.24. |
| ww243 —— w162 | w .512 —— √.29 —+— √480 |

### *Examples of Binomialles with Residualles.*

| | | |
|---|---|---|
| √.80. —+— 6. | 30. —+— √. 20 | .561 —— √512 |
| √. 5. —— 2. | 12. —— √.5. | √288 —+— 340 |
| √.125 —+— 4. | 42 —+— √.5. | 901 —— √1568 |

| | |
|---|---|
| √.63. —— w 160 | w.320 —— √.56. |
| √. 7. —+— w.20. | w.40 —+— √..24. |
| √.112 —— w684 | w1680 —— √.80 —— √5376 |

Scholar. I see that you make seueralle Additions in all these nombers. For you adde still like nombers with their matches. So that here is nothyng diuerse from the woorkes of simple *Surdes*. Although in euery thirde example, there appeare moare difficultie, then there is in deede: When I consider the like transposition in *Cossike* nombers. For the woorke addeth like nombers together.

## *Of Subtraction.*

Master. In Subtraction there is as litle diuersitie. As these examples will sufficiētly declare: whiche be set as trialles of the former Additions.

## Examples of Binomialles.

| | |
|---|---|
| √.72 + 18 | 36 + √2844 |
| √. 2 + 8. | √.1264 + 8. |
| √.50 + 10 | 28 + √.316. |

| | |
|---|---|
| 33 + √.135 | ww.1875 + √.80. |
| 15 + √.15. | ww. 48. + √.5. |
| 18 + √.60 | ww. 243 + √.45. |

ww.108 + √.29. + √.760.
ww. 4 + √.19.
ww. 32 + √.10.

## Examples of Reſidualles.

| | |
|---|---|
| √.108 − 5 | 174. − √.275. |
| √. 3 − 1 | √.44 − 76 |
| √. 75 − 4 | 250. − √.108 |

| | |
|---|---|
| 30. − √.12. | ww.243 − w.162. |
| 14. − √.3. | ww. 9 − w.6. |
| 16. − √.27. | ww. 72 − w.96. |

w.512 − √.29. + √.480.
w. 32 − √.5.
w. 32 − √.24.

## Examples of bothe together.

| | |
|---|---|
| √.125 + 4 | 901 − √1568 |
| √. 5 − 2 | √.288 + 340 |
| √. 80 + 6 | 561 − √.512. |

| | |
|---|---|
| 42 — + — √.5. | √.112 — — — ww.648. |
| 12 — — — — √.5. | √. 7 — + — ww.20. |
| 30 — + — √.20. | √.63 — — — ww.160. |

ww.1080. — — — √.80. — — — √.5376.
ww. 40. — + — √.24.

ww. 320. — — — √.56.

Scholar. This is as easie as Addition, saue for. 3. examples, whiche I vnderstande not. For although I see the laste example, of eche of the sortes of nombers, to bee agreable with the like examples in Addition, yet I can not so well perceiue, the order of their Subtraction, as I doe knowe the maner of their Additiõ. For by the arte of simple *Surdes*, I see that. √.10 and. √.19. doe make. √.29 — + — √.760. But when √.29. — + — . √.760. is set as a totalle, and. √.19. to be Subtracted out of it, how I shall woorke that, and leaue. √.10. for the remainer, I see not.

So in the *residualles*, I knowe how . √.5. and √.24. doe make. √.29 — + — √.480. But I knowe not how √.5 abated out of. √.29 — + — √.480. doeth make for the remainer. √.24.

And the like doubte is in the thirde sorte of *Surdes*, whiche are mixte nombers. For where I see in Addition — + — √.24. added with — — — — . √.56. And the totalle to bee. — — — . √.80. — — — √.5376. I knowe the reason of the woorke, for the signes . — + — . and — — — — . by that I learned in *Cossike* nombers: And the reaste is manifeste by Addition of simple *Surdes*. For it is wrought by abatyng √.24. out of. √.56. But then in Subtraction, how — + — √.24. beyng Subtracted from — — — √.80 — — — √.5376 shall leaue — — √.56 I can not iudge. And yet by the signes I gesse (as I learned in *Cossike* nombers) that it is doen by Additiõ, bicause the signes doe disagree.

Q.q.y. Master.

Master. In that you remember the former rules, to conferre them aptly with these later worke, I can praise you well. But in that you can not vnderstande the reason of that, whiche was not yet taughte you, I can not greately blame you. Although I can not praise you, for that you thinke your self to be cunnynger then you are. For in those Additions, that you thinke your self to be experte inough, I dare saie, that you bee disceiued, if you take them to bee nombers of any soche, as hetherto hath been taughte vnto you.

Scholar. I take them for compounde *Surdes*.

Master. Thei are not so: Nother is their worke agreable, with the worke of compounde *Surdes*. But thei are the rootes of compounde *Surdes*: And therfore are called *vniuersalle rootes* of *Surdes*. And accordyng to their proper nature, thei ought to bee called rootes of *Surdes*, and not *Surde* rootes. As I will tell you anon. When I will also discusse your doubte.

But before I speake any moare of theim, I will eande the worke of these compounde *Surdes*: whereof. 2. kindes yet remaine behinde.

## *Of Multiplication.*

Multiplicotion of compounde *Surdes*, is as easie as can bee. And differeth in nothyng, frō the worke of simple *Surdes*. Onely this must you marke, as reason would, that you muste multiplie euery parte of the one nōber, by euery parte of the other nōber: as you remember the worke of compounde *Cossike* nombers.

Scholar. I praie you giue me some examples.

Master. That shall you haue. And that maie suffice for this worke. Marke them well therfore.

Examples

## Examples of Binomialles.

23 + √.15.
6 + √.8.

138 + √.120.
+ √.540. + √.4232.

138 + √.4232 + √.540 + √.120.

√.120 + √.12.
√. 12 + √.7.

√1440 + √.84.
+ √.840 + 12.

12 + √.1440 + √.840 + √.84.

## Examples of Reſidualles.

5. — √.10.
5. — √.10.

25 + 10.
— √.250 — 250.

35 — √.1000.

√. 24 — √.20.
√. 30 — √.24.

√.720. + √.480.
— 24 — √.600.

√.720 + √.480 — 24 — √.600.

## Examples of bothe together.

32 + √.14.
√.124 — 6.

√.126976 + √.1736.
— 192. — √.504.

√.126976 + √.1736 — 192 — √.504

√.52 + 17.
17 — √.52.

√.15028 + 289.
— 52 — √.15028.

37.

Scholar. Multiplication, as I see, is the easiest woorke of all the other. So that I dooe marke the reduction, in gatheryng the totalle: whiche is easie inough to vnderstand, by that I haue learned in *Cossike* nombers. And Diuision be no harder, it maie lone be learned.

## *Of Diuision.*

Maister.

DIuision by one simple nomber, is no moare difficulte: as these examples doe declare. Where the diuisor is a nomber vncompounde.

√.26 + 15 diuided by.5. doeth make. √.1 $\frac{1}{25}$ + 3.

Againe. √.56 + √.24. diuided by. √.6. doeth yelde. √.9 $\frac{1}{3}$ + 2.

And so √.75 — √.48. diuided by. √.3. dooeth byyng so, the.5. — 4. that is. 1.

Likewaies. √.320. + √.180. beyng parted by √.5. doeth make the *quotiente*. 14.

Scholar. I see it so. For at the firste it is. √.64. + √.36. that is. 8. + 6. whiche maketh. 14.

Maister. So maie you woorke all like diuisions. But when the diuisor is a compounde nomber, then must you vse an other meane: that is to reduce that compounde nōber, to a simple nomber: whiche thing you maie easily doe, by multiplying any *Binomialle*, by his *Residualle*, or contrarywaies, the *Residualle* by his *Binomialle*.

As

As 6 + √.10. multiplied by. 6 − √.10 doeth make. 26.

And so. √.8 − √.5, multiplied by √.8 + √.5 doeth yelde. 8. − 5. that is. 3.

Scholar. I perceiue a briefe waie in this multiplication: For I neade not in the firste example, to multiplie 6. by. √.10. sith it would amounte to nothyng. In so moche as at one multiplication, it would bee +, and at an other. −. And so the one would abate the other, and leaue nothyng for them bothe.

Maister. That is well marked. And it is so generally. Wherefore (as you see) the diuisor by this meanes, maie lightly be tourned into a simple nomber, or a plaine absolute nomber.

And now to make the diuidende, in thesame proportion, to this newe diuisor, that it was vnto the old diuisor, you shall multiplie it by thesame nomber, by whiche the diuisor was multiplied. For if any nombers bee multiplied, by one common nomber, their newe totalles kepe thesame proportion, that was betwene the firste nombers.

Scholar. That must neades be so. For as. 3. is *sesquialtera* vnto. 2. so if you multiplie them by. 5. thei will make. 15. and. 10. whiche be in *sesquialtera* proportion and likewaies will their prospotion remain, by what so euer nomber thei be multiplied. Wherfore it must neades be reasonable, that if the diuidende and the diuisor, be multiplied by any one nomber, simple or compounde, thei shall kepe the same proportion, that thei had before.

Maister. For more certain vnderstandyng of this rule, take these examples. The firste is, where √.68 + √.5 + is sette to bee diuided by. √.6 + √.3.

Here firste I multiplie the diuisor by his contrarie, that is his *Binomi-*

| |
|---|
| √.6 + √.3. |
| √.6 − √.3 |
| 6 − 3. |
| That is. 3. |

*alle*

alle. √.6 —— 3. And there riseth. 6 —— 3. that is. 3 whiche I shall kepe for the newe diuisor.

Then doe I multiplie the diuidende √.68 — + — √54 by the same *Residualle*.

√.68. — + — √54.
√. 6. —— √3.

√.408 — + — √.324.
—— √.204 —— √.162.

√.408 — + — √.324 —— √.204 —— √.162.

And there doth amounte, as here in worke is expressed.

√.408 — + — √.324 —— √.204 —— √.162.

whiche nomber shall be taken for the newe diuidende: and must be diuided by. 3. that is the newe diuisor. In whose stede I set. √.9. for moare redinesse in worke. Therfore I set the doune in order, as here foloweth.

√.408 — + — √.324 — √.204 — √.162 (√.45 1/3 — + — 6 — √22 2/3 — √18.
√. 9 √. 9. √. 9 √. 9

And then doe I seke how often. √.9. maie bee founde in. √.408. whiche maie bee. 45 1/3 of tymes. Wherefore I set. √.45 1/3 in the *quotiente*. And then doe I reiterate the diuisor, and sette it vnder. √. 324. where I finde it. 36. tymes: and therefore set I. 6. for it, bicause the *quotiente* els would bee. √. 36. whiche is iustly. 6. Thirdly, I remoue the diuisor vnder √.204. where it maie bee founde. 22 2/3 tymes. For whiche I sette √.22 2/3 in the *quotiente*. And then set I the diuisor last of all vnder. 162. where it is founde. 18. tymes: and for that cause I set √.18. in the *quotiente*: And so is the whole *quotiente* √.45 1/3 — + — 6 — √.22 2/3 — √.18.

Scholar. This diuision is straunge to credite, although it be not difficulte to worke.

Master. If you doubte of it, you maie vse the accustomable trialle by the contrary kinde.

Scholar.

Scholar. So must it folowe, that if I dooe multiplie this *quotiente* by the firste diuisor, the firste dividende will resulte thereof.

And for the proofe of that, I dooe multiplie, $\surd.45\frac{1}{3}. - + - .6. - - - - .\surd.22\frac{2}{3} - - - - .\surd.18.$ by $\surd.6. - + - \surd.3.$ But for the moare ease, I doe tourne all the mirte nombers into onely fractions. And then doe I multiplie them orderly.

$$\begin{array}{l}
\surd.\frac{136}{3} - + - 6 - - - - \surd.\frac{68}{3} - - - - \surd.18. \\
\surd.6 - + - \surd.3. \\
\hline
\surd.\frac{816}{3} - + - \surd.216 - - - - \surd.\frac{408}{3} - - - - \surd.108. \\
\surd.\frac{408}{3} - - - - \surd.108 - - - - \surd.\frac{204}{3} - - - - \surd.54. \\
\hline
\surd.272 - + - \surd.216 - + - \surd.136 - + - \surd.108. \\
- \surd.68 - - - - \surd.54 - - - - \surd.136 - - - - \surd.108. \\
\hline
\surd.68 - + - \surd.54.
\end{array}$$

First I multiplie $\surd.\frac{136}{3}$ by $\surd.6.$ and there commeth $\surd.\frac{816}{3}$ that is. $\surd.272.$ Again I doe multiplie. 6. or $\surd.36$ by $\surd.6.$ and it maketh $\surd.216.$ Then I multiplie $\surd.\frac{68}{3}$ by. $\surd.6.$ & it giueth. $\surd.\frac{408}{3}$, whiche is. $\surd.136.$ Fourthly $\surd.18.$ multiplied by. $\surd.6.$ dooeth make. $\surd.108.$ All whiche I set doune with their conuenient signes.

After that I multiplie. $\surd.\frac{136}{3}$ by. $\surd.3.$ and it yeldeth $\surd.\frac{408}{3}$ that is. $\surd.136.$ whiche I sette doune with his signe $- + -$. Then $\surd.36$ by. 3. maketh $\surd.108.$ Thirdly. $\surd.\frac{68}{3}$ by $\surd 3.$ dooeth giue. $\surd.68.$ and last of all, $\surd.18.$ multiplied by. $\surd.3.$ bryngeth forthe. $\surd.54.$

When all these be placed conueniently, I doe consider that $- + -$ $\surd.136.$ and $- - - -$ $\surd.136.$ maie bee bothe cancelled, bicause the one dooeth abate the other. And likewaies, $- + -$ $\surd.108.$ and $- - - -$ $\surd.108.$ eche abate other: so that thei must bothe be reiected.

Then I see, that $\surd.68.$ beyng abated out of $\surd.272$ there will remain. $\surd.68.$ And in like. $\surd.54.$ beyng abated out of. $\surd.216.$ dooeth leaue. $\surd.54.$ So that the whole multiplicatiõ doth make iustly $\surd 68 - + - \surd 54$

whiche is the firste diuidende. And so is that diuision approued good.

*An other example.* Master. Yet for you exercise, you shall haue some examples moare of diuision.

√.456.——.√.72. is sette to bee diuided by √.18—+—√.6.

Scholar. That diuisor must I multiplie by his contrarie, whiche is the *Residualle*. √.18——√.6. And so, as you maie sone perceiue, there will rise. 18.——6. that is 12. whiche must be kepte for the newe diuisor.

√.18.—+—√.6.
√.18.——√.6.
18——.6.
That is. 12.

Then shall I multiplie the former diuidende, that is √.456——√.72 by thesame *residualle* √.18——√6

√.456——√.72.
√. 18——√. 6.
√.8208——√.1296.
√.432.——√.2736.

√.8208—+—√.432——√.2736——√.1296.

And there will rise of that multiplication, as here by example appereth √.8208—+—√.432——√.2736 ——1296. whiche nober I shall diuide by. 12. that was founde for the newe diuisor. And then will the *quotiente* bee. √.57—+—√.3——√.19.——√.9. As here in woorke doeth appeare.

√.8208—+—√.432——√.2736——√.1296(√.57+√.3——√19——√
√.144. √144 √.144 √.144.

Where I haue set. √.144. for. 12. seyng thei be all one: but that. √.144. is moare apte for this woorke. And I haue repeated it as often tymes, as the diuisor should be remoued.

*The proofe.* But now to trie this woorke, whether it bee well wrought, I shall multiplie this *quotiente* by the firste diuisor, & then ought the firste diuidende to amounte.

As

As here in example, you see wrought.

√.57.—+—√.3.——√.19.———√.9.
√.18.—+—√.6.

√.1026—+—√.54——√.342——√.162.
√.342—+—√.18——√.114——√.54.

√.1026—+—√.18——√.114——√.162.

Where —+—√.54. doeth cancell ——√.54. and is cancelled by it.

So —+—√.342. and ——√.342. exclude one an other, and therefore must bee bothe reiected. And then remaineth onely,

√.1026—+—√.18.——√.114——√.162.

Whiche nombers I dooe well examine: and finde that √.114. beyng abated out of. √.1026. there will remaine. √.456. Againe if —+—√.18. be subtracted out of——√.162. there will reste——√.72. And so is that whole multiplicatiõ onely. √.456——√72 agreable to the firste diuidende. Wherby it is manifeste, that the former diuision was good.

Maister. How can you worke this example? Where. 24. is set to be diuided by. 3.—+—√.8.

*The thirde example.*

Scholar. I must still obserue the generalle rule. And multiplie bothe those nombers, by the contrarie of the diuisor, that is, by the *residualle*. 3——√.8. And of the firste multiplication of it, with the diuidende. 24, there riseth. 72——√.4608. Of the seconde multiplica-

| |
|---|
| 24. |
| 3——√.8 |
| 72——√.4608 |

tion, where the *Binomialle* is multiplied by the *Residualle*, that is his contrary, the totalle will be. 9——8. that is but. 1. And therfore seyng. 1. doeth nother multiplie nor diuide, the former nomber.

| |
|---|
| 3.—+—√.8 |
| 3.——√.8 |
| 9.——.8. |

That is. 1.

That is. 72.——.√.4608. is the *quotiente*, when 24. is diuided by. 3.—+—√.8.

*The proofe.* For proke whereof, I multiplie 72 —— √.4608 that is the *quotiente*, by. 3. —+— √.8. And there riseth 216 —+— √.41472 —— √.41472 —— √.36864. whereof. 2. nombers differyng but by —+— ¢ —— muste bothe bee reiected, as nombers superfluouse.

72 —— √.4608.
3 —+— √.8.

216 —— √.41472
√.41472 —— √.36864

216 —— .192.

That is. 24.

Then. 36864. is a *square* nomber, and hath. 192 for his roote. Wherfore the whole nomber is, 216 —— 192 that is (as it is manifeste inough) 24. And so is the whole worke proued good.

*The fourthe example.* Master. You shall haue one example moare, and then will I make an ende of diuision.

When √.6570. —+— √.254. is propounded to bee diuided by √.54 —— √.6. I would knowe the *quotiente*.

Scholar. I see the newe diuisor will be. 54 —— 6. that is. 48.

And then for to finde a dividende conueniente, I shall multiplie the firste dividende, by the contrarie of the firste diuisor, that is by √.54 —+— √6 And there will rise, as you see. √.354780. —+— √.13716 —+— √.39420. —+— √.1524.

√. 6570. —+— √.254.
√. 54 —+— √. 6.

√.354780 —+— √.13716.
√.39420 —+— √. 1524.

That dividende must be diuided by. 48. or moare aptly by. √.2304. And the *quotiente* will bee.

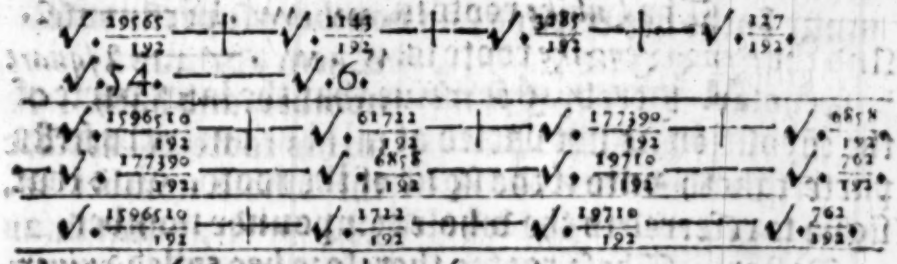
√.153 $\frac{189}{192}$ —+— √.5 $\frac{183}{192}$ —+— √.17 $\frac{63}{576}$ —+— √. $\frac{127}{192}$.

As here appeareth in worke.

√.354780 —+— √.13716 —+— √39420 +√1524 (√153 $\frac{189}{192}$ +√.5 $\frac{183}{192}$ +√17 $\frac{63}{576}$ +√ $\frac{127}{192}$
√.2304. √.2304 √.2304 √2304.

*The proofe.* And that this worke is good, I will proue it by multiplication.

multiplication. As the example folowyng dooeth declare. Where by the firste multiplication there commeth. 8. nombers, that is. 4. with. —+—. and. 4. with —— .



And bicause the firste nōber with ——, is equalle to the thirde with —+—, therfore thei bothe must be reiected. Again in as moche as the seconde nōber with —— is equalle to the fourthe nomber with —+—, thei bothe shall bee cancelled. And then remaineth. 2. nombers with —+—, and other. 2. with ——.

So if you abate the thirde —— out of the firste —+—, the *quotiente* will be. ✓.6570.

Likewaies if you abate the fourthe —— out of the seconde —+—, the *quotiente* will yelde. ✓.254.

And thei bothe will make the firste diuidende. ✓.6570. Wherby the former diuisiō is approued good.

Master. This shall suffice for diuision.

## *Of extraction of rootes.*

The nexte woorke is extraction of rootes: whiche you maie very easilie woorke, by puttyng the signe of the roote, that you desire, before the whole nomber. As if you would haue the square roote of ✓.10 —+— ✓.5. this is it ✓.10 —+— ✓.5. The *Cubike* roote of the same nōber is. ✓✓.✓.10 —+— ✓.5. And the *Zenzizenzike* roote of it is ✓✓.✓.10. —+— ✓.5. But if you will haue the *Square* roote of. 10 —+— ✓.5

it is. √.10 + √.5. And his *Cubike* roote is. ww.10 + √.5. Likewaies his *zenzizenzike* roote is ww.10 + √.5.

So of. ww.18 —— 2. the *Square* roote is √.ww.18 —— 2. The *Cubike* roote is. ww.ww.18 —— 2. And the *zenzizenzike* roote is. w.ww. —— 2.

Scholar. Hereby I perceiue that the later parte of the cõpoſition, is not varied at all, but onely the firſte parte taketh vnto it the ſigne of the roote. And that ſigne is referred to the whole compounde nomber.

*Vniuerſalle rootes.*

Maſter. Theſe rootes therefore bee called *vniuerſalle rootes*, bicauſe thei are the rootes, not of the ſeueralle partes of the compounde nõber, but of the whole compounde nomber. And that is the differenee, betwene the common *Surde* nombers, and *vniuerſalle rootes*. For if √.24 + √.144. be ſette for a common *Surde* nomber, then doeth it betoken, that I muſt take 2. rootes, that is. √.24. and √.144, and ioyne theim together. But if it ſtande for an *vniuerſalle roote*, it repreſenteth the roote of this whole nomber. 24 + √.144. whiche is. 6. for the whole *Square* is. 36.

Scholar. I perceiue it well. For. √.144. beeyng 12, that. 12. with. 24. dooeth make. 36. And therefore muſt the *vniuerſalle roote* of. 24 + √.144. bee. 6. And ſo √.24 + √.144. is iuſt. 6.

But if. √.24 + √.144. doe ſtande for a common *Surde* nomber compounde: then is it made of. 2. rootes, that is √.24. whiche is almoſte. 5. and √.144 beyng. 12. And ſo the whole compounde roote, in that ſorte is almoſte. 17. And is nighe. 3. tymes ſo moche as the ſame nomber, beyng an *vniuerſalle roote*.

Maſter. Bicauſe you maie perceiue it the better, I will put an example in *Square* nombers, made like *Surdes*. As this. √.81 + √.361 If it be an *vniuerſalle roote*, then it is equalle to 10. For I muſt take firſt the roote of the laſte nomber, whiche is. 19. And adde it

with

with.81. wherby there amounteth.100. whose roote is.10. But if it stand after the common sorte of *Surde* nombers, it betokeneth the roote of.81. and the roote of.361. (that is.9. and.19) to bee added together. And so thei make.28. whiche is farre aboue.10.

But farther now, if it stande for a common *Surde* nomber: And I would haue the *Square* roote of it, then is that. √√.81.+√.361. And betokeneth the *Square* roote of the square roote of.81. and the Square roote of.361. added together, that is the square roote of.28. But moste generally and moste aptly, it betokeneth the roote of the *vniuersalle roote* of.81.+√.361.

Scholar. Now I perceiue that in Addition, and Subtraction of *Surdes*, the last nombers that did result of that woorke, were *vniuersalle rootes*.

Maister. You saie truthe. But harke what meaneth that hastie knockyng at the doore?

Scholar. It is a messenger.

Maister. What is the message? tel me in mine eare Yea sir is that the mater? Then is there noe remedie, but that I must neglect all studies, and teaching, for to withstande those daungers. My fortune is not so good, to haue quiete tyme to teache.

Scholar. But my fortune and my fellowes, is moche worse, that your vnquietnes, so hindereth our knowledge. I praie God amende it.

Maister. I am inforced to make an eande of this mater: But yet will I promise you, that whiche you shall chalenge of me, whē you see me at better laiser: That I will teache you the whole arte of *vniuersallē rootes*. And the extraction of rootes in all *Square Surdes*: with the demonstration of theim, and all the former woorkes.

If I mighte haue been quietly permitted, to reste but a litle while lōger, I had determined not to haue ceased, till I had ended all these thinges at large. But

now

now farewell. And applie your studie diligently in this that you haue learned. And if I maie gette any quietnesse reasonable, I will not forget to performe my promise with an augementation.

Scholar. My harte is so oppressed with pēsiuenes, by this sodaine vnquietnesse, that I can not expresse my griefe. But I will praie, with all theim that loue honeste knowledge, that God of his mercie, will sone ende your troubles, and graunte you soche reste, as your trauell doeth merite. And al that loue learnyng: saie therto. Amen.

Maister. Amen, and Amen.

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